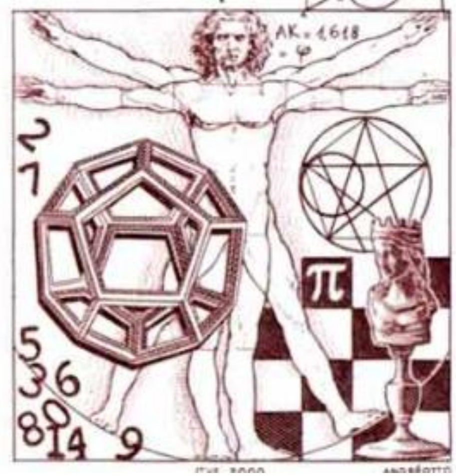
CALCULUS

Week 1

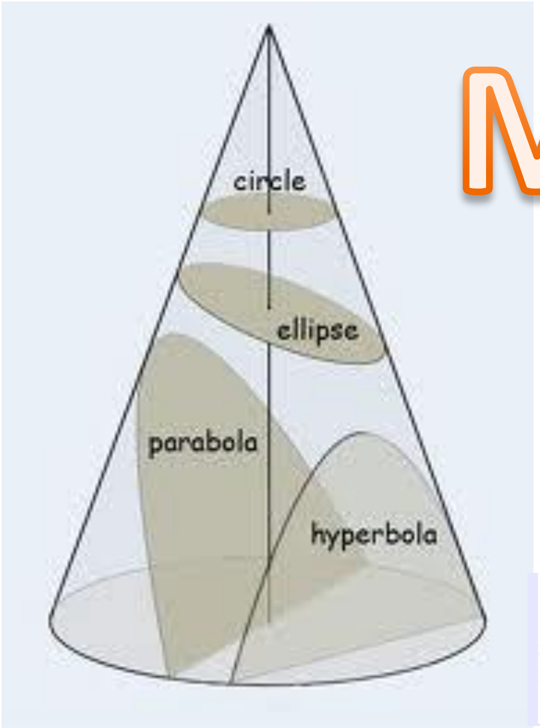
PRELIMINARIES



CALCULUS

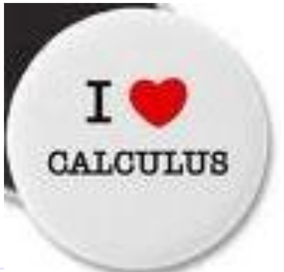
Math 101.3

Hypatia
of
Alexandria

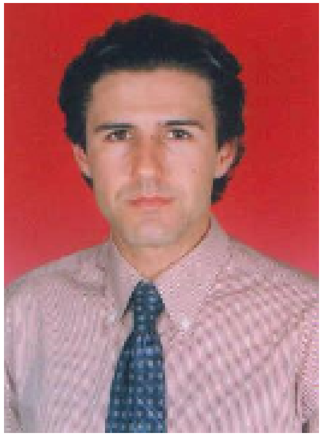


$$\int_a^b f(x)\,dx = F(b) - F(a)$$

$$\phi = \sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1+\dots}}}}}$$



Mustafa Cem Çelik



- Mechanical Engineering
- Energy

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Türkçe | Engl

Marmara Üniversitesi

AR

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World Conference on Financial Crisis and Impact 2011 (WCFC 2011) Konferansı

Duyurular

Kayıt Yenileme (Ders Seçme – Harç Ödeme) Süreleri (YENİ)

Özmen Aktar ve Handan Ertuğrul Kız Yurdunda Barınmaya Hak Kazananlar

Yabancı Uyruklu Öğrenciler İçin Türkçe Kursları

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- Biyomühendislik
- Çevre Mühendisliği
- Elektrik ve Elektronik Müh.
- Endüstri Mühendisliği
- Kimya Mühendisliği
- Makine Mühendisliği
- Metalurji ve Malzeme Müh.

Duyurular

» YAP/ÇAP PROGRAMINA KABUL EDİLENLER

Tüm Duyurular

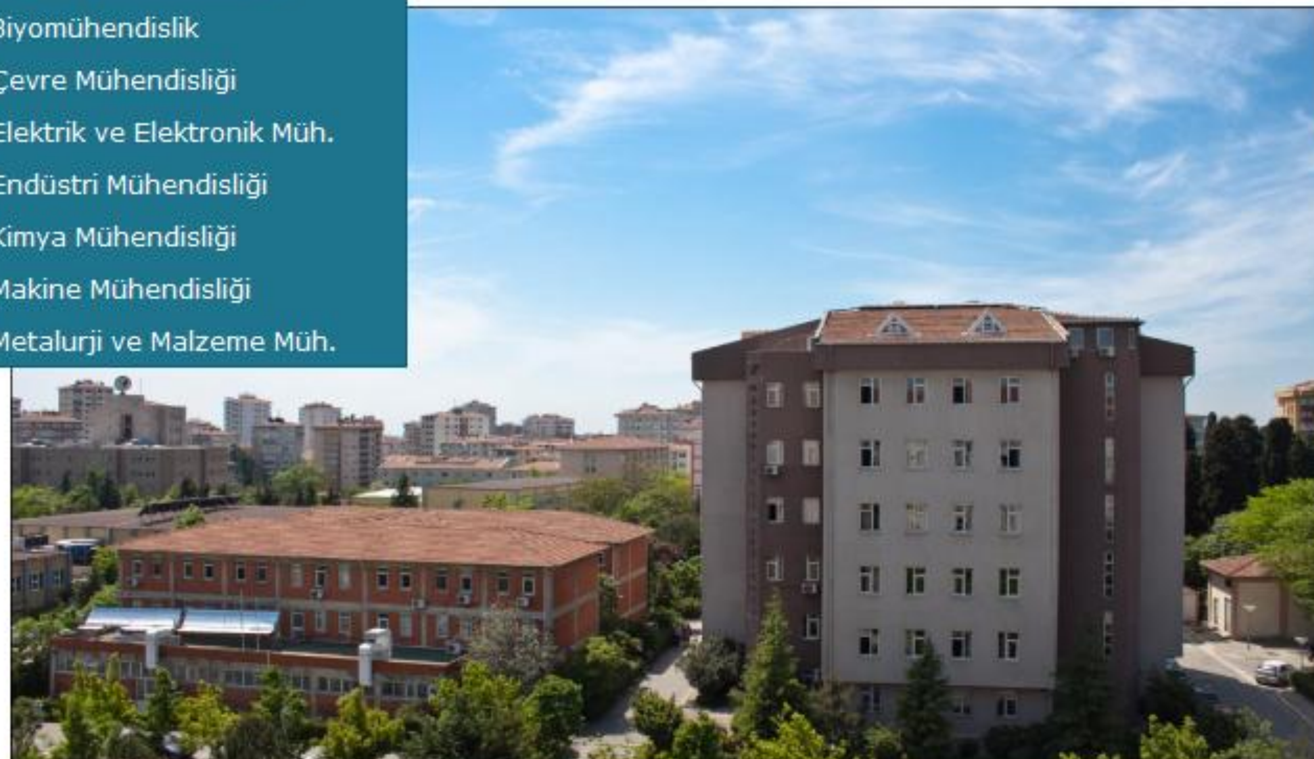
Öğrenci Duyuruları

» Yeni kayıt yaptıran öğrenciler için Oryantasyon Kitapçığı

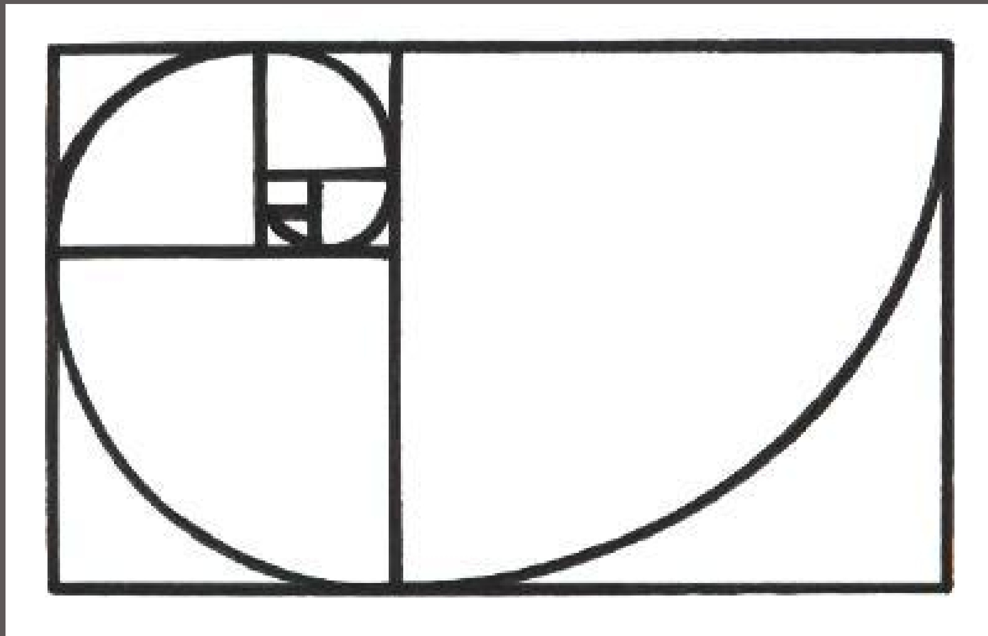
» 2010-2011 EĞİTİM ÖĞRETİM YILI MEZUNİYET TÖRENİ FOTOĞRAFLARI

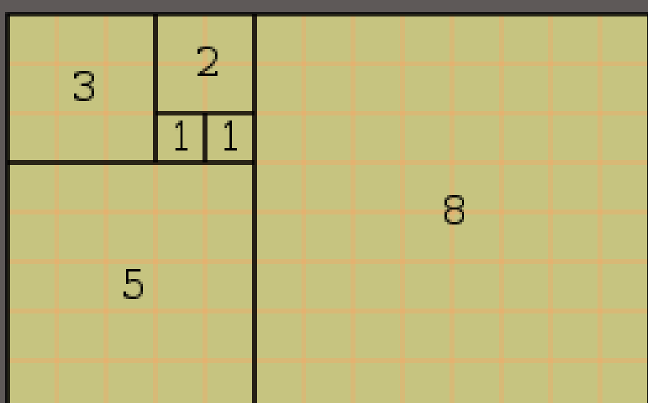
Tüm Öğrenci Duyuruları

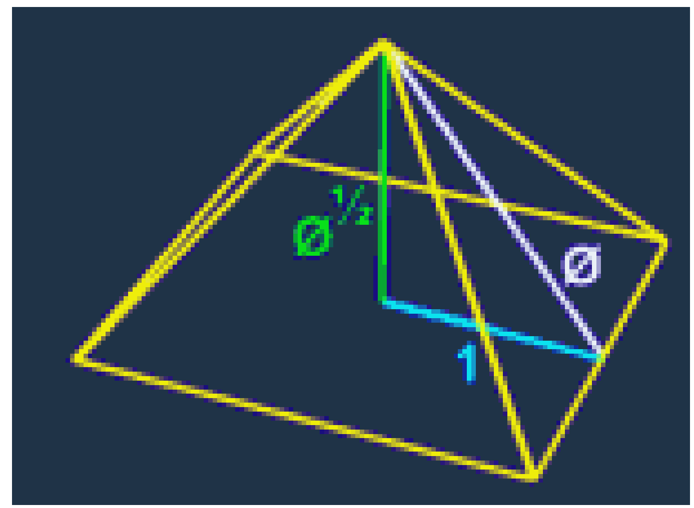
ARA



The Fibonacci Spiral: The Golden Section is a ratio based on a phi



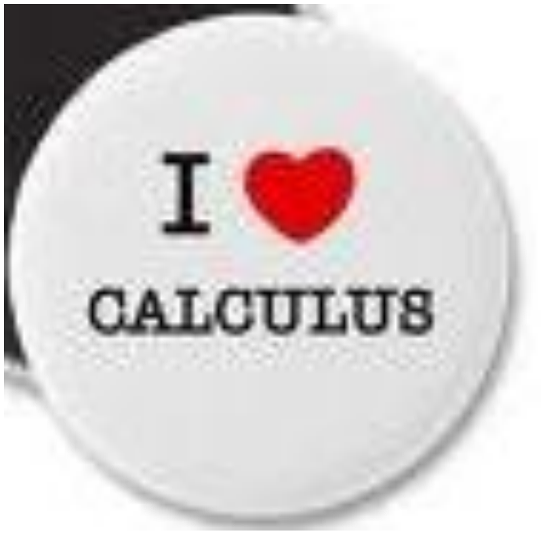


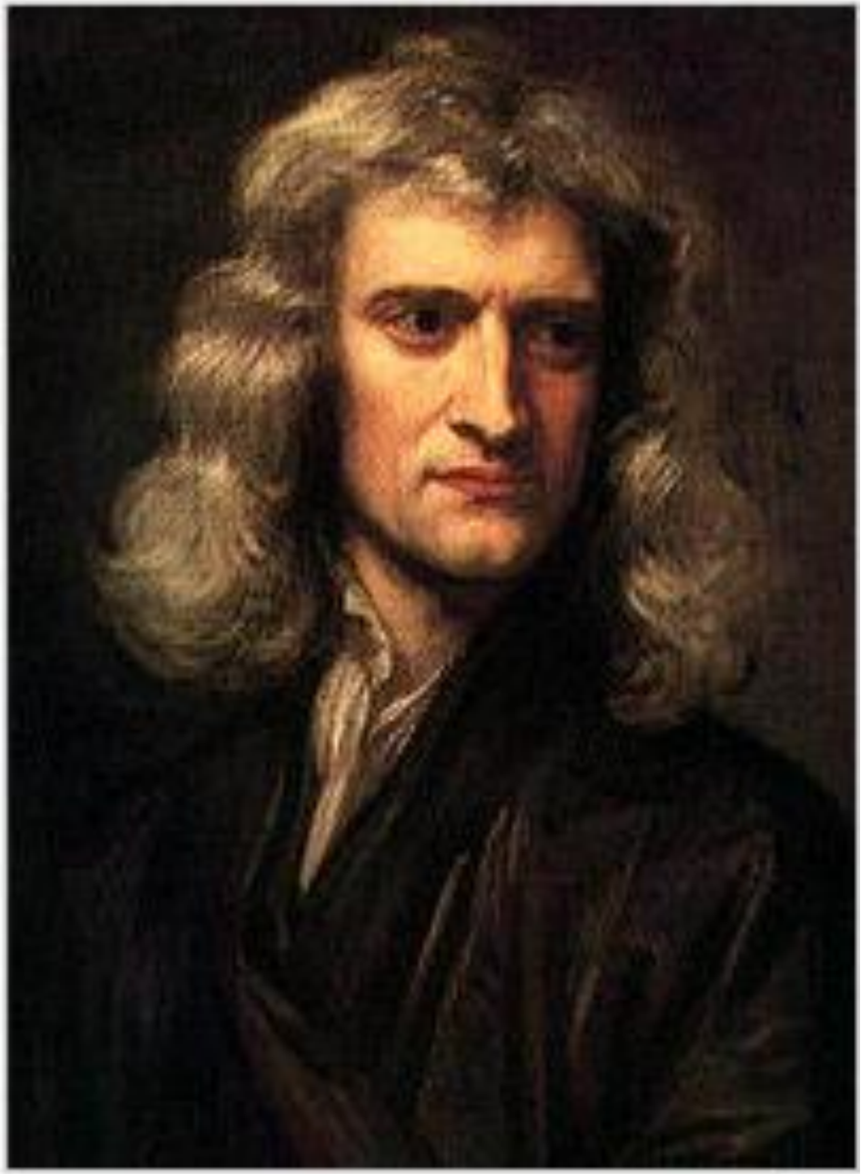


$$\phi = \sqrt{1 + \sqrt{1 + \sqrt{1 + \sqrt{1 + \sqrt{1 + \ldots}}}}}$$

CALCULUS

Calculus (Latin, calculus, a small stone used for counting) is a branch of mathematics focused on limits, functions, derivatives, integrals, and infinite series.





Isaac Newton developed the use of calculus in his laws of motion and gravitation.

Gottfried Wilhelm Leibniz was the first to publish his results on the development of calculus.

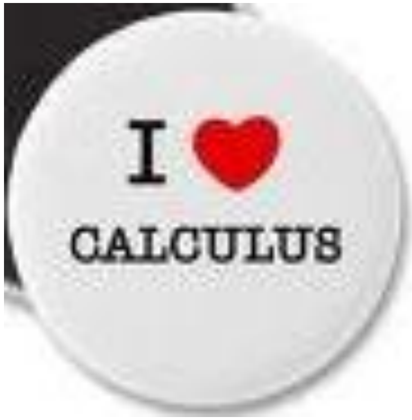
THOMAS' CALCULUS

1 - Preliminaries
2 - Limits and Continuity
3 - Differentiation
4 - Applications of Derivatives
5 - Integration
6 - Applications of Definite Integrals
7 - Transcedental Functions

Math 101

8 - Techniques of Integration
9 - Further Applications of Integration
10 - Conic Sections and Polar Coordinates
11 - Infinite Sequences and Series
12 - Vectors and the Geometry of Space
~~13 - Vector-valued Functions and Motion in Space~~
14 - Partial Derivatives
15 - Multiple Integrals

Math 102



http://www.eksisozluk.com/show.asp?t=calculus

4. yaz okulu nedeni

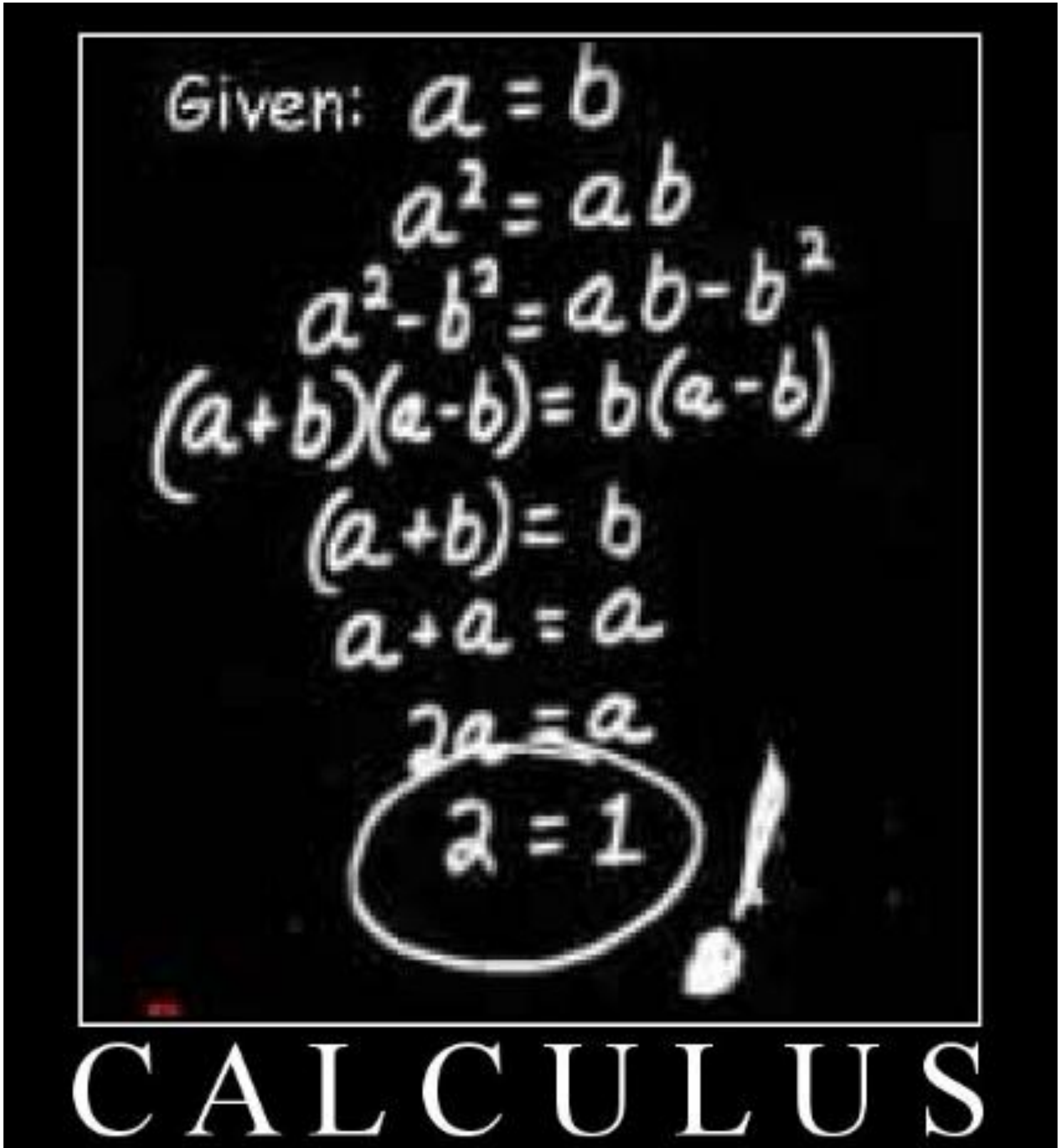
12. newtonun çıkardığı belalardan bir diğeri

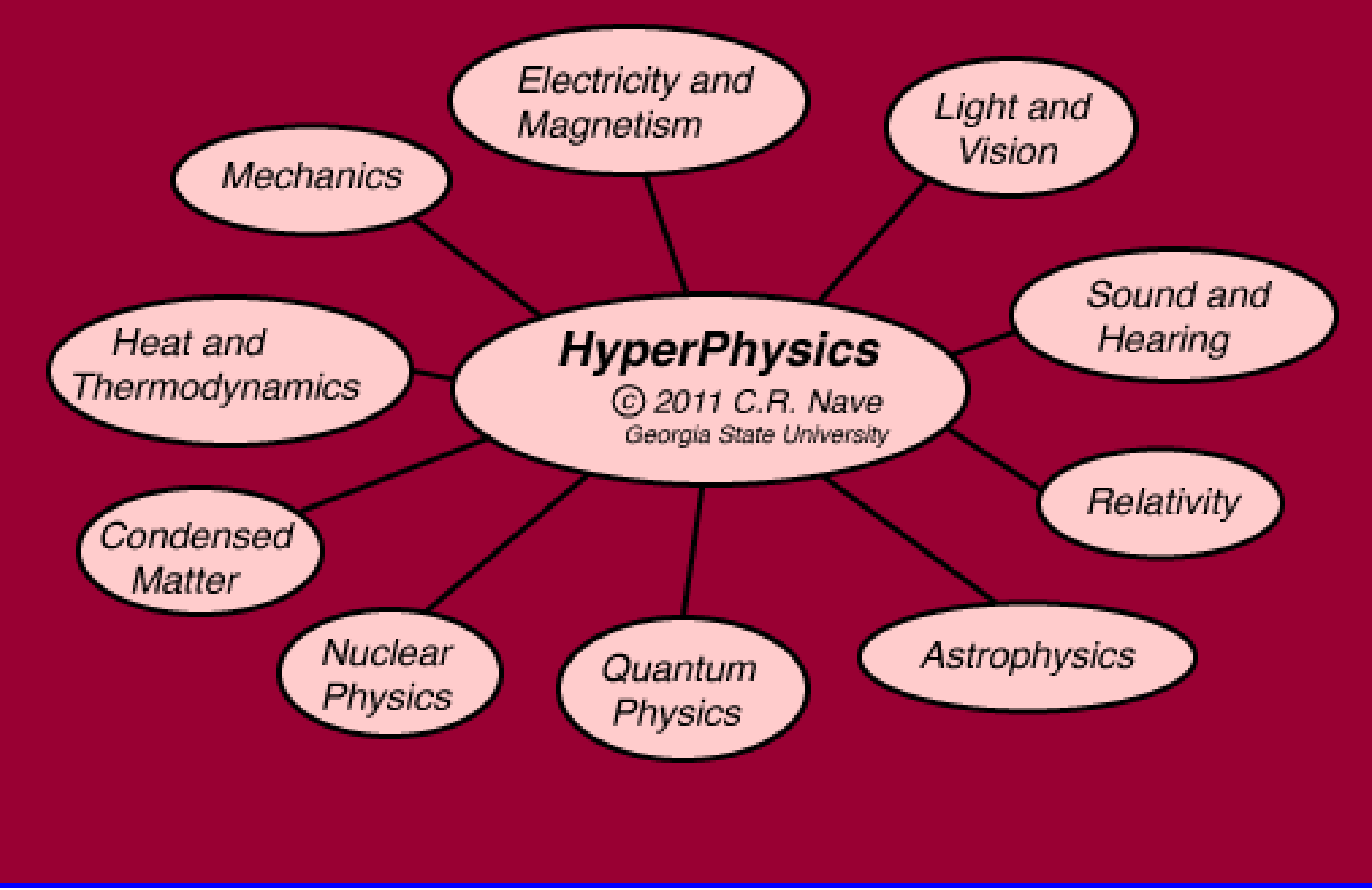
14. eski oys matematigini beceren turk genclerinin zorlanmadan gectikleri, lise son matematiginin muhendis kafasina uyarlanmis seklidir calculus. uzun sure alipta bir turlu veremeyenlerin sayisi ne kadar coksa, o kadar sure sonunda konulari anlayip bunca yil niye gecemedim diye kafasini duvarlara vuranlarin sayisi da o kadar coktur.

15. yalnizca zorlugundan diil, devamsizlik nedeniylede kalinabilecek ders.

16. yaz okulunda alınınca isilik yapan bu ders için:"hangi notla geçersen geç,amman sakın arkana dönme ve kaç olum kaç" derim ben

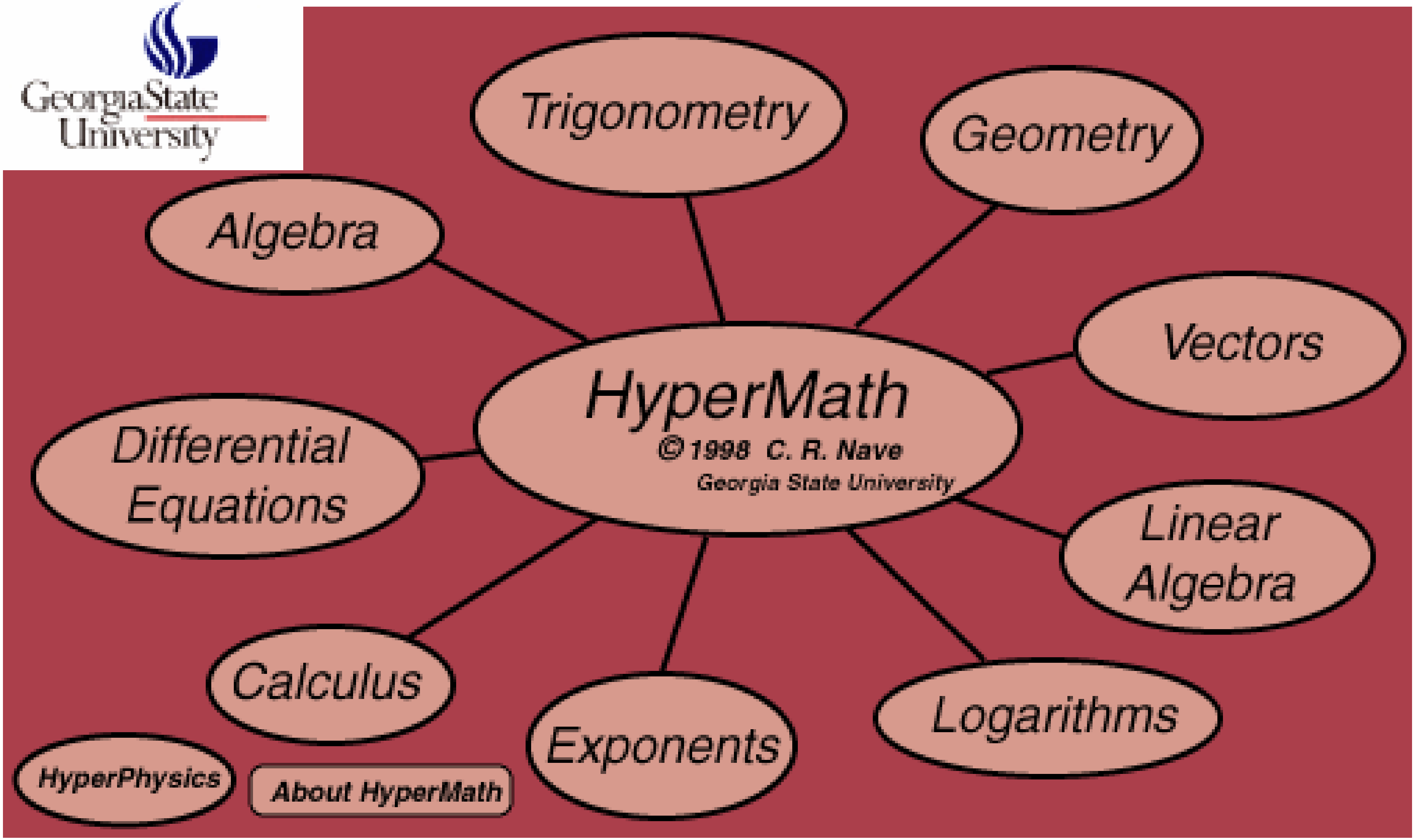
35. mühendislikteki çoğu hesaplama için gereken matematik bilgisi...
mühendisliğin kullandığı araç denilebilir


Given: a = b
a² = ab
a² - b² = ab - b²
(a+b)(a-b) = b(a-b)
(a+b) = b
a + a = a
2a = a
2 = 1 !
CALCULUS

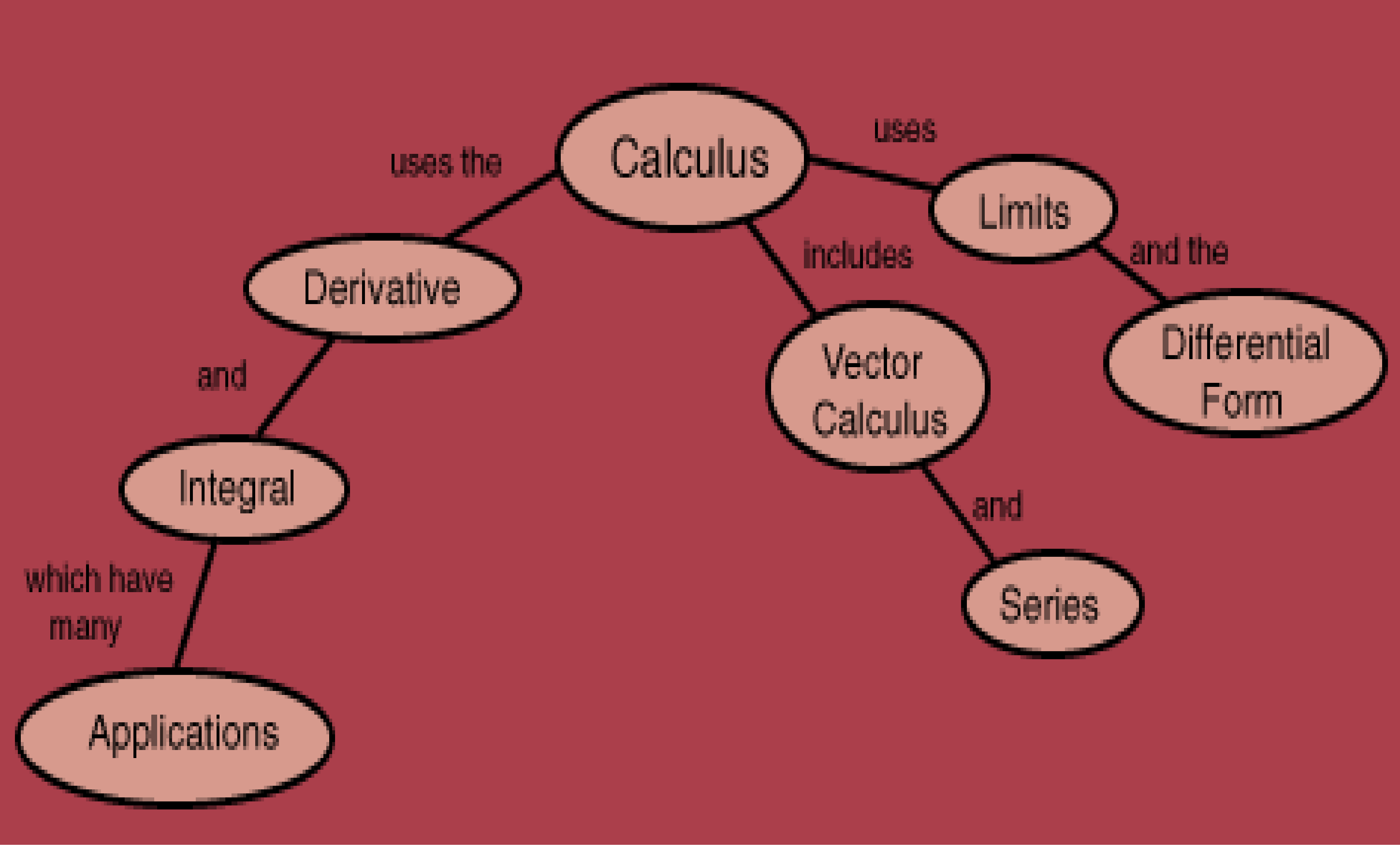


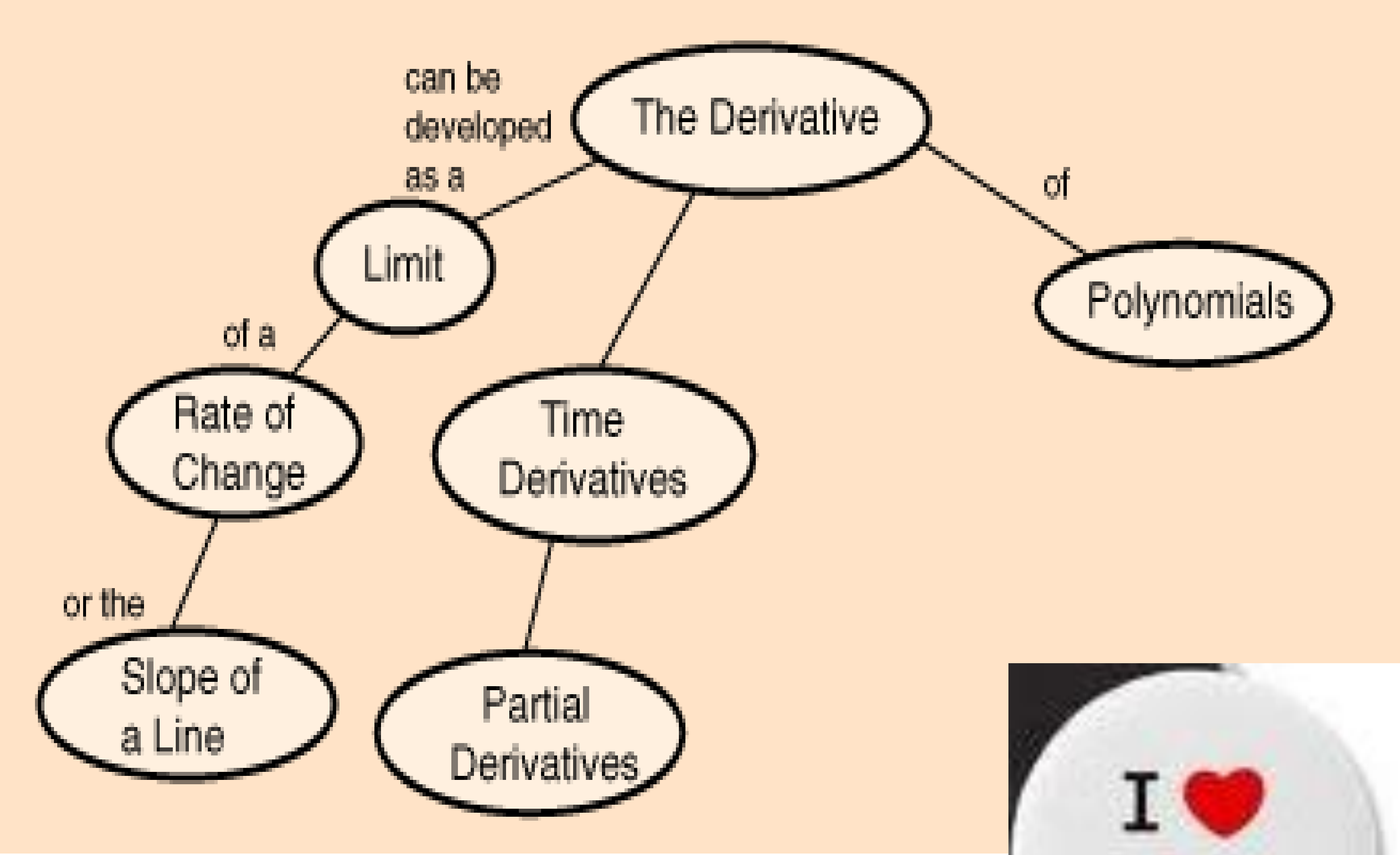
HyperPhysics is hosted by the
Department of Physics and Astronomy

Georgia State University

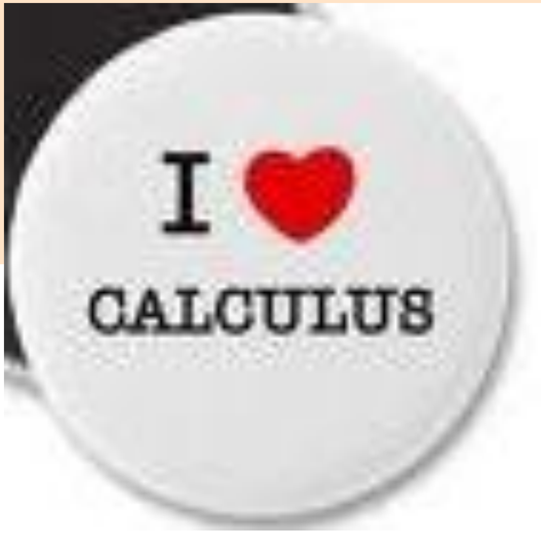


http://hyperphysics.phy-astr.gsu.edu/hbase/hframe.html

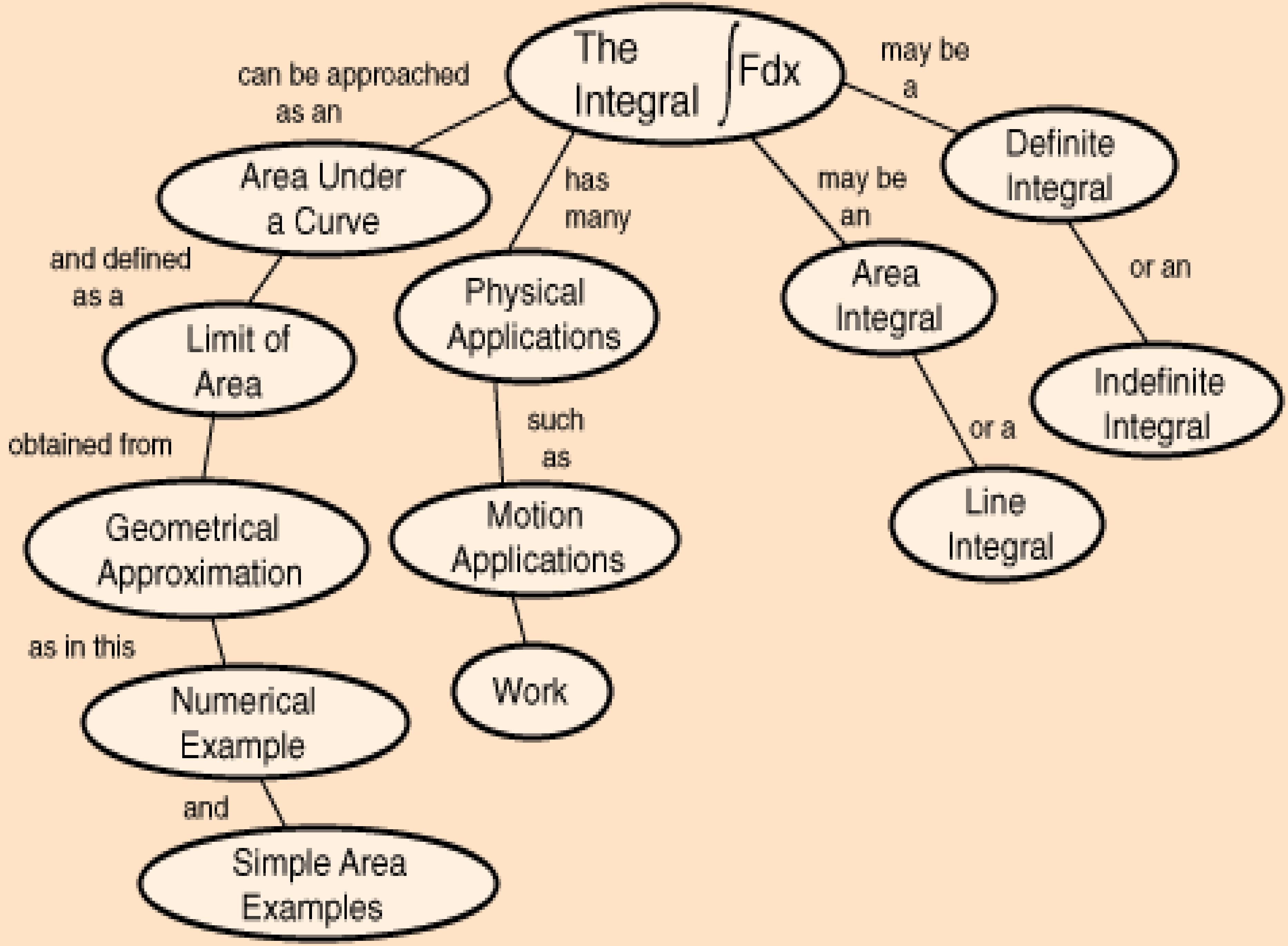

Calculus
uses the
uses
Limits
and the
Derivative
includes
Vector
Calculus
Differential
Form
and
Integral
and
Series
which have
many
Applications



The Derivative
can be
developed
as a
Limit
of a
Rate of
Change
or the
Slope of
a Line
Time
Derivatives
Partial
Derivatives
of
Polynomials



I
CALCULUS


The Integral ∫Fdx
can be approached as an
Area Under a Curve
and defined as a
Limit of Area
obtained from
Geometrical Approximation
as in this
Numerical Example
and
Simple Area Examples
has many
Physical Applications
such as
Motion Applications
Work
may be an
Area Integral
or a
Line Integral
may be a
Definite Integral
or an
Indefinite Integral

Applications

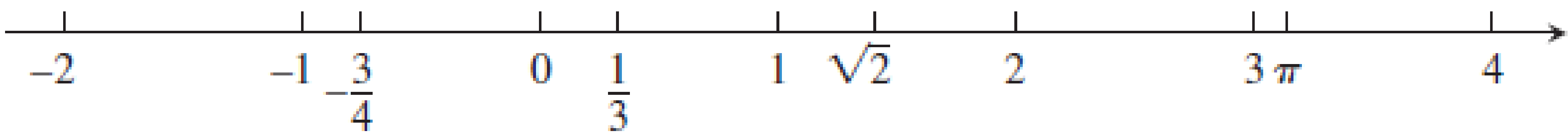
Calculus is used in physical sciences;

computer science, statistics, **engineering**, economics, business, medicine, demography, and in other fields wherever a problem can be **mathematically modeled** and an **optimal solution** is desired.

WEEK	Date	TOPICS	Re
Week 1	22.02.2010	Preliminaries	1
Week 2	01.03.2010	Limits and Continuity, Rates of Change, Vertical Asymptotes	1
Week 3	08.03.2010	Differentiation, Rate of Change, Trigonometric Functions, Chain Rule	2
Week 4	15.03.2010	Derivatives, Parametric Equations, Implicit Differentiation	2
Week 5	22.03.2010	Applications of Derivatives: Extreme Values, Curve Sketching	3
Week 6	29.03.2010	Optimization Problems, L'Hôpital's Rule, Newton's Method	3
Week 7	05.04.2010	Integration: Antiderivatives	4
Week 8	12.04.2010	Study Week	
Week 9	19.04.2010	Midterm	
Week 10	22.04.2010	Definite Integral, Indefinite Integrals and the Substitution Rule	4
Week 11	26.04.2010	Applications of Integrals: Area Between Curves, Volumes by Slicing & Shells	5
Week 12	03.05.2010	Moments and Centers of Mass, Fluid Pressures and Forces	5
Week 13	10.05.2010	Transcendental Functions: Natural Logarithms, Exponential Function	6
Week 14	17.05.2010	Exponential Growth and Decay, Inverse Trigonometric Functions,	6
Week 15	24.05.2010	Hyperbolic Functions, Techniques of Integration: Partial Fractions	7
Week 16	31.05.2010	Numerical Integration, Linear Differential Equations	7
Week 17	07.06.2010	Study Week	7
Week 18	14.06.2010	Final	

PRELIMINARIES

Real Numbers and the **real line**.



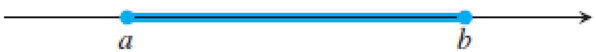
We distinguish three special subsets of real numbers.

1. The **natural numbers**, namely 1, 2, 3, 4, . . .
2. The **integers**, namely 0, ±1, ±2, ±3, . . .
3. The **rational numbers**, namely the numbers that can be expressed in the form of a fraction m/n, where m and n are integers and $n \neq 0$. Examples are

$$\frac{1}{3}, \quad -\frac{4}{9} = \frac{-4}{9} = \frac{4}{-9}, \quad \frac{200}{13}, \quad \text{and} \quad 57 = \frac{57}{1}.$$

Types of intervals

TABLE 1.1 Types of intervals

	Notation	Set description	Type	Picture
Finite:	(a, b)	$\{x \mid a < x < b\}$	Open	
	$[a, b]$	$\{x \mid a \leq x \leq b\}$	Closed	
	$[a, b)$	$\{x \mid a \leq x < b\}$	Half-open	
	$(a, b]$	$\{x \mid a < x \leq b\}$	Half-open	
Infinite:	(a, ∞)	$\{x \mid x > a\}$	Open	
	$[a, \infty)$	$\{x \mid x \geq a\}$	Closed	
	$(-\infty, b)$	$\{x \mid x < b\}$	Open	
	$(-\infty, b]$	$\{x \mid x \leq b\}$	Closed	
	$(-\infty, \infty)$	$\mathbb{R}$ (set of all real numbers)	Both open and closed	

Example:

Solve the inequality and show the solution set on the real line

$$|2x - 3| \leq 1$$

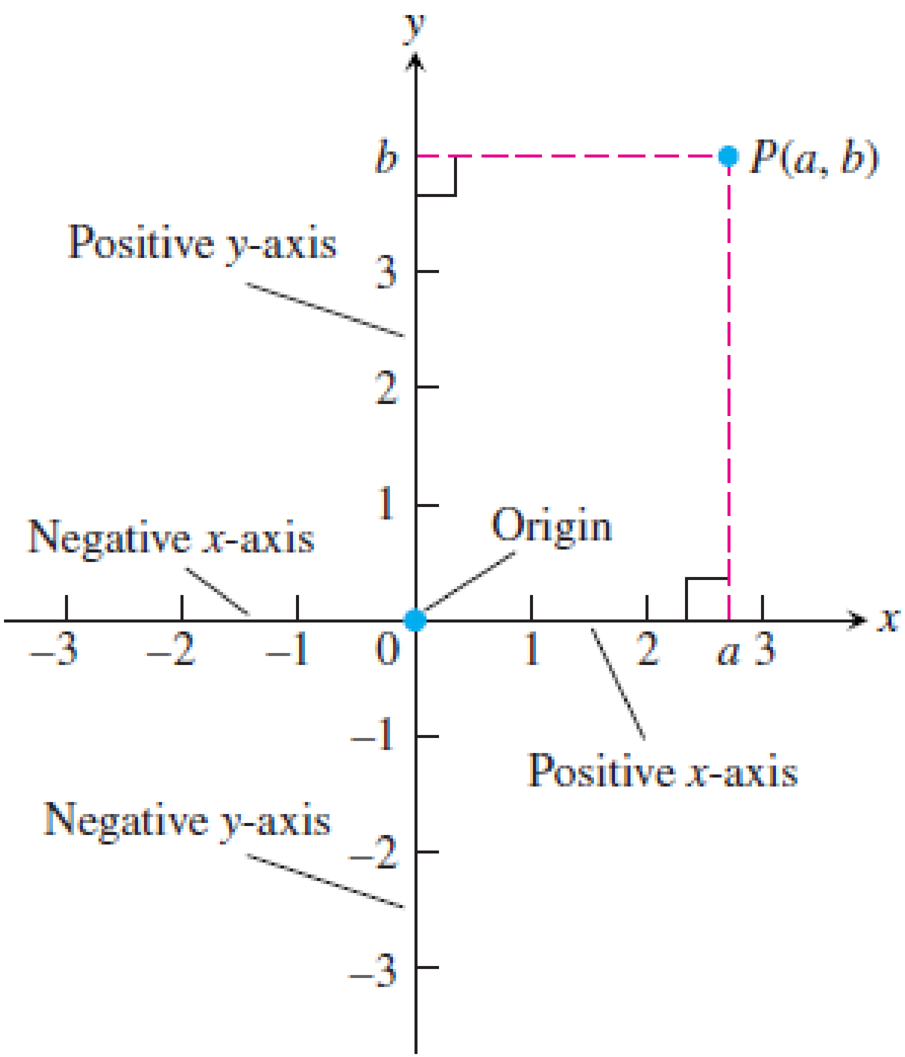
Example: **Quadratic Inequalities**

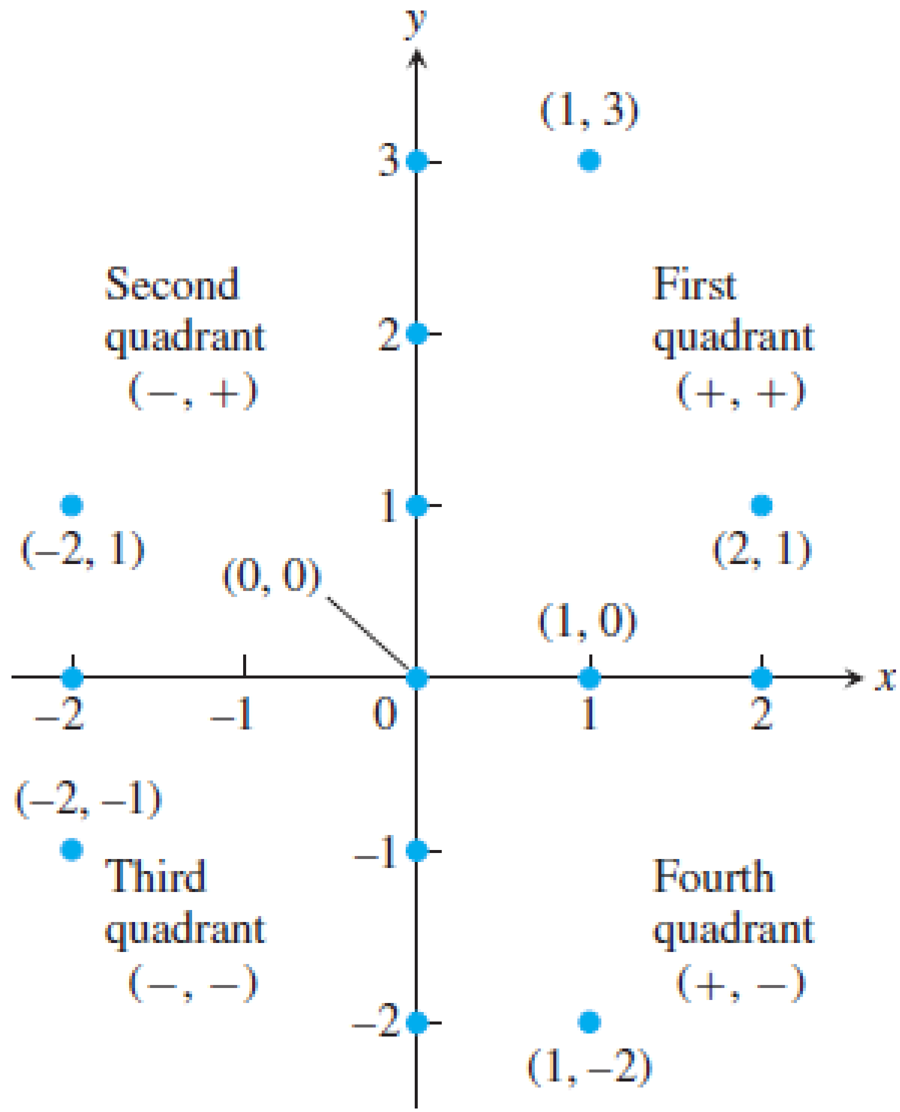
Solve the inequality. Express the solution sets as an interval or union of intervals.

$$x^2 - x - 2 \geq 0$$

The solution interval is (-∞, -1] [2, ∞)

Lines, Circles, and Parabolas



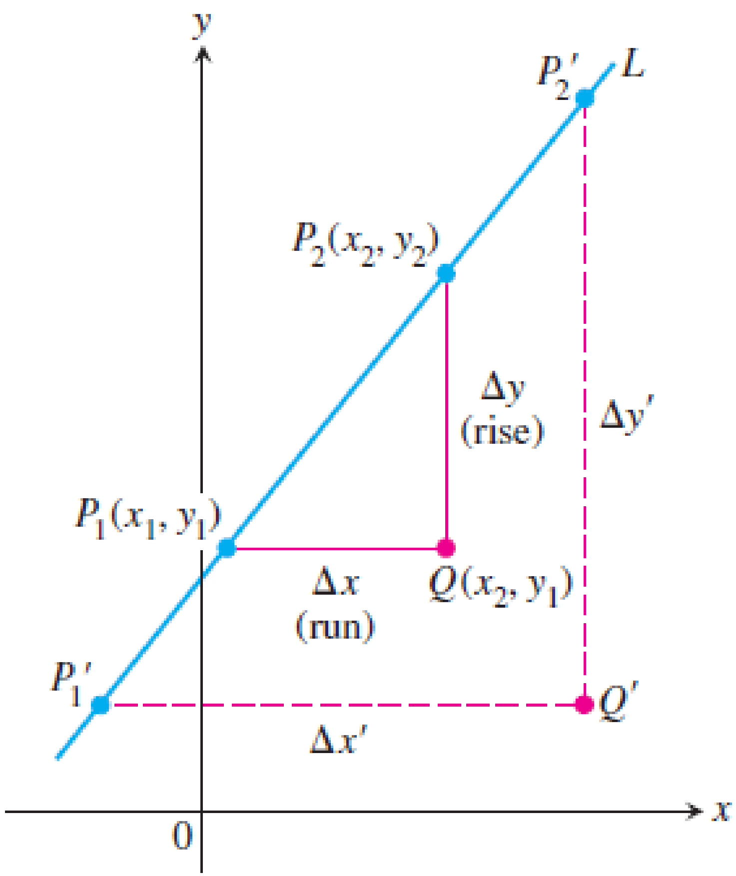


DEFINITION Slope

The constant

$$m = \frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

is the **slope** of the nonvertical line P_1P_2.

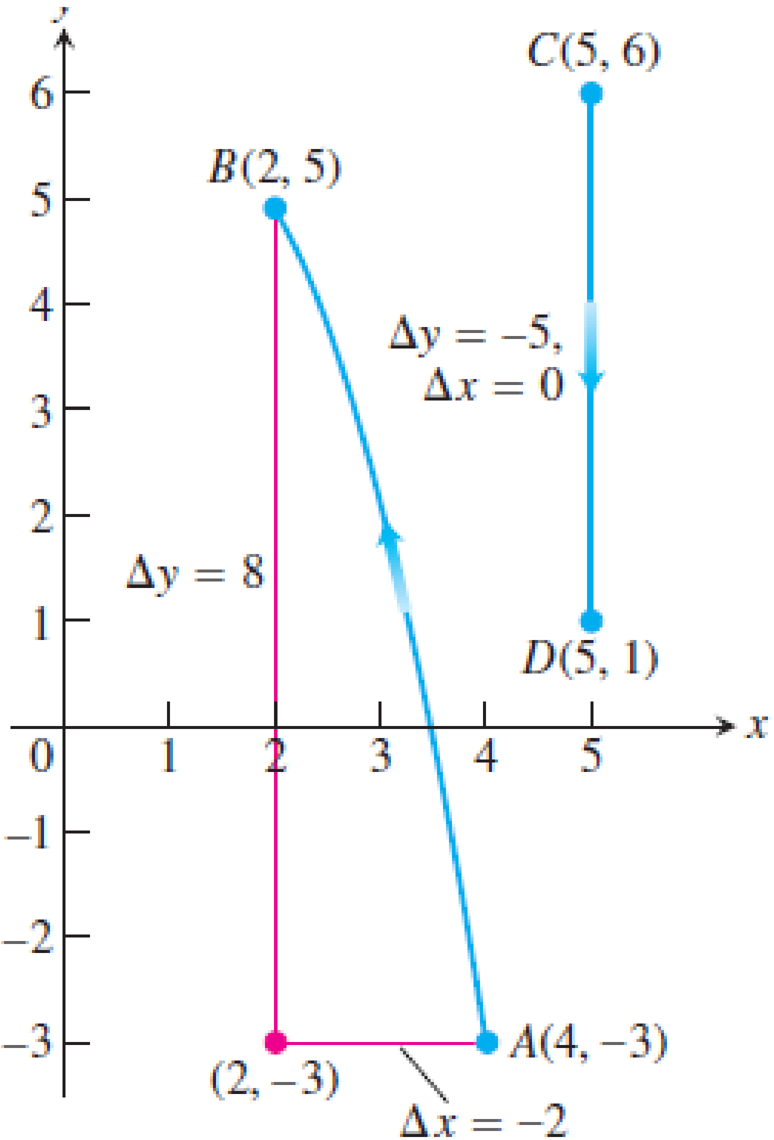


Common form of the

equation of a line

$$\mathbf{y} = m\mathbf{x} + n$$

In going from the point $A(4, -3)$ to the point $B(2, 5)$ the increments in the x- and y-coordinates are



Slope m = -4

$\mathbf{y} = m\mathbf{x} + n$

Applying point B

$\mathbf{y} = -4\mathbf{x} + 13$

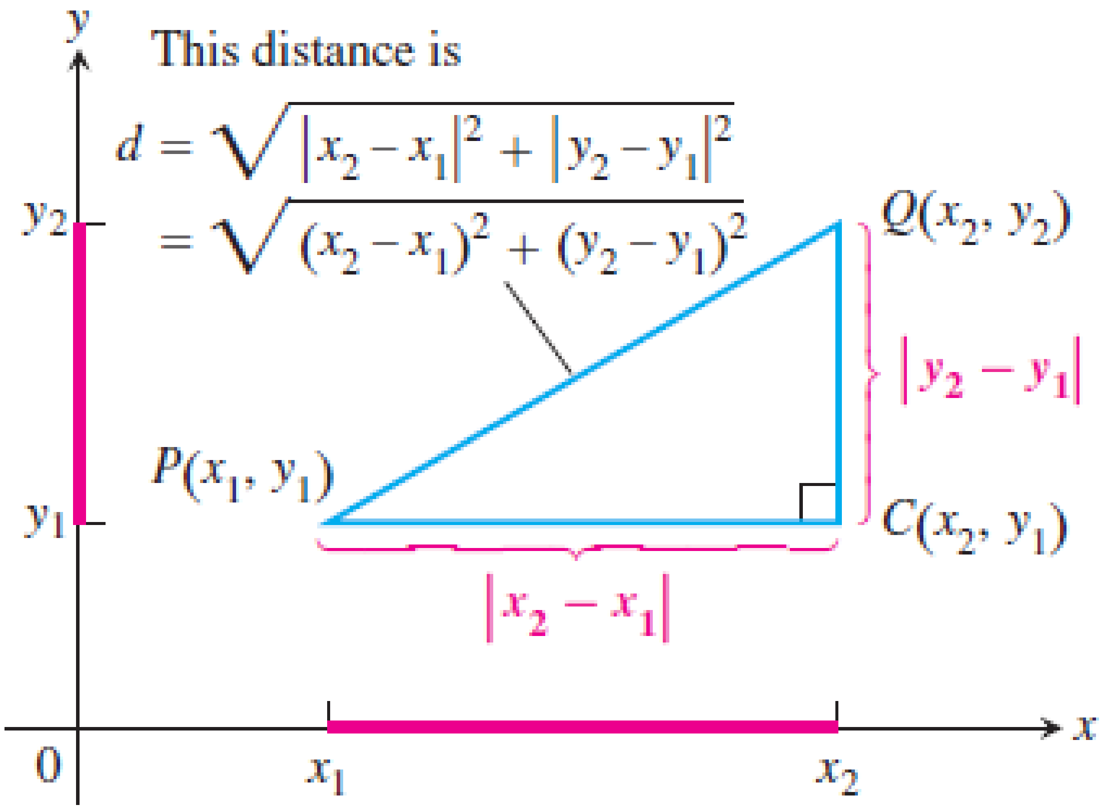
Parallel and Perpendicular Lines

- Lines that are parallel have equal angles of inclination, so they have the same slope

$$\mathbf{m_1 = m_2}$$

- If two lines are perpendicular, their slopes satisfy

$$\mathbf{m_1 . m_2 = -1}$$



Distance Formula for Points in the Plane

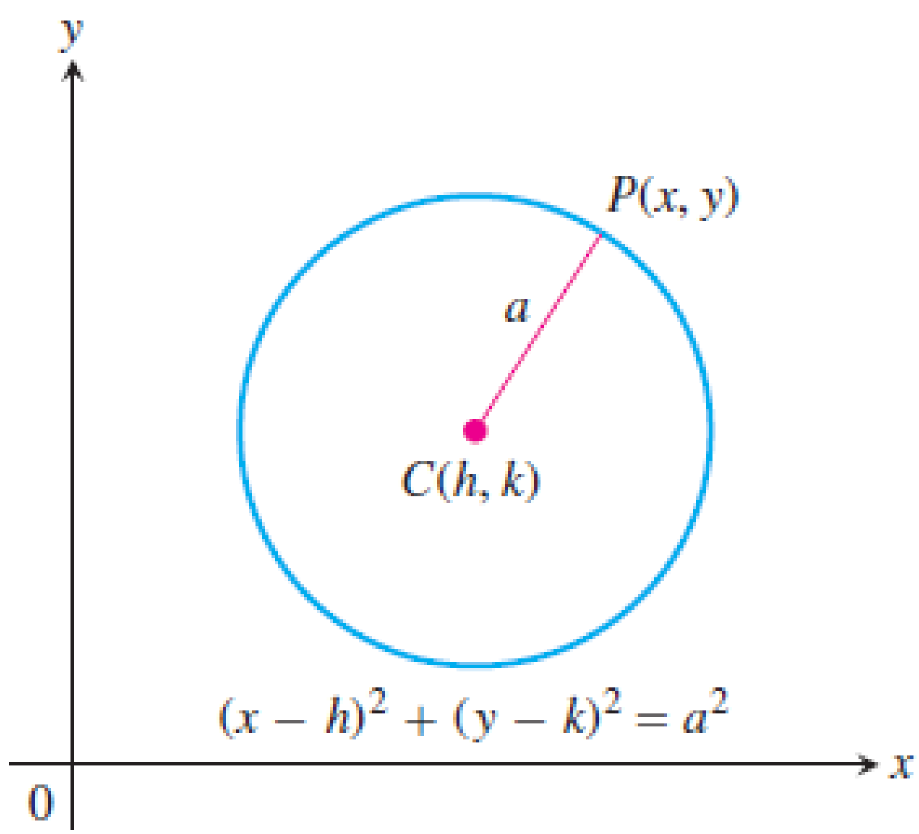
The distance between $P(x_1, y_1)$ and $Q(x_2, y_2)$ is

$$d = \sqrt{(\Delta x)^2 + (\Delta y)^2} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

$$(x - h)^2 + (y - k)^2 = a^2. \quad (1)$$

Equation (1) is the **standard equation** of a circle with center (h, k) and radius a. The circle of radius $a = 1$ and centered at the origin is the **unit circle** with equation

$$x^2 + y^2 = 1.$$



write an equation for each line described

1. Passes through $(3, 4)$ and $(-2, 5)$
2. Passes through $(-8, 0)$ and $(-1, 3)$
3. Has slope $-5/4$ and y-intercept 6
4. Has slope $1/2$ and y-intercept -3
5. Passes through $(-12, -9)$ and has slope 0
6. Passes through $(1/3, 4)$, and has no slope
7. Has y-intercept 4 and x-intercept -1

Graph the circles whose equations are given, each circle's center and intercepts (if any) with their coordinate pairs.

1. $x^2 + y^2 - 4x + 4y = 0$
2. $x^2 + y^2 + 2x = 3$

graph the two equations and find the points in which the graphs intersect.

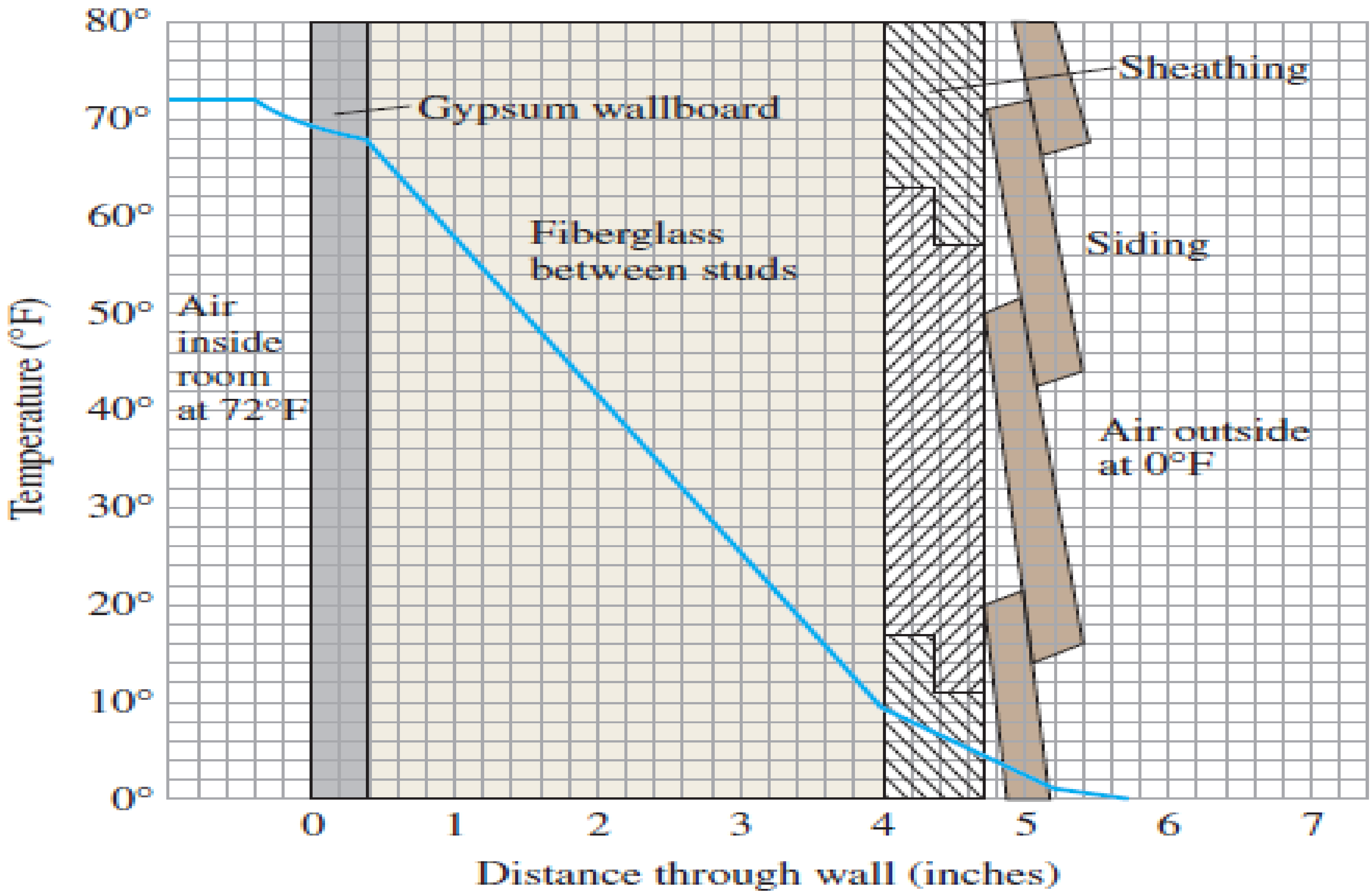
$y = 2x, \quad x^2 + y^2 = 1$

$y - x = 1, \quad y = x^2$

$y = -x^2, \quad y = 2x^2 - 1$

$x^2 + y^2 = 1, \quad x^2 + y = 1$

Insulation By measuring slopes in the accompanying figure, estimate the temperature change in degrees per inch for (a) the gypsum wallboard; (b) the fiberglass insulation; (c) the wood sheathing.



Functions and Their Graphs

DEFINITION **Function**

A **function** from a set D to a set Y is a rule that assigns a *unique* (single) element $f(x) \in Y$ to each element $x \in D$.

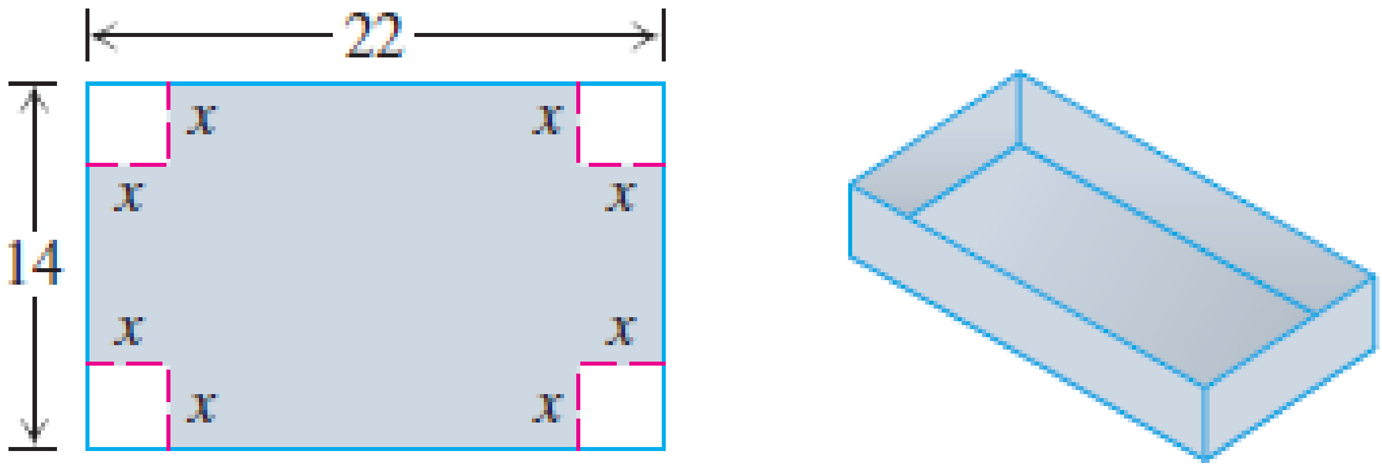
A diagram showing a function as a kind of machine.

Identifying Domain and Range

Verify the domains and ranges of these functions.

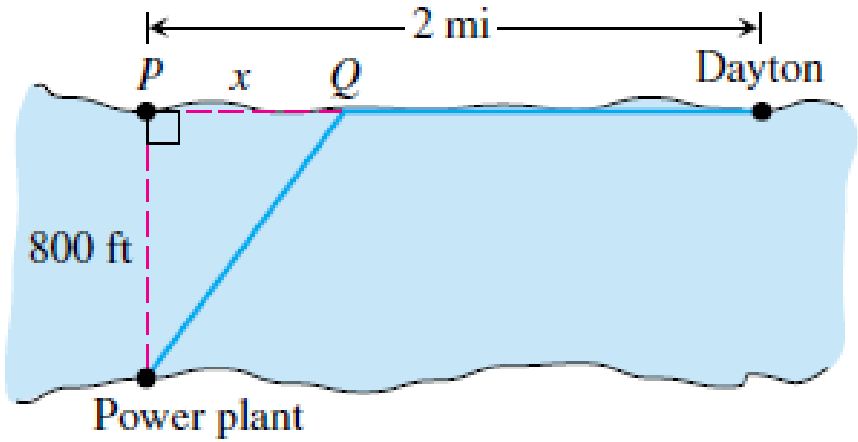
Function	**Domain (x)**	**Range (y)**
$y = x^2$	$(-\infty, \infty)$	$[0, \infty)$
$y = 1/x$	$(-\infty, 0) \cup (0, \infty)$	$(-\infty, 0) \cup (0, \infty)$
$y = \sqrt{x}$	$[0, \infty)$	$[0, \infty)$
$y = \sqrt{4 - x}$	$(-\infty, 4]$	$[0, \infty)$
$y = \sqrt{1 - x^2}$	$[-1, 1]$	$[0, 1]$

A box with an open top is to be constructed from a rectangular piece of cardboard with dimensions 14 in. by 22 in. by cutting out equal squares of side x at each corner and then folding up the sides as in the figure. Express the volume V of the box as a function of x.



$$v = f(x) = x(14 - 2x)(22 - 2x) = 4x^3 - 72x^2 + 308x;\ 0 < x < 7.$$

Industrial costs Dayton Power and Light, Inc., has a power plant on the Miami River where the river is 800 ft wide. To lay a new cable from the plant to a location in the city 2 mi downstream on the opposite side costs \$180 per foot across the river and \$100 per foot along the land.



(Not to scale)

a. Suppose that the cable goes from the plant to a point Q on the opposite side that is x ft from the point P directly opposite the plant. Write a function $C(x)$ that gives the cost of laying the cable in terms of the distance x.

b. Generate a table of values to determine if the least expensive location for point Q is less than 2000 ft or greater than 2000 ft from point P.

(a) Note that 2 mi = 10,560 ft, so there are $\sqrt{800^2 + x^2}$ feet of river cable at \$180 p
cable at \$100 per foot. The cost is $C(x) = 180\sqrt{800^2 + x^2} + 100(10,560 - x)$

(b) $C(0) = \$1,200,000$

$C(500) \approx \$1,175,812$

$C(1000) \approx \$1,186,512$

$C(1500) \approx \$1,212,000$

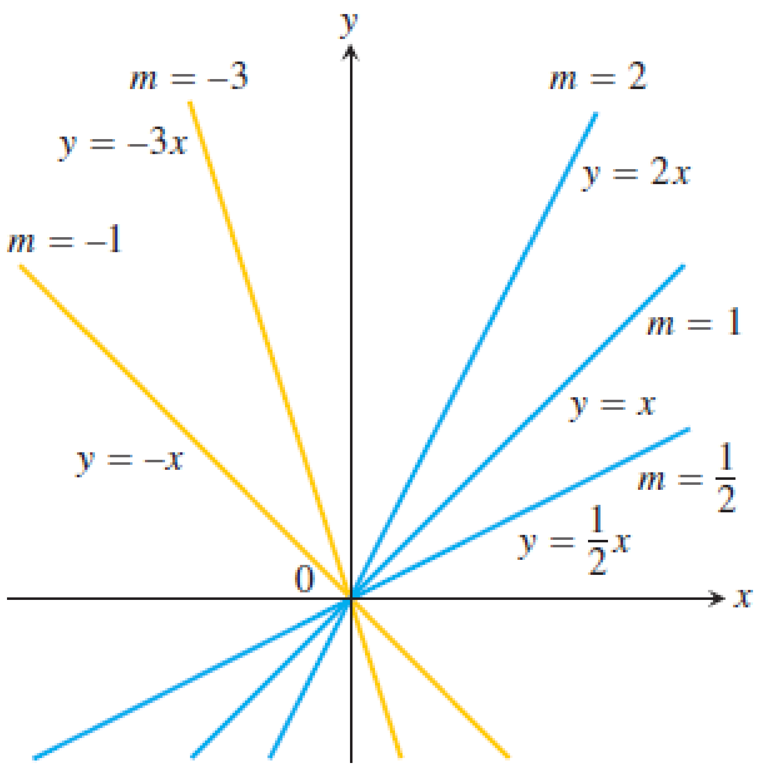
$C(2000) \approx \$1,243,732$

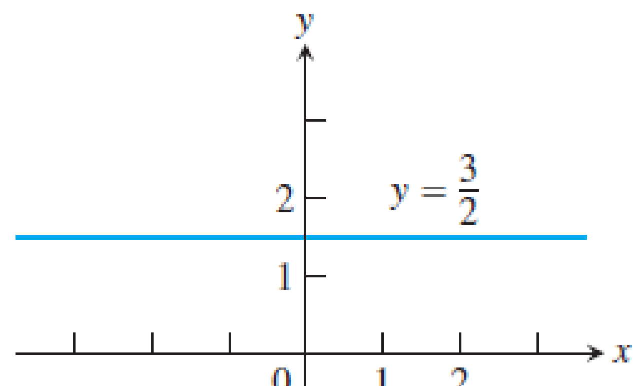
$C(2500) \approx \$1,278,479$

$C(3000) \approx \$1,314,870$

Identifying Functions; Mathematical Models

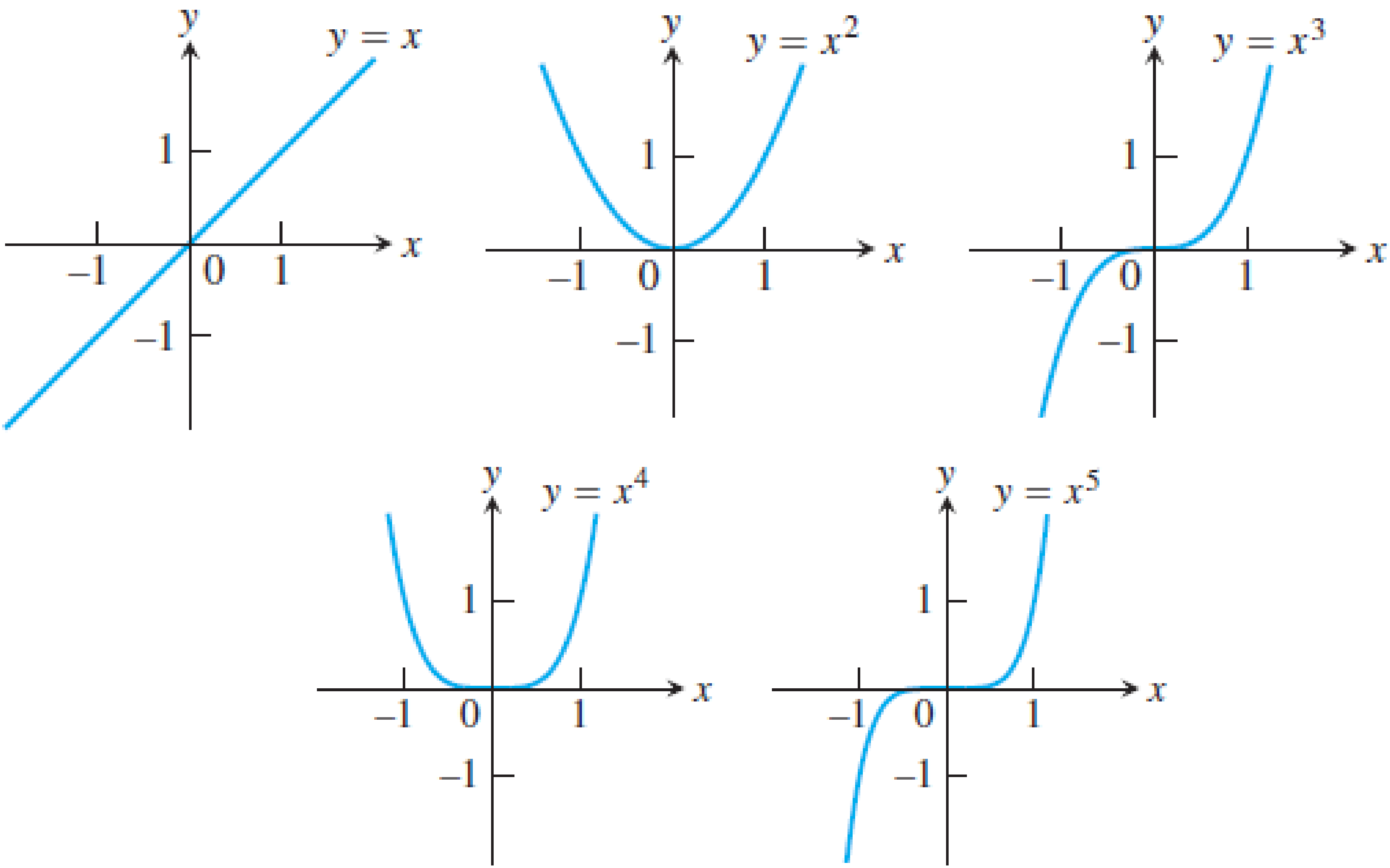
Linear Functions A function of the form $f(x) = mx + b$, for constants m and b, is called a **linear function**. Figure 1.34 shows an array of lines $f(x) = mx$ where $b = 0$, so these lines pass through the origin. Constant functions result when the slope $m = 0$ (Figure 1.35).



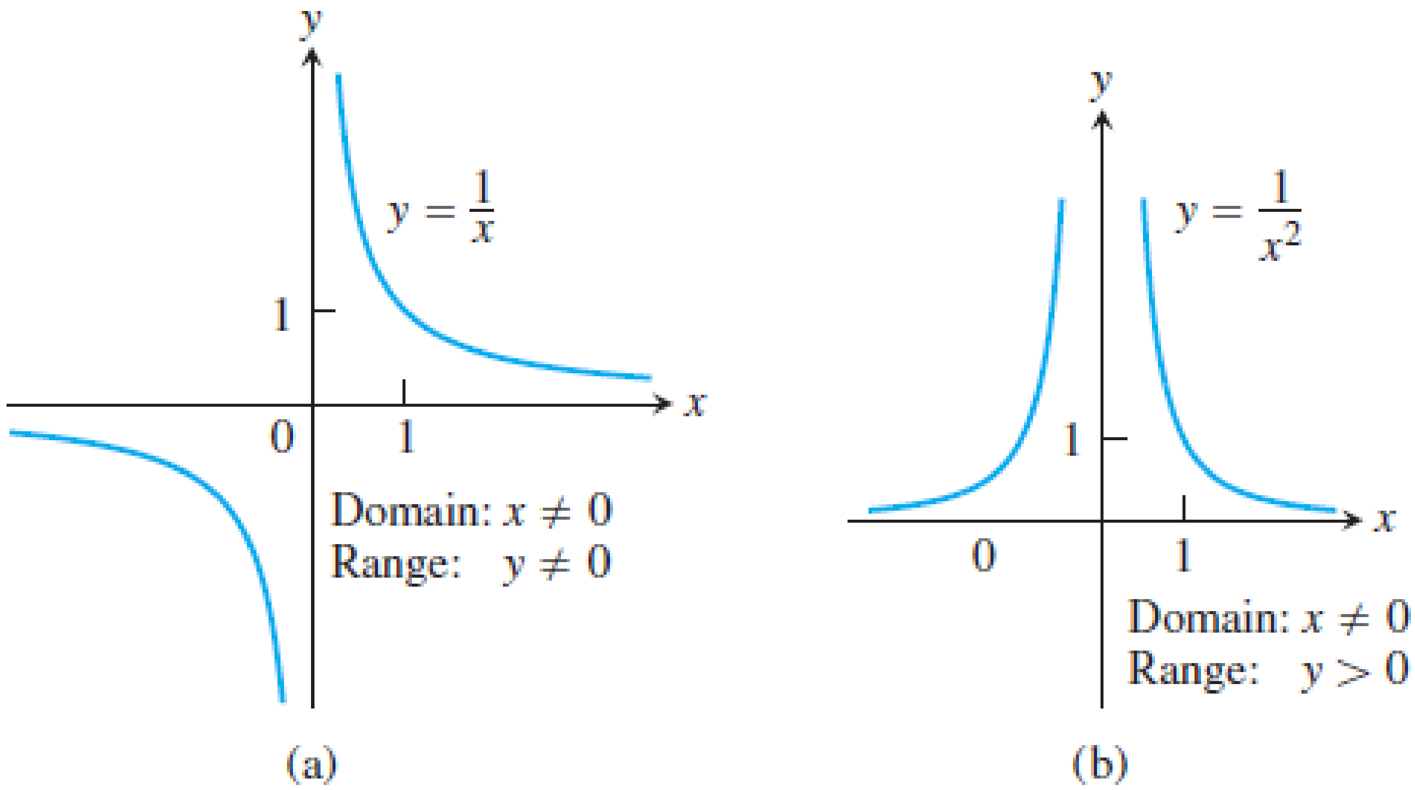


Power Functions A function $f(x) = x^a$, where a is a constant, is called a **power function**. There are several important cases to consider.

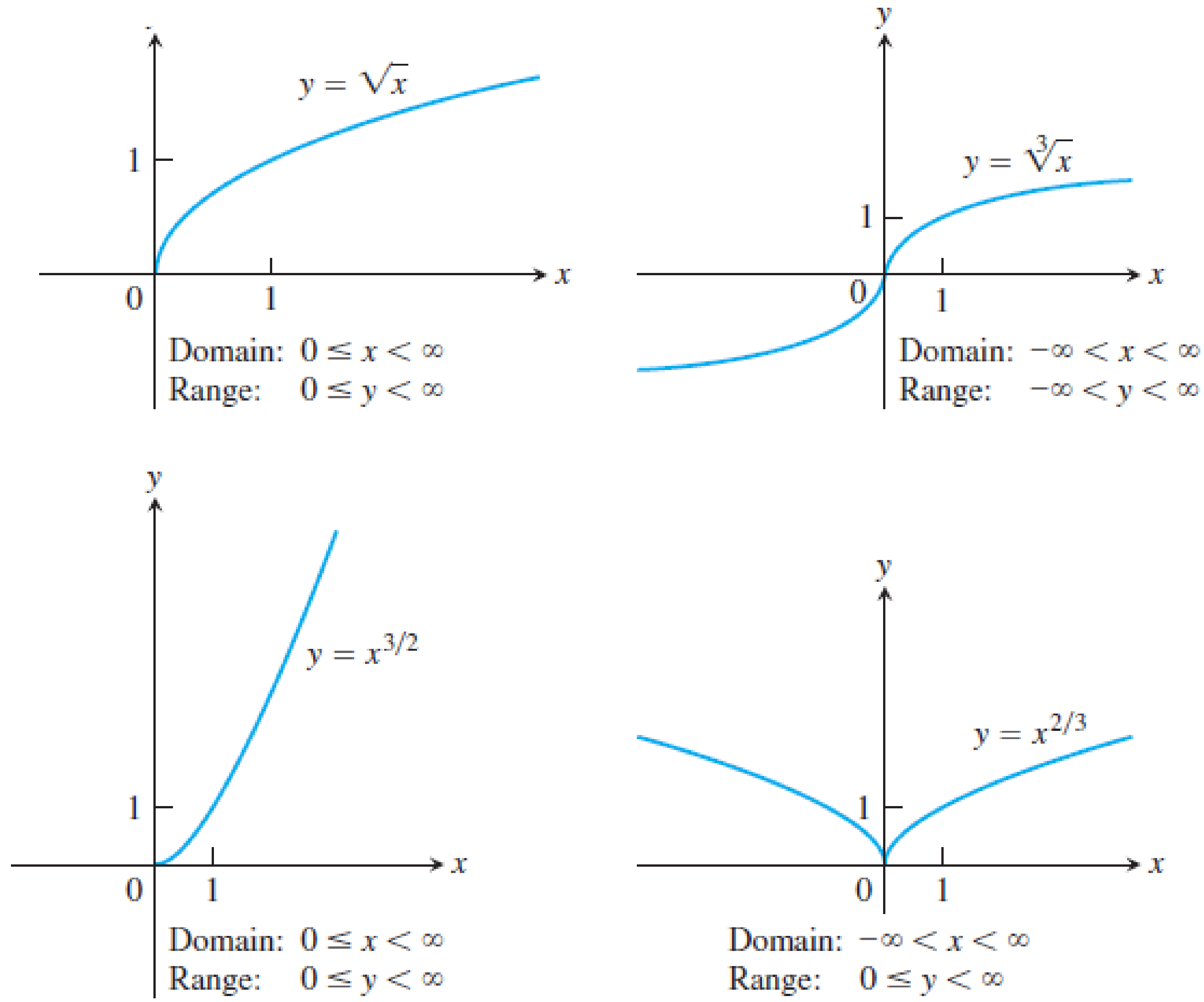
(a) $a = n$, a positive integer.

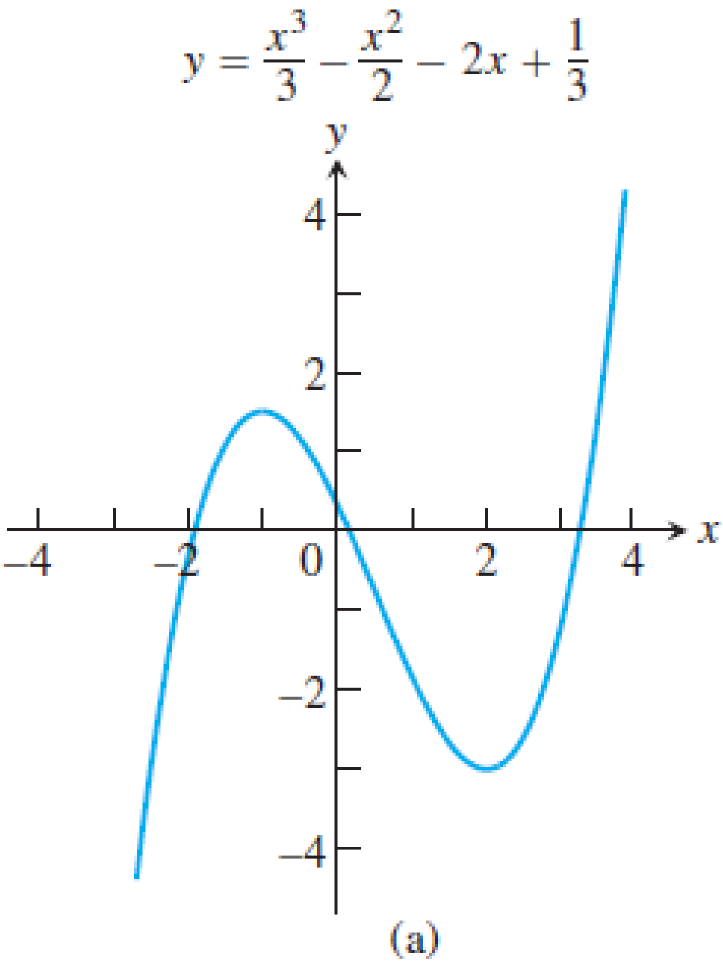


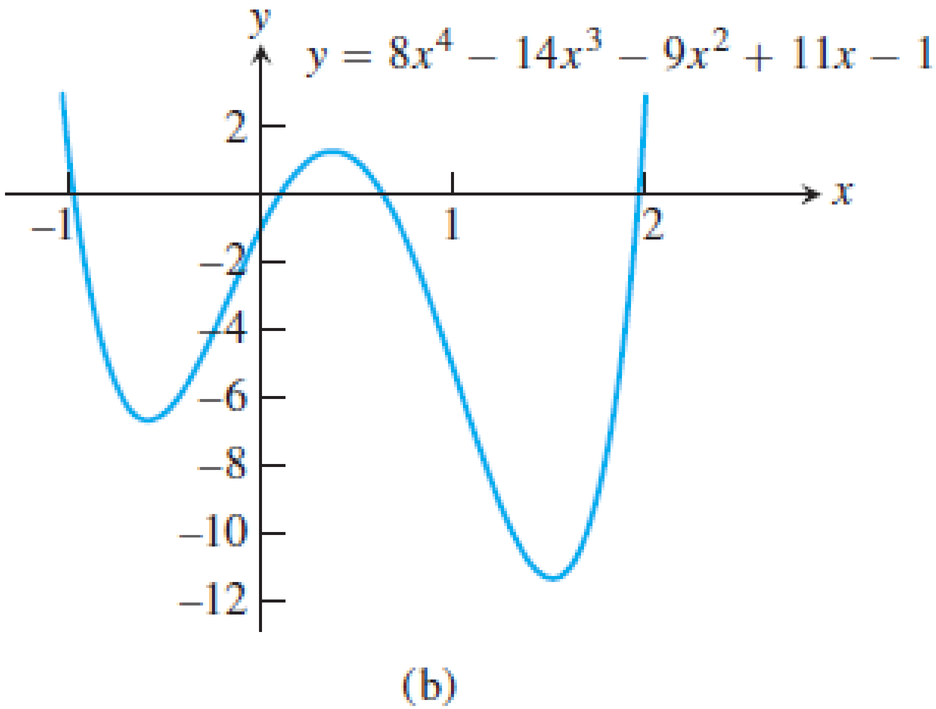
(b) $a = -1$ or $a = -2$.


y
x
$y = \frac{1}{x}$
1
0
1
Domain: $x \neq 0$
Range: $y \neq 0$
(a)
y
x
$y = \frac{1}{x^2}$
1
0
1
Domain: $x \neq 0$
Range: $y > 0$
(b)

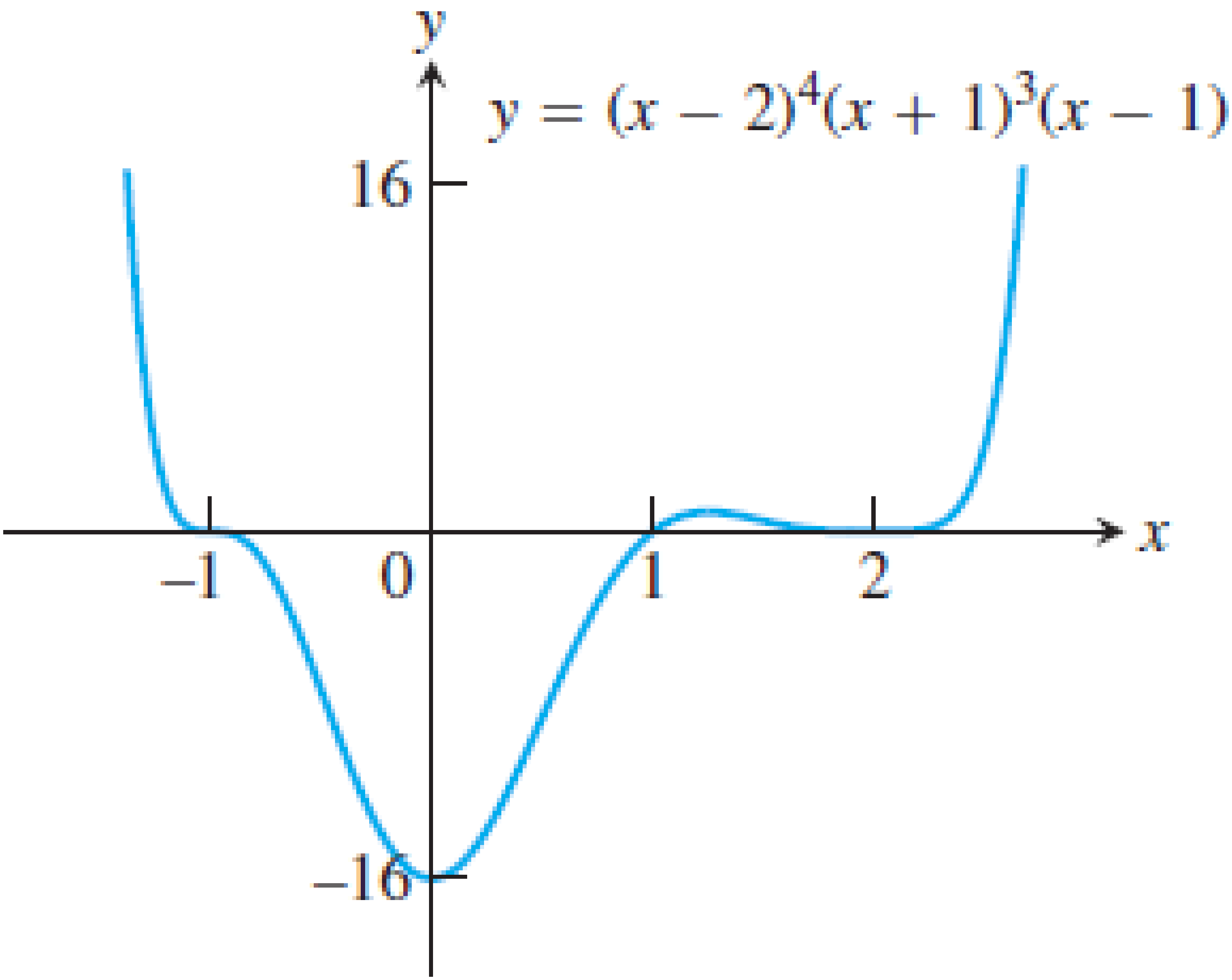
(c) $a = \frac{1}{2}, \frac{1}{3}, \frac{3}{2}, \text{ and } \frac{2}{3}$.





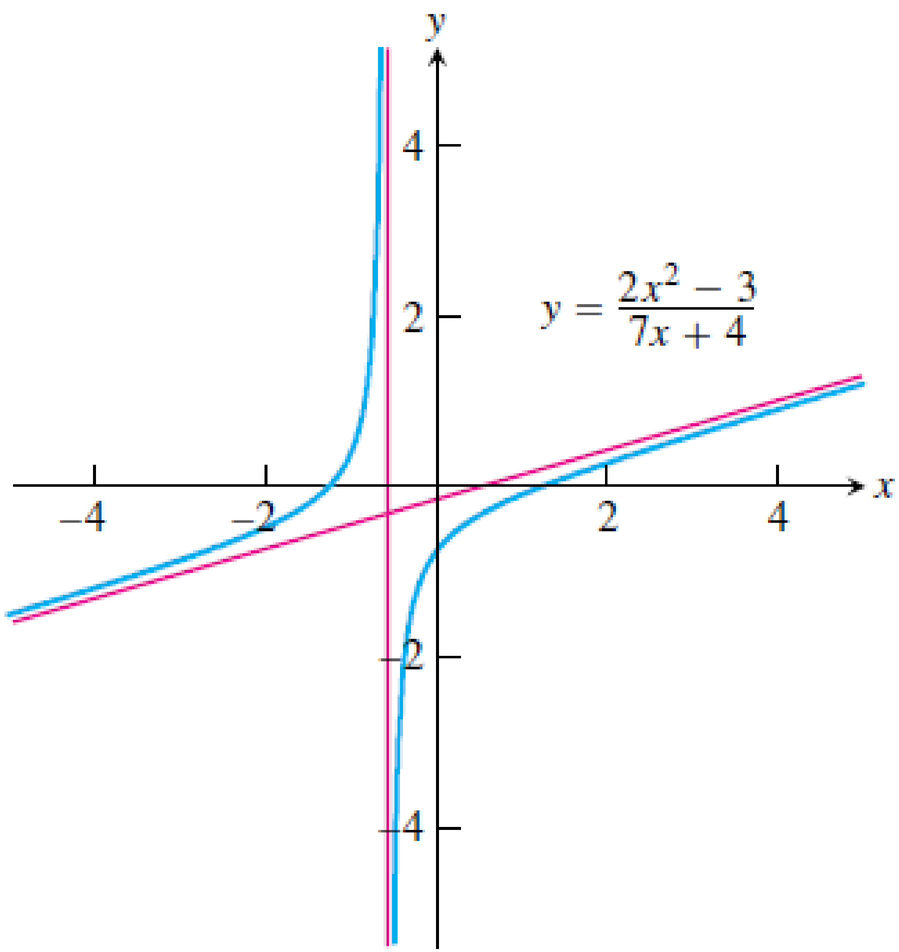


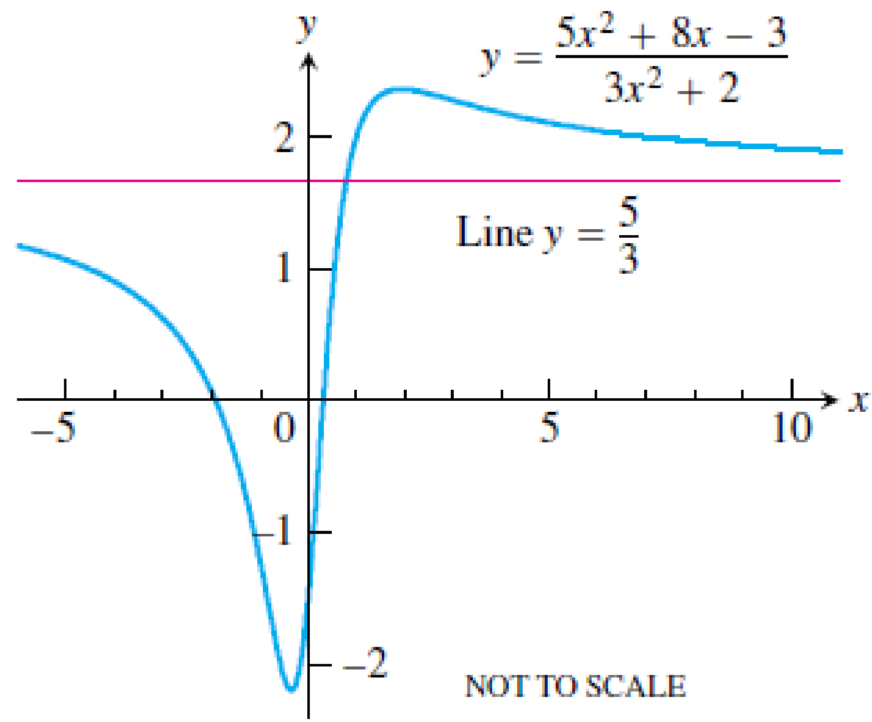
Graphs of three polynomial functions.


y
$y = (x - 2)^4(x + 1)^3(x - 1)$
16
−1
0
1
2
x
−16

Rational Functions A **rational function** is a quotient or ratio of two polynomials:

$$f(x) = \frac{p(x)}{q(x)}$$



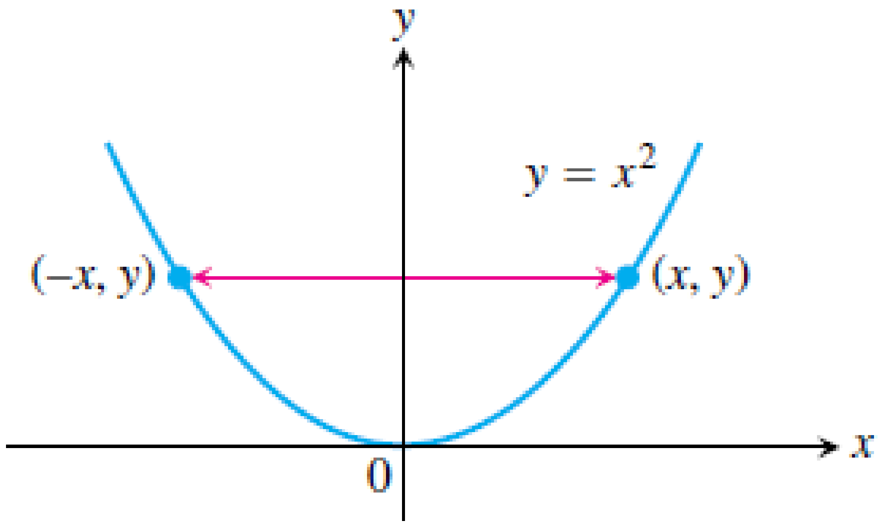


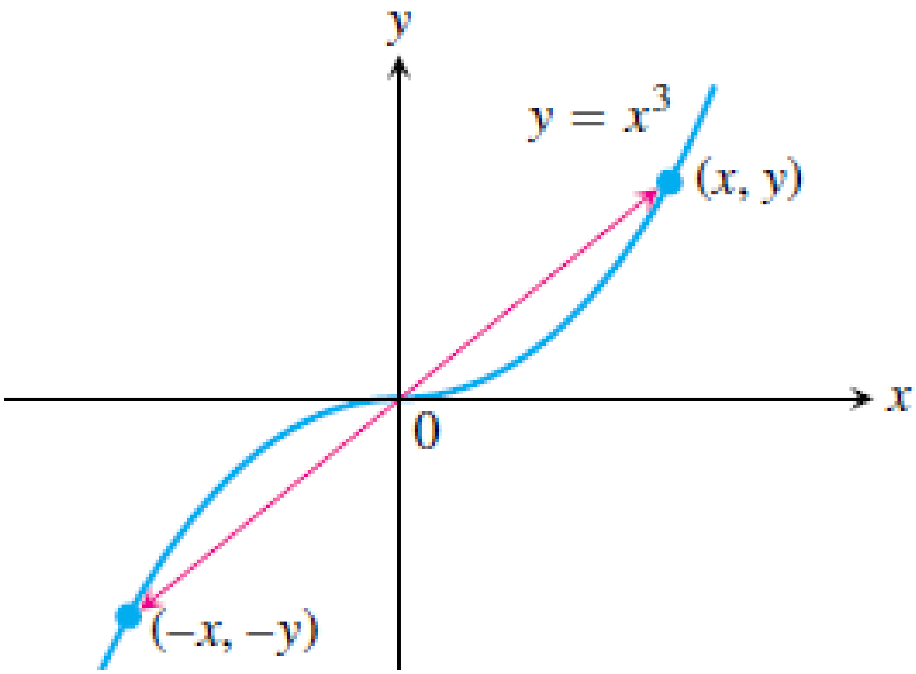
DEFINITIONS Even Function, Odd Function

A function $y = f(x)$ is an

$$\textbf{even function of } x \quad \text{if } f(-x) = f(x),$$
$$\textbf{odd function of } x \quad \text{if } f(-x) = -f(x),$$

for every x in the function's domain.



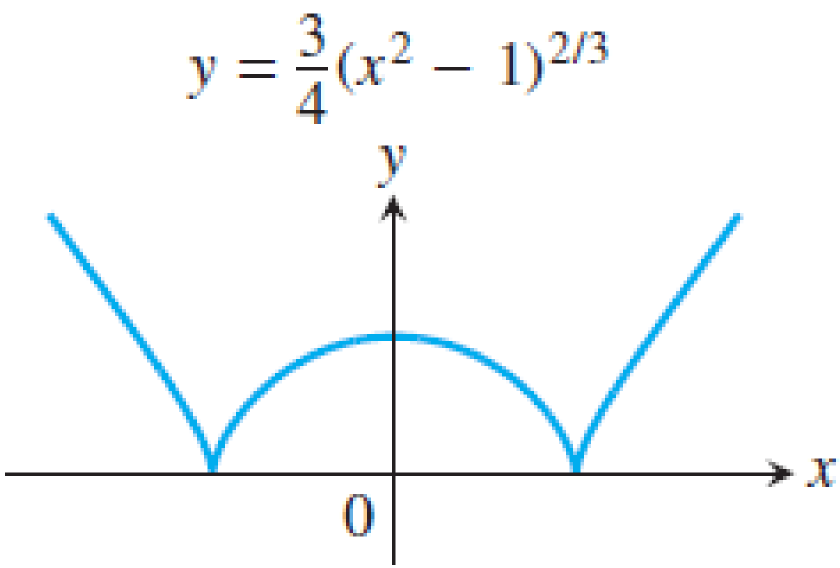


Algebraic Functions An **algebraic function** is a function constructed from polynomials using algebraic operations (addition, subtraction, multiplication, division, and taking roots). Rational functions are special cases of algebraic functions.

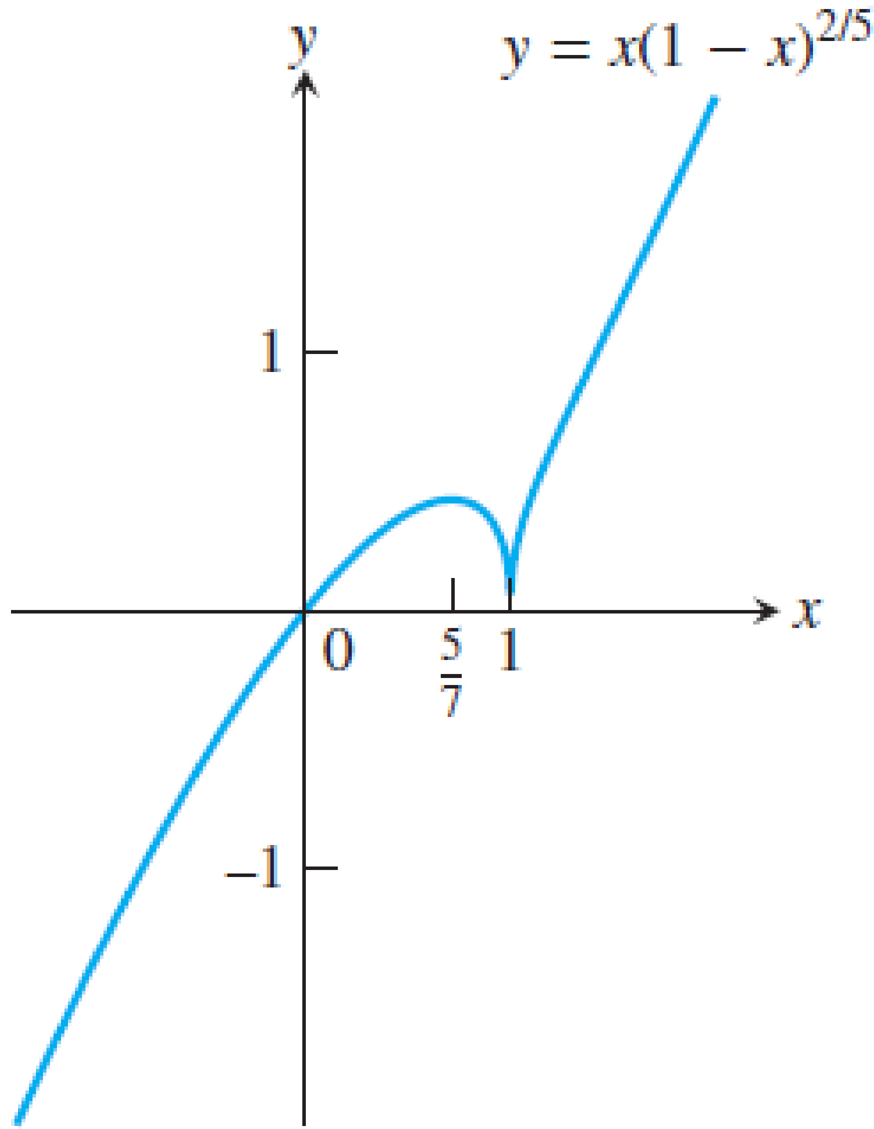
Trigonometric Functions

Exponential Functions Functions of the form $f(x) = a^x$, where the base $a > 0$ is a positive constant and $a \neq 1$, are called **exponential functions**. All exponential functions have domain $(-\infty, \infty)$ and range $(0, \infty)$. So an exponential function never assumes the value 0.

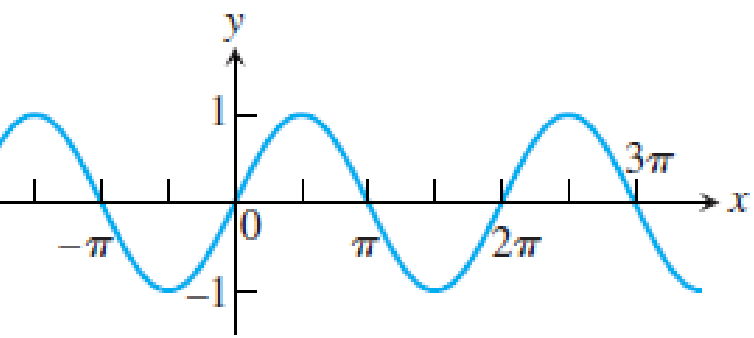
Logarithmic Functions These are the functions $f(x) = \log_a x$, where the base $a \neq 1$ is a positive constant. They are the *inverse functions* of the exponential functions,



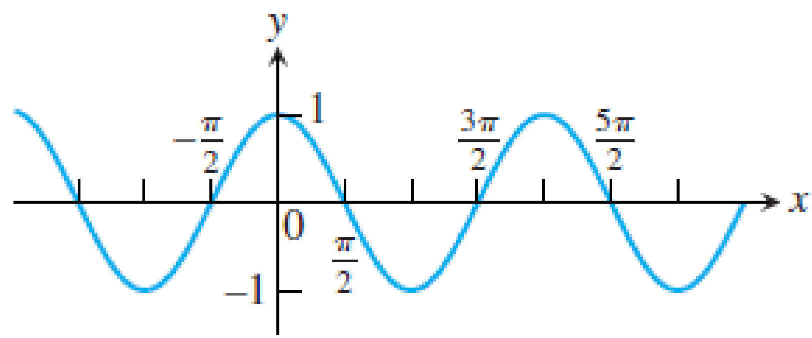
$y = \frac{3}{4}(x^2 - 1)^{2/3}$
y
x
0



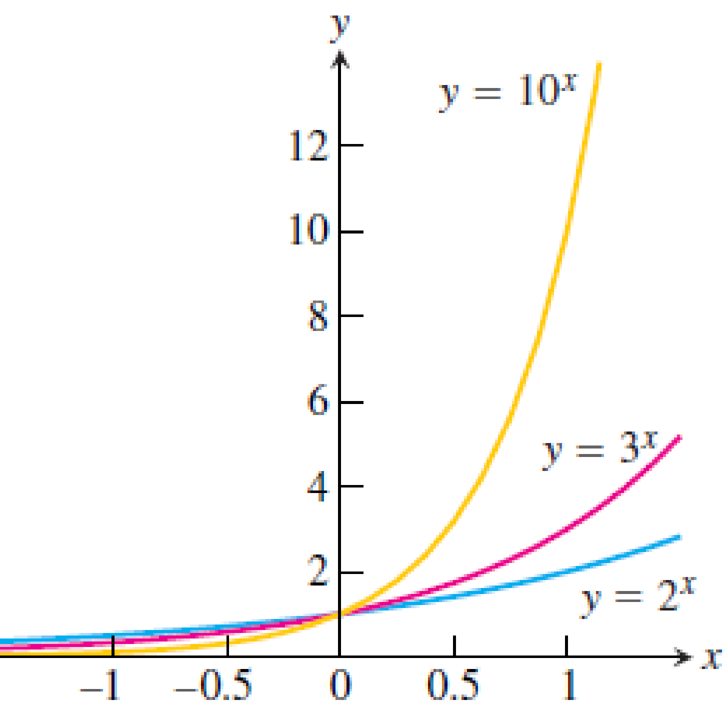
y
$y = x(1 - x)^{2/5}$
1
x
0
$\frac{5}{7}$
1
−1



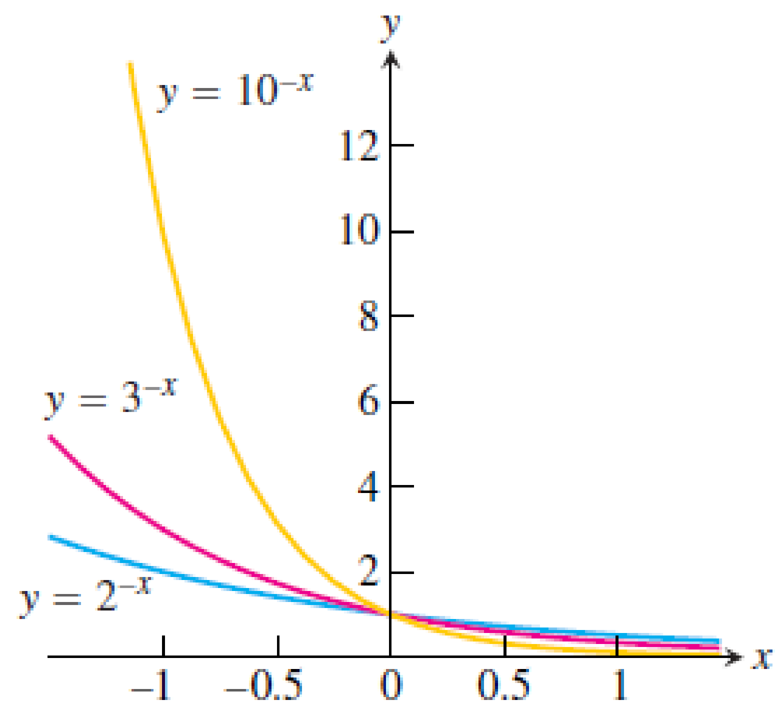
(a) $f(x) = \sin x$



(b) $f(x) = \cos x$



(a) $y = 2^x, y = 3^x, y = 10^x$



(b) $y = 2^{-x}, y = 3^{-x}, y = 10^{-x}$

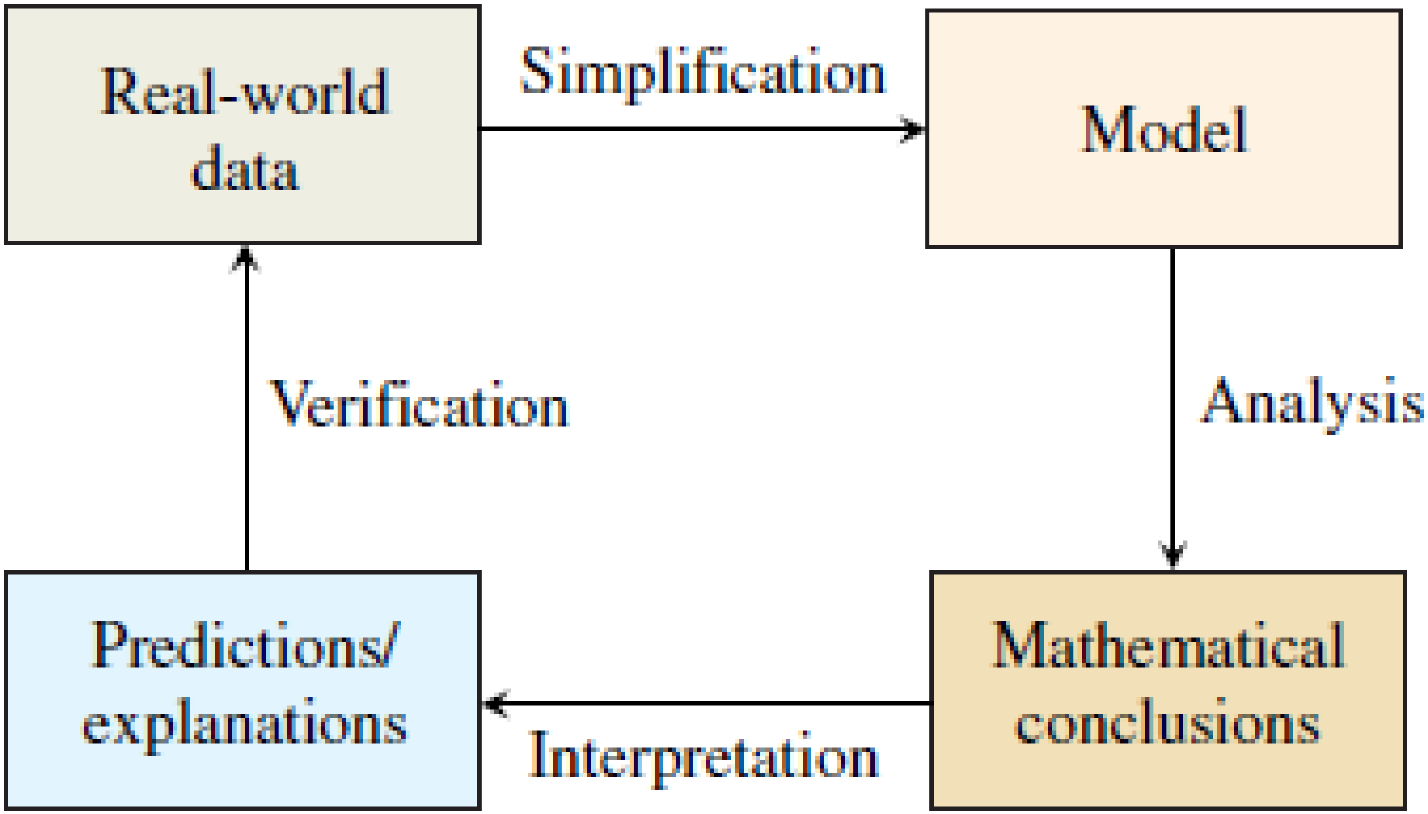
Transcendental Functions

These are functions that are not algebraic. İnclude;

- trigonometric,
- inverse trigonometric,
- exponential,
- logarithmic functions,
- hyperbolic functions,
- and many other

Mathematical Models

To help us better understand our world, we often describe a particular phenomenon mathematically (by means of a function or an equation, for instance). Such a **mathematical model** is an idealization of the real-world phenomenon and is seldom a completely accurate representation. Although any model has its limitations, a good one can provide valuable results and conclusions.



A flow of the modeling process beginning with an examination of real-world data.

Combining Functions; Shifting and Scaling Graphs

Combining Functions Algebraically
Sums, Differences, Products, and Quotients

$$(f + g)(x) = f(x) + g(x).$$

$$(f - g)(x) = f(x) - g(x).$$

$$(fg)(x) = f(x)g(x).$$

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} \qquad (\text{where } g(x) \neq 0).$$

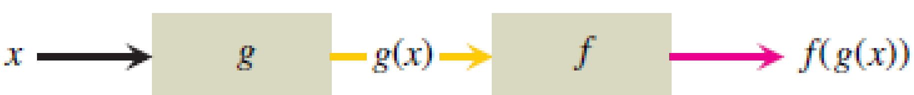
$$(cf)(x) = cf(x)$$

DEFINITION Composition of Functions

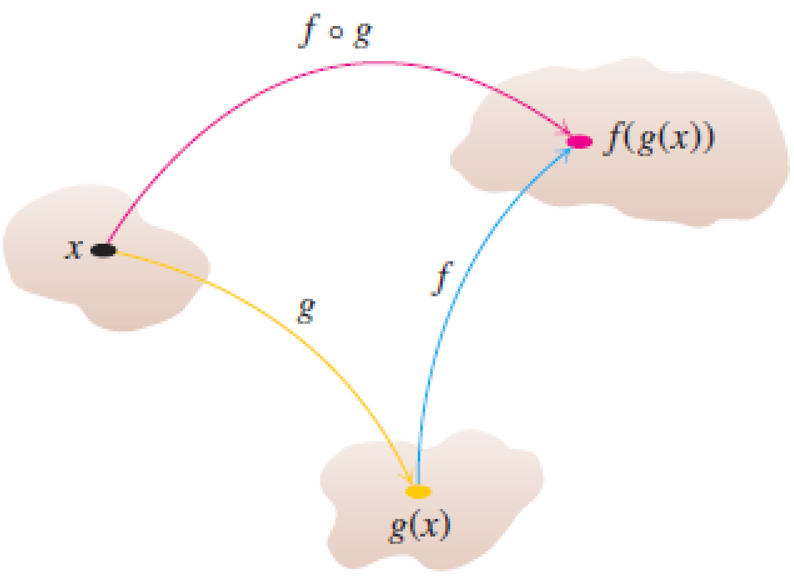
If f and g are functions, the **composite** function $f \circ g$ ("f composed with g") is defined by

$$(f \circ g)(x) = f(g(x)).$$

The domain of $f \circ g$ consists of the numbers x in the domain of g for which $g(x)$ lies in the domain of f.



Two functions can be composed at x whenever the value of one function at x lies in the domain of the other. The composite is denoted by $f \circ g$.



Arrow diagram for $f \circ g$.

EXAMPLE Finding Formulas for Composites

If $f(x) = \sqrt{x}$ and $g(x) = x + 1$, find

(a) $(f \circ g)(x)$ **(b)** $(g \circ f)(x)$ **(c)** $(f \circ f)(x)$ **(d)** $(g \circ g)(x)$.

Solution

	Composite	**Domain**
(a)	$(f \circ g)(x) = f(g(x)) = \sqrt{g(x)} = \sqrt{x + 1}$	$[-1, \infty)$
(b)	$(g \circ f)(x) = g(f(x)) = f(x) + 1 = \sqrt{x} + 1$	$[0, \infty)$
(c)	$(f \circ f)(x) = f(f(x)) = \sqrt{f(x)} = \sqrt{\sqrt{x}} = x^{1/4}$	$[0, \infty)$
(d)	$(g \circ g)(x) = g(g(x)) = g(x) + 1 = (x + 1) + 1 = x + 2$	$(-\infty, \infty)$

To see why the domain of $f \circ g$ is $[-1, \infty)$, notice that $g(x) = x + 1$ is defined for all real x but belongs to the domain of f only if $x + 1 \geq 0$, that is to say, when $x \geq -1$. ■

Notice that if $f(x) = x^2$ and $g(x) = \sqrt{x}$, then $(f \circ g)(x) = \left(\sqrt{x}\right)^2 = x$. However, the domain of $f \circ g$ is $[0, \infty)$, not $(-\infty, \infty)$.

Shifting a Graph of a Function

Shift Formulas

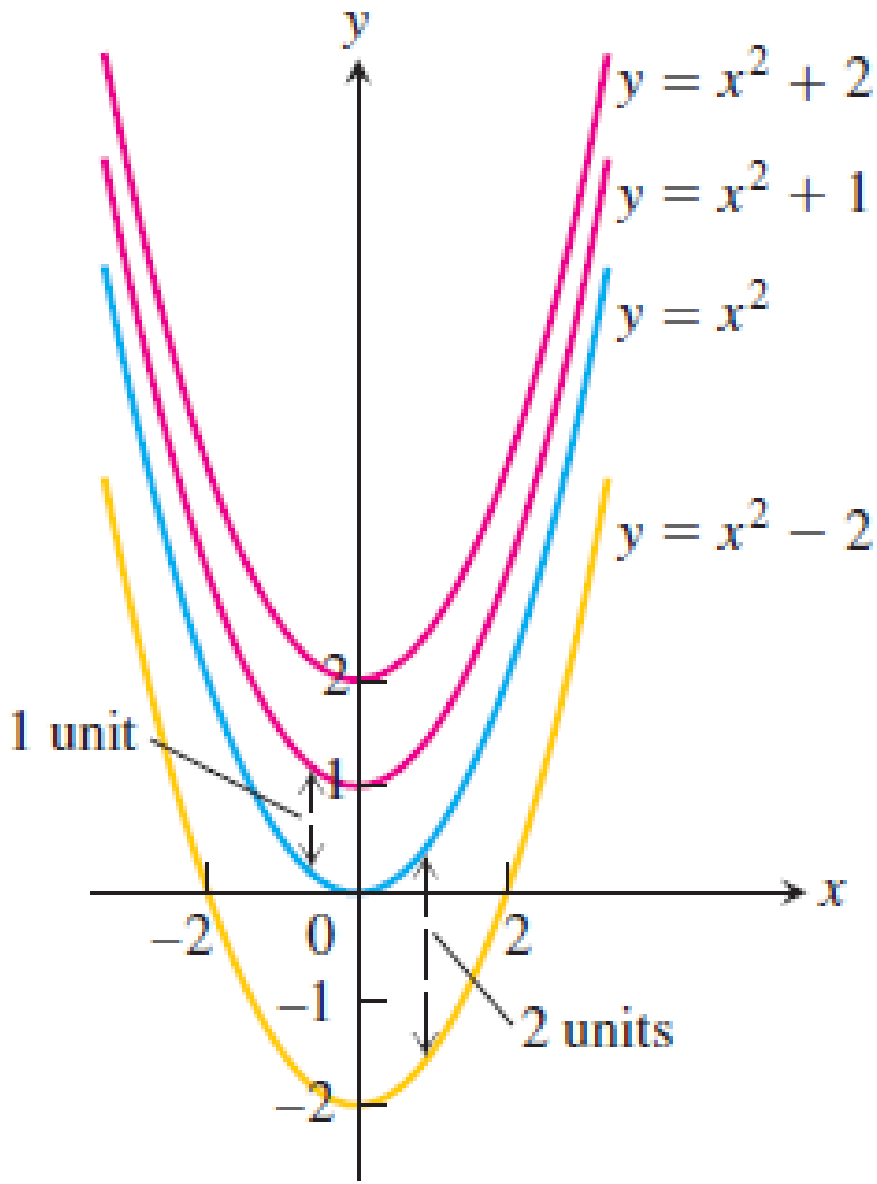
Vertical Shifts

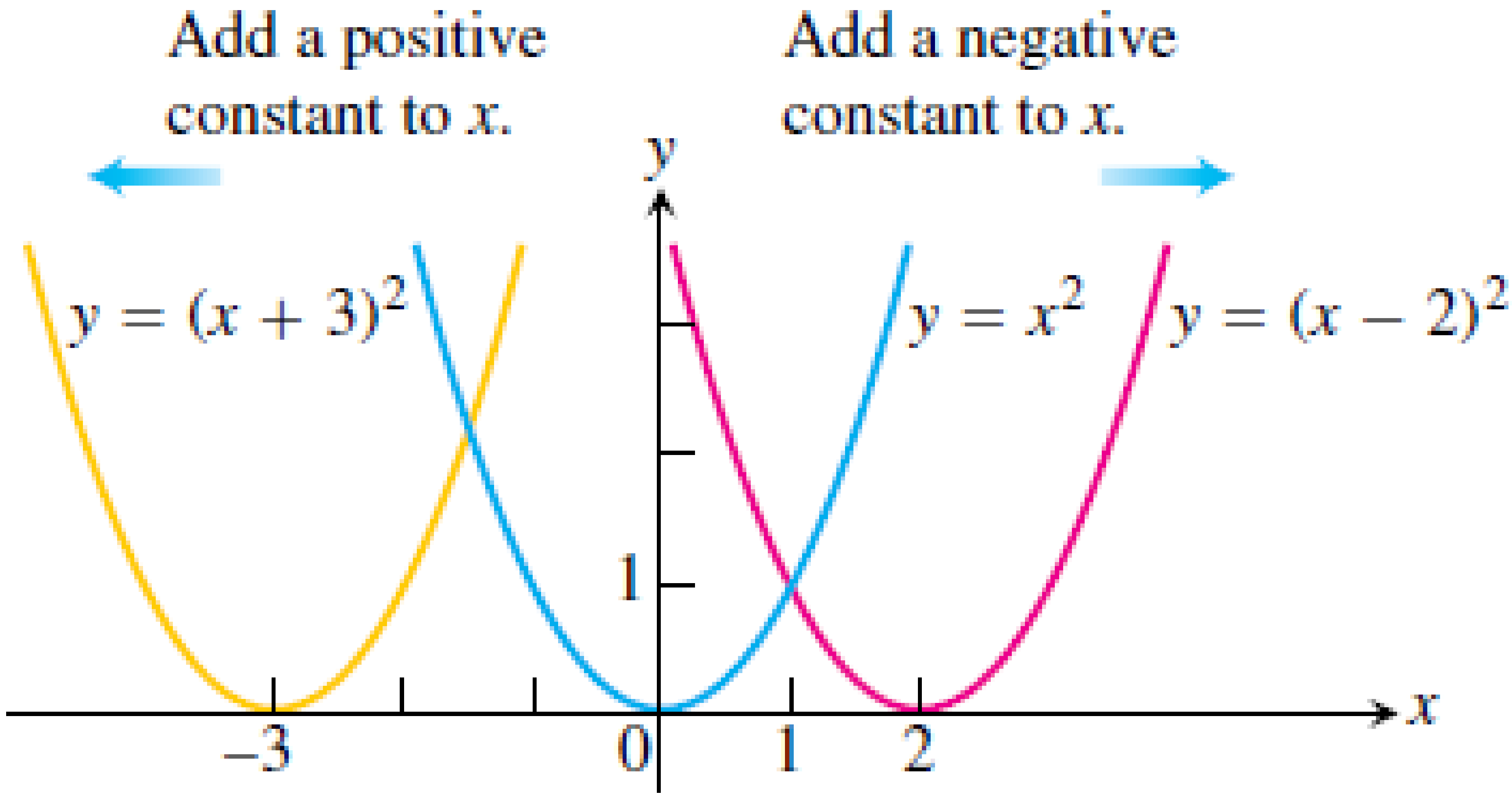
$y = f(x) + k$	Shifts the graph of f *up* k units if $k > 0$
	Shifts it *down* $\lvert k \rvert$ units if $k < 0$

Horizontal Shifts

$y = f(x + h)$	Shifts the graph of f *left* h units if $h > 0$
	Shifts it *right* $\lvert h \rvert$ units if $h < 0$

Shifting a Graph of a Function





Add a positive
constant to x.
Add a negative
constant to x.
y
$y = (x + 3)^2$
$y = x^2$
$y = (x - 2)^2$
1
x
−3
0
1
2

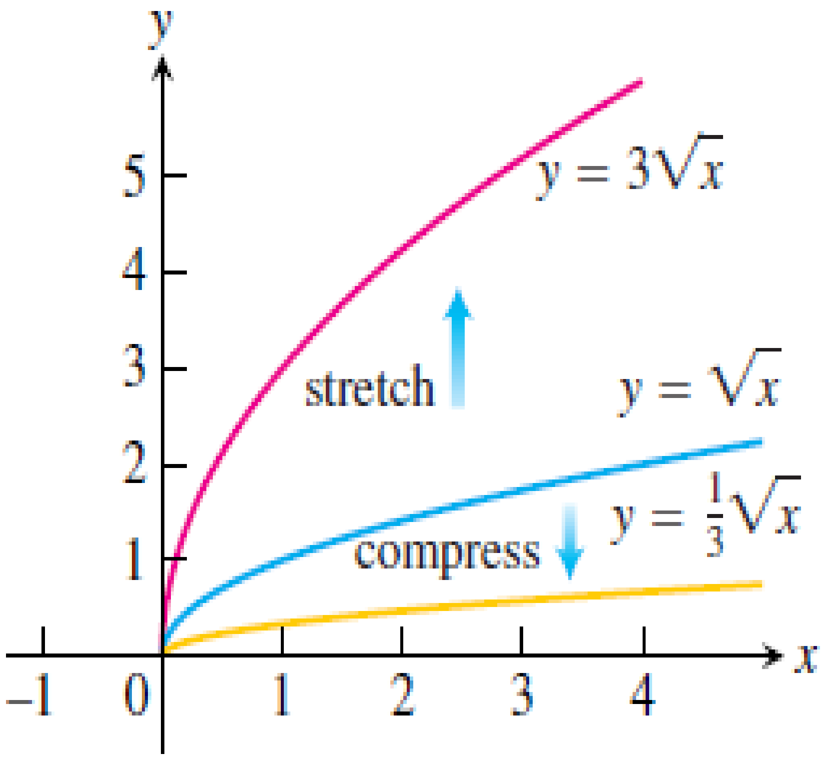
Vertical and Horizontal Scaling and Reflecting Formulas

For $c > 1$,

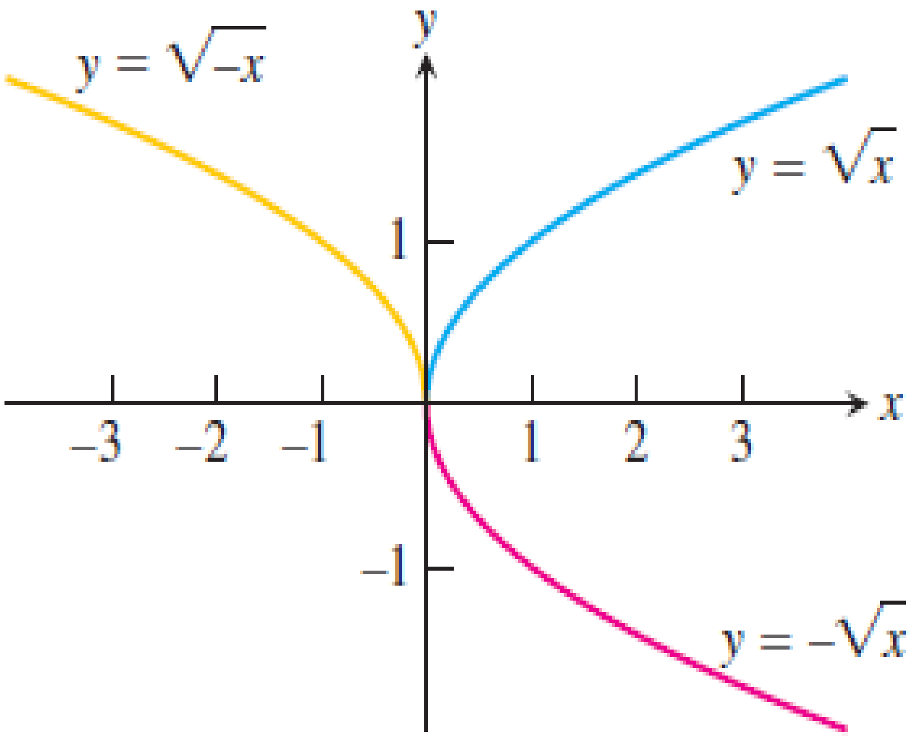
$y = cf(x)$	Stretches the graph of f vertically by a factor of c.
$y = \frac{1}{c}f(x)$	Compresses the graph of f vertically by a factor of c.
$y = f(cx)$	Compresses the graph of f horizontally by a factor of c.
$y = f(x/c)$	Stretches the graph of f horizontally by a factor of c.

For $c = -1$,

$y = -f(x)$	Reflects the graph of f across the x-axis.
$y = f(-x)$	Reflects the graph of f across the y-axis.



y
x
5
4
3
2
1
–1
0
1
2
3
4
$y = 3\sqrt{x}$
stretch
$y = \sqrt{x}$
$y = \frac{1}{3}\sqrt{x}$
compress



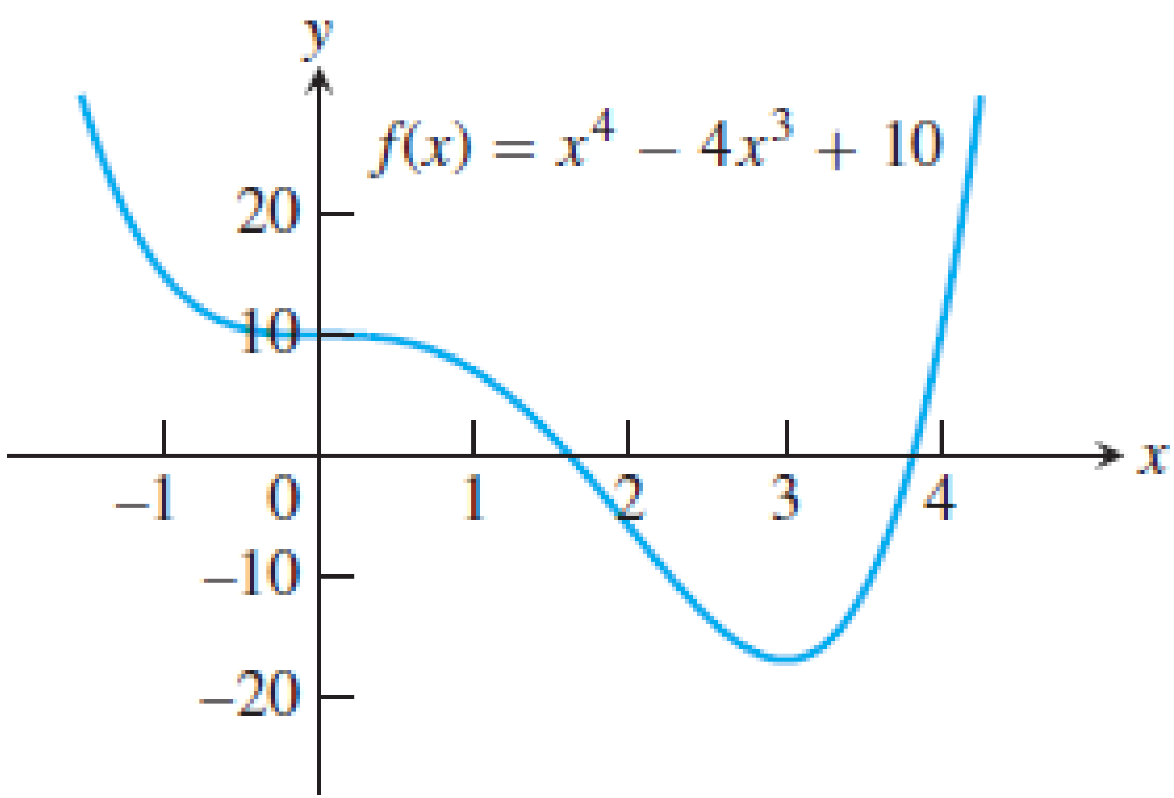
y
x
$y = \sqrt{-x}$
$y = \sqrt{x}$
$y = -\sqrt{x}$
1
–1
–3
–2
–1
1
2
3

Combining Scalings and Reflections

Given the function $f(x) = x^4 - 4x^3 + 10$ find formulas to

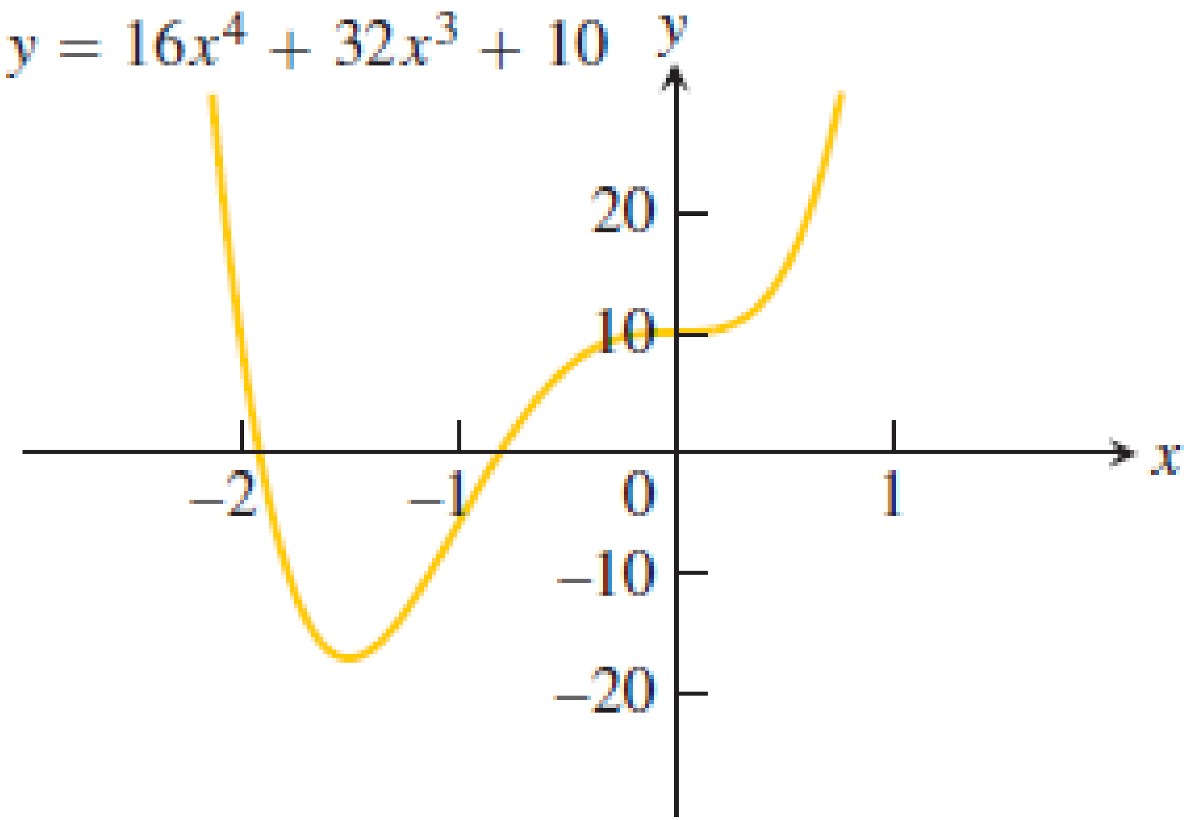
(a) compress the graph horizontally by a factor of 2 followed by a reflection across the y-axis (Figure 1.60b).

(b) compress the graph vertically by a factor of 2 followed by a reflection across the x-axis (Figure 1.60c).



(a) The formula is obtained by substituting $-2x$ for x in the right-hand side of the equation for f

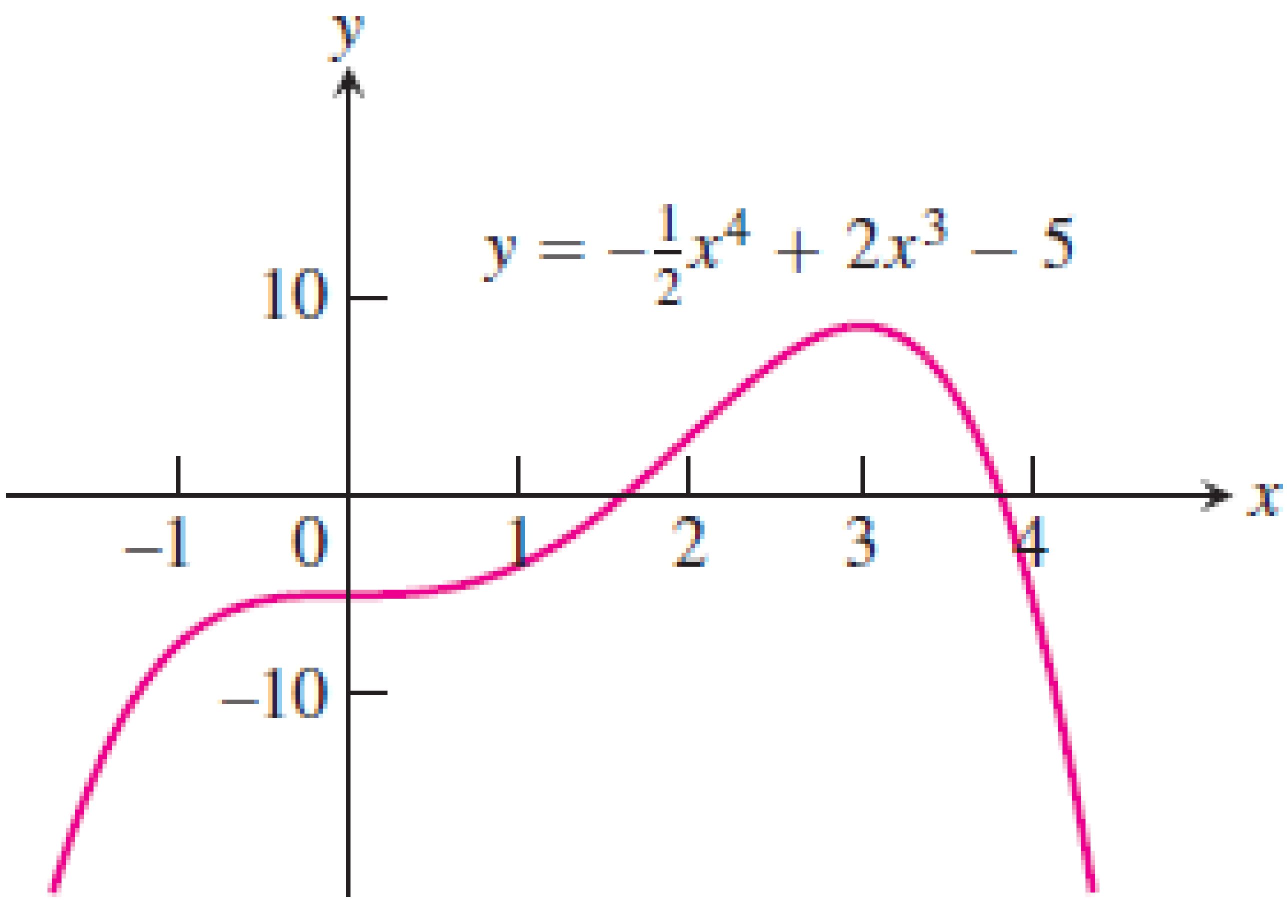
$$y = f(-2x) = (-2x)^4 - 4(-2x)^3 + 10$$
$$= 16x^4 + 32x^3 + 10.$$



(b)

(b) The formula is

$$y = -\frac{1}{2}f(x) = -\frac{1}{2}x^4 + 2x^3 - 5.$$

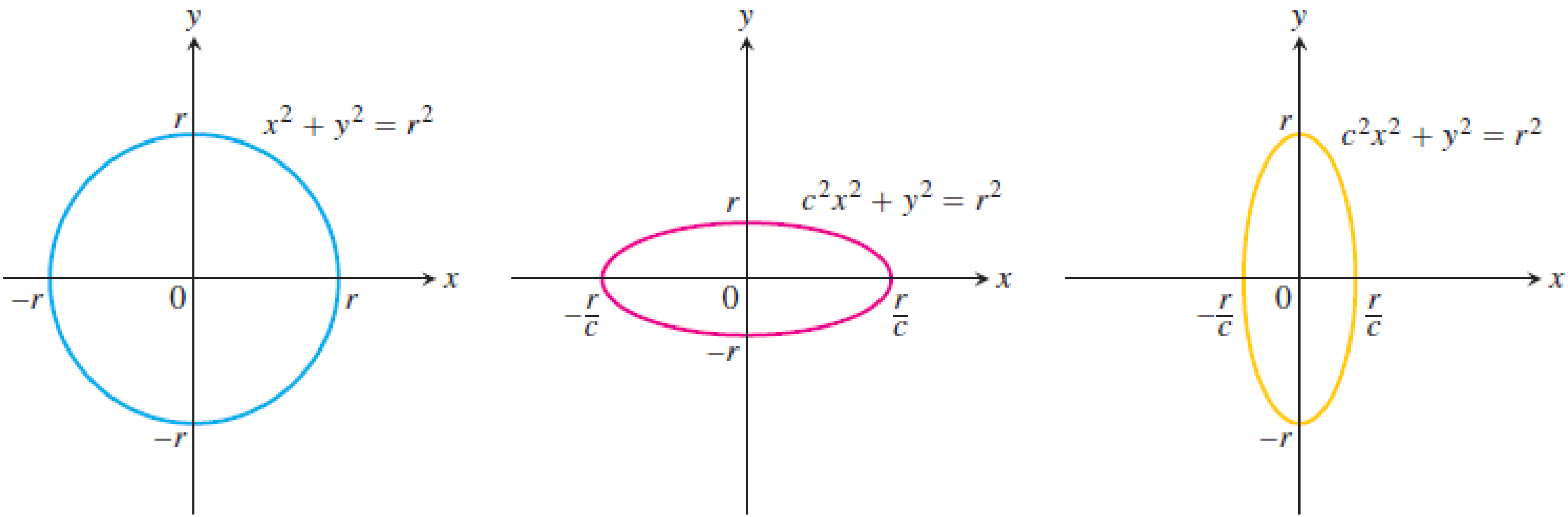


(c)

Ellipses

Substituting cx for x in the standard equation for a circle of radius r (
gives

$$c^2x^2 + y^2 = r^2.$$



(a) circle (b) ellipse, $0 < c < 1$ (c) ellipse, $c > 1$

If we divide both sides of Equation (1) by r^2, we obtain

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$$

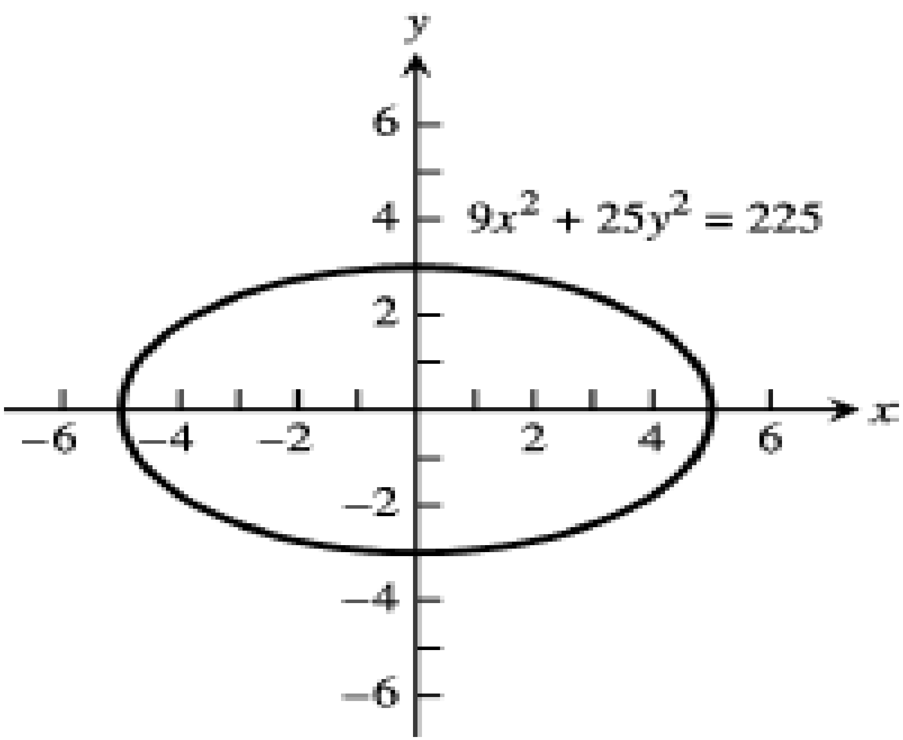
$$\frac{(x - h)^2}{a^2} + \frac{(y - k)^2}{b^2} = 1. \tag{3}$$

Equation (3) is the **standard equation of an ellipse** with center at (h, k). The geometric definition and properties of ellipses are reviewed in Section 10.1.

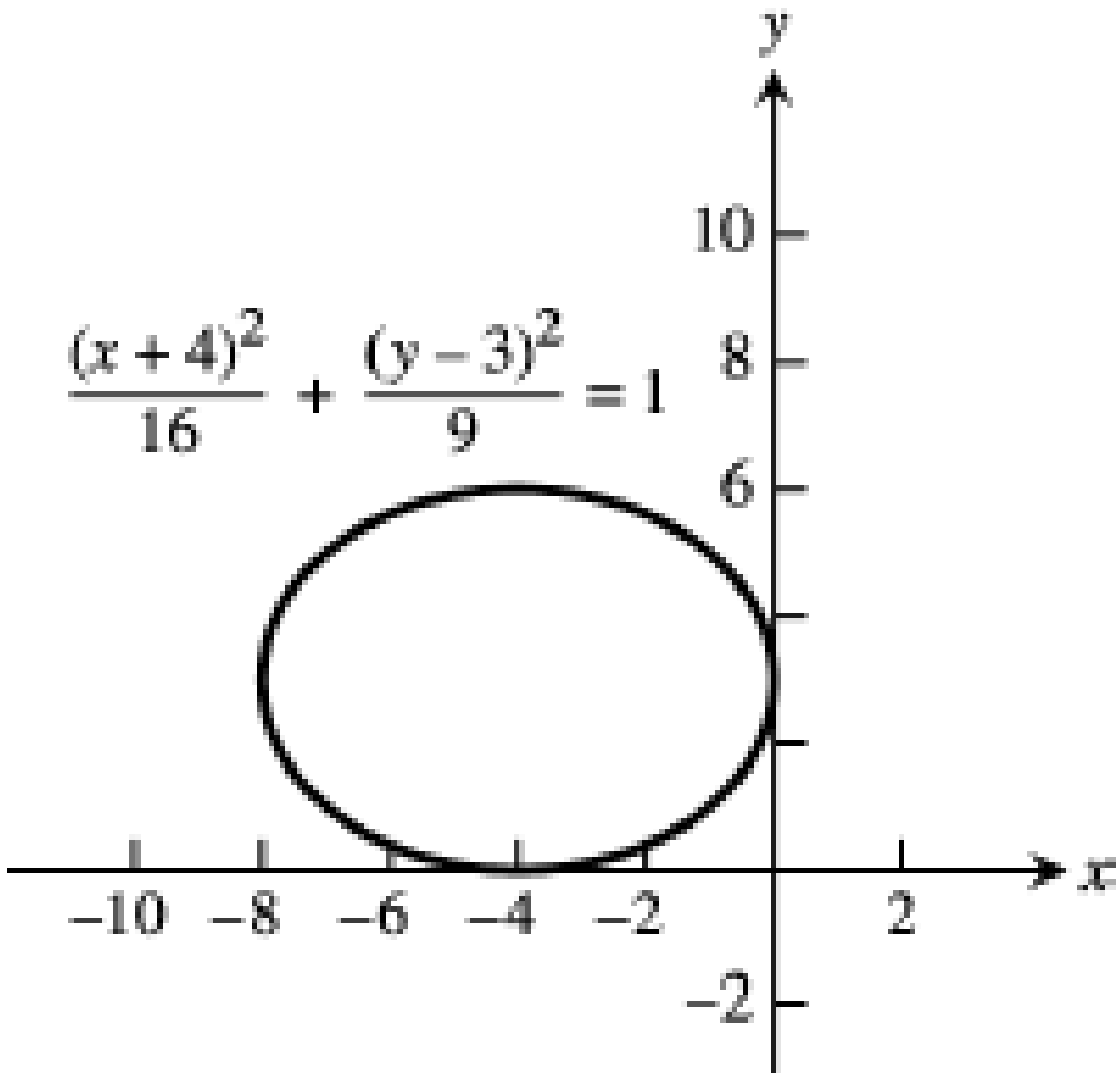
Put the equation in standard form and sketch the ellipse.

$$9x^2 + 25y^2 = 225$$

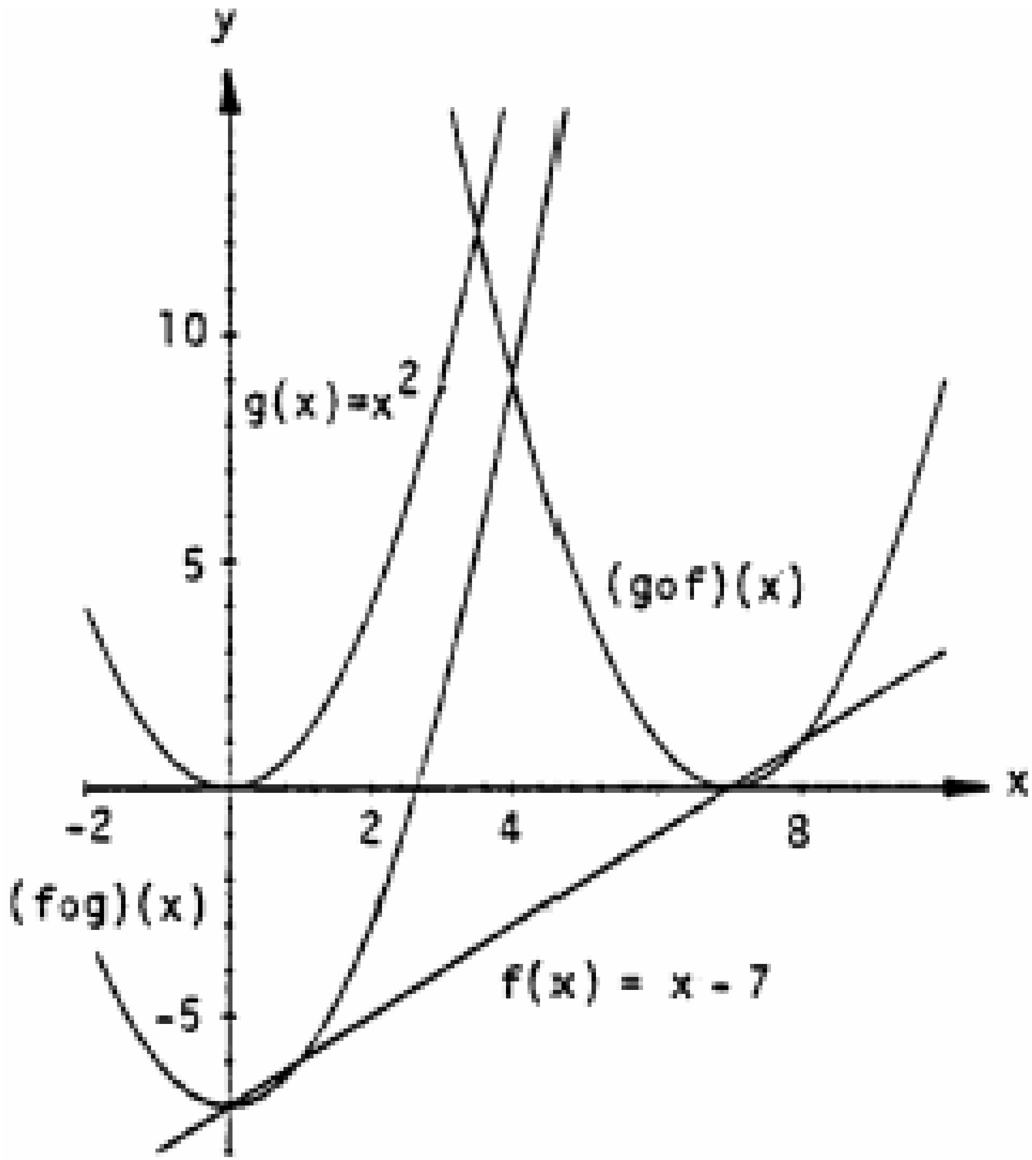
. $9x^2 + 25y^2 = 225 \Rightarrow \frac{x^2}{5^2} + \frac{y^2}{3^2} = 1$



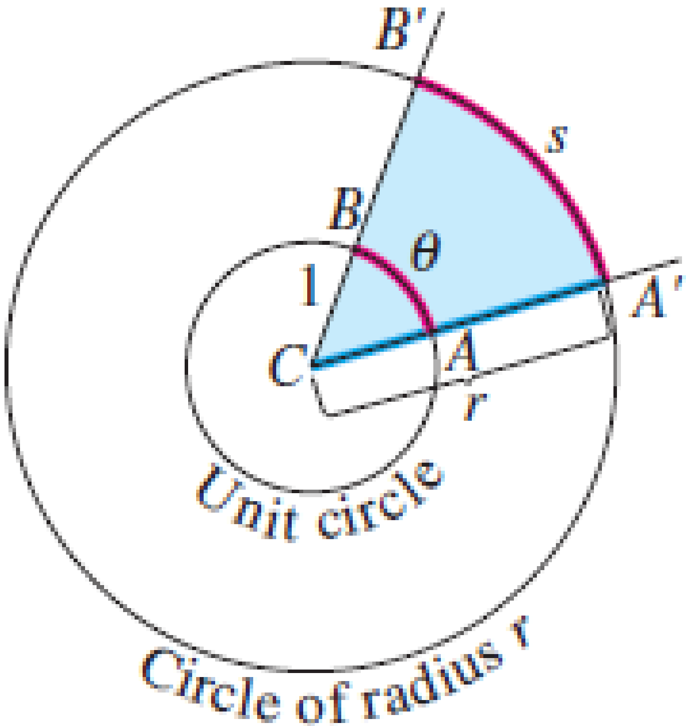
Write an equation for the ellipse $(x^2/16) + (y^2/9) = 1$ shifted 4 units to the left and 3 units up. Sketch the ellipse and identify its center and major axis.



Let $f(x) = x - 7$ and $g(x) = x^2$. Graph f and g together with $f \circ g$ and $g \circ f$.



Trigonometric Functions



$$\pi \text{ radians } = 180°.$$

45° in radian measure is

$$45 \cdot \frac{\pi}{180} = \frac{\pi}{4} \text{ rad},$$

and $\pi/6$ radians is

$$\frac{\pi}{6} \cdot \frac{180}{\pi} = 30°.$$

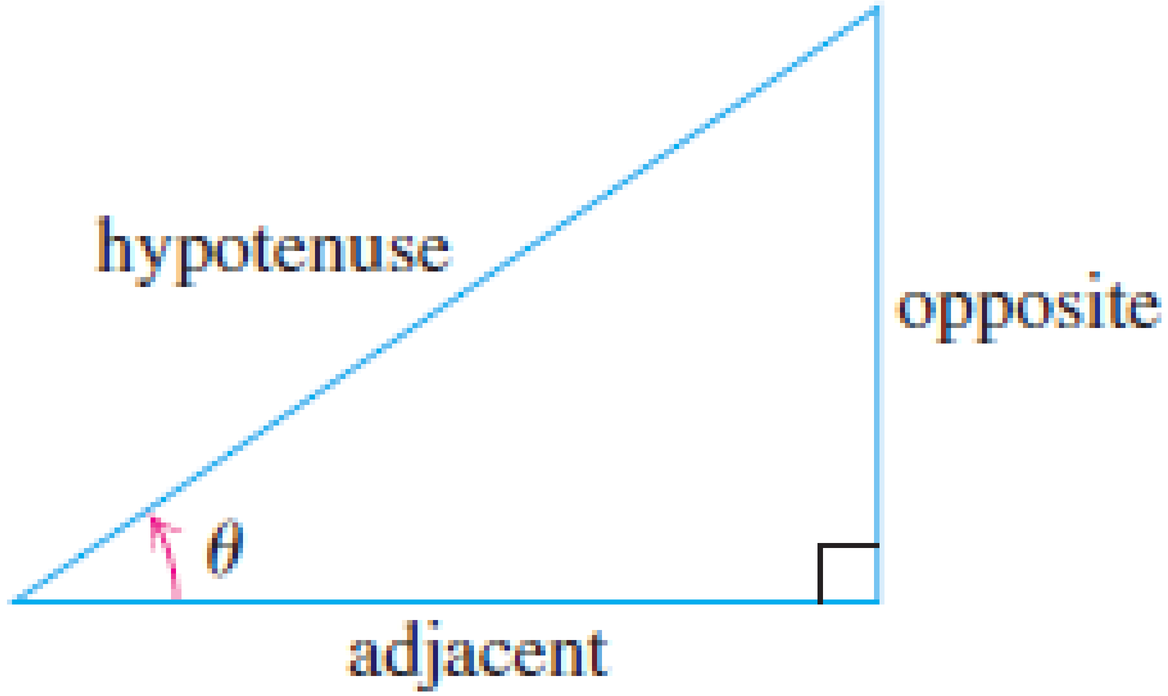
Conversion Formulas

$$1 \text{ degree} = \frac{\pi}{180} (\approx 0.02) \text{ radians}$$

$$\text{Degrees to radians: multiply by } \frac{\pi}{180}$$

$$1 \text{ radian} = \frac{180}{\pi} (\approx 57) \text{ degrees}$$

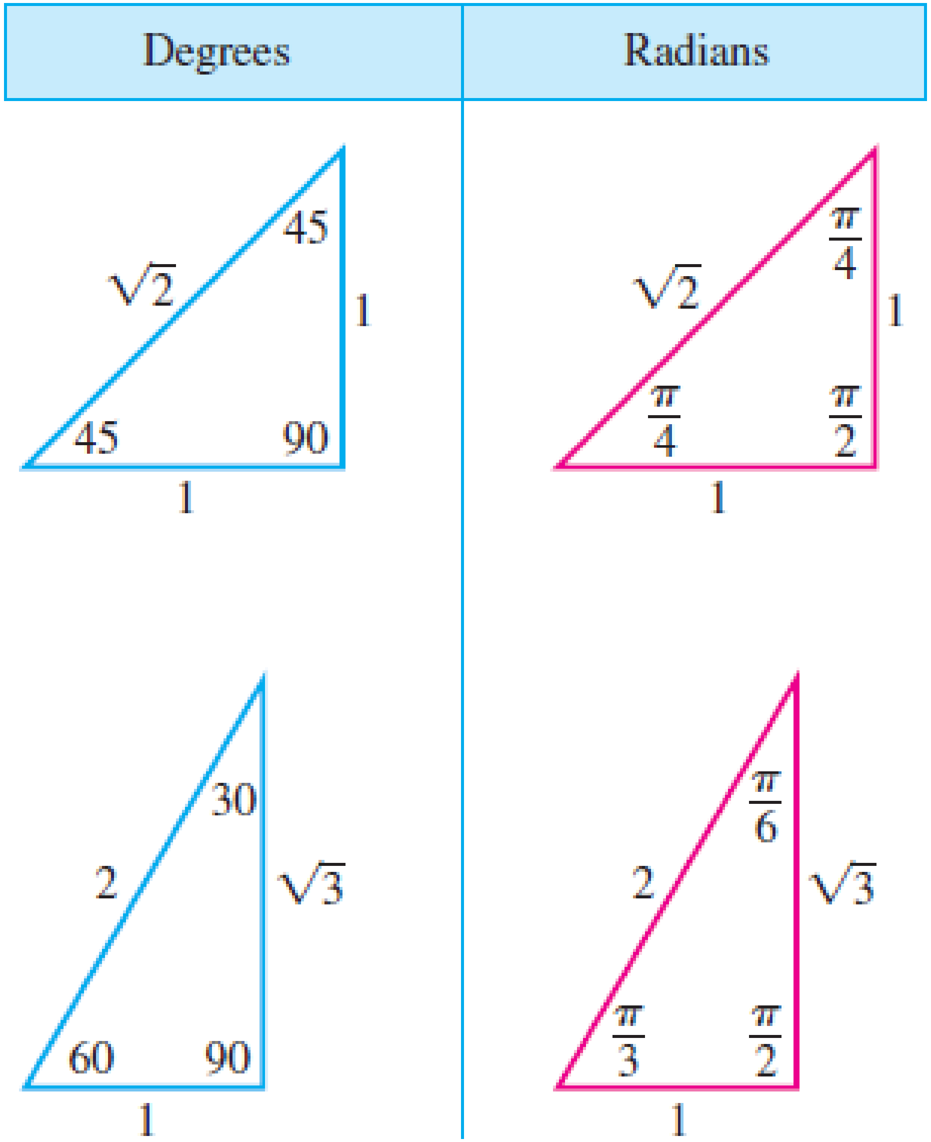
$$\text{Radians to degrees: multiply by } \frac{180}{\pi}$$

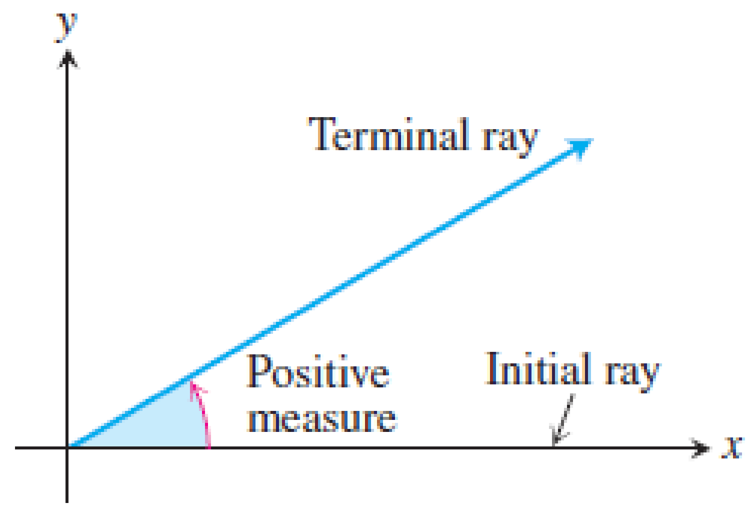


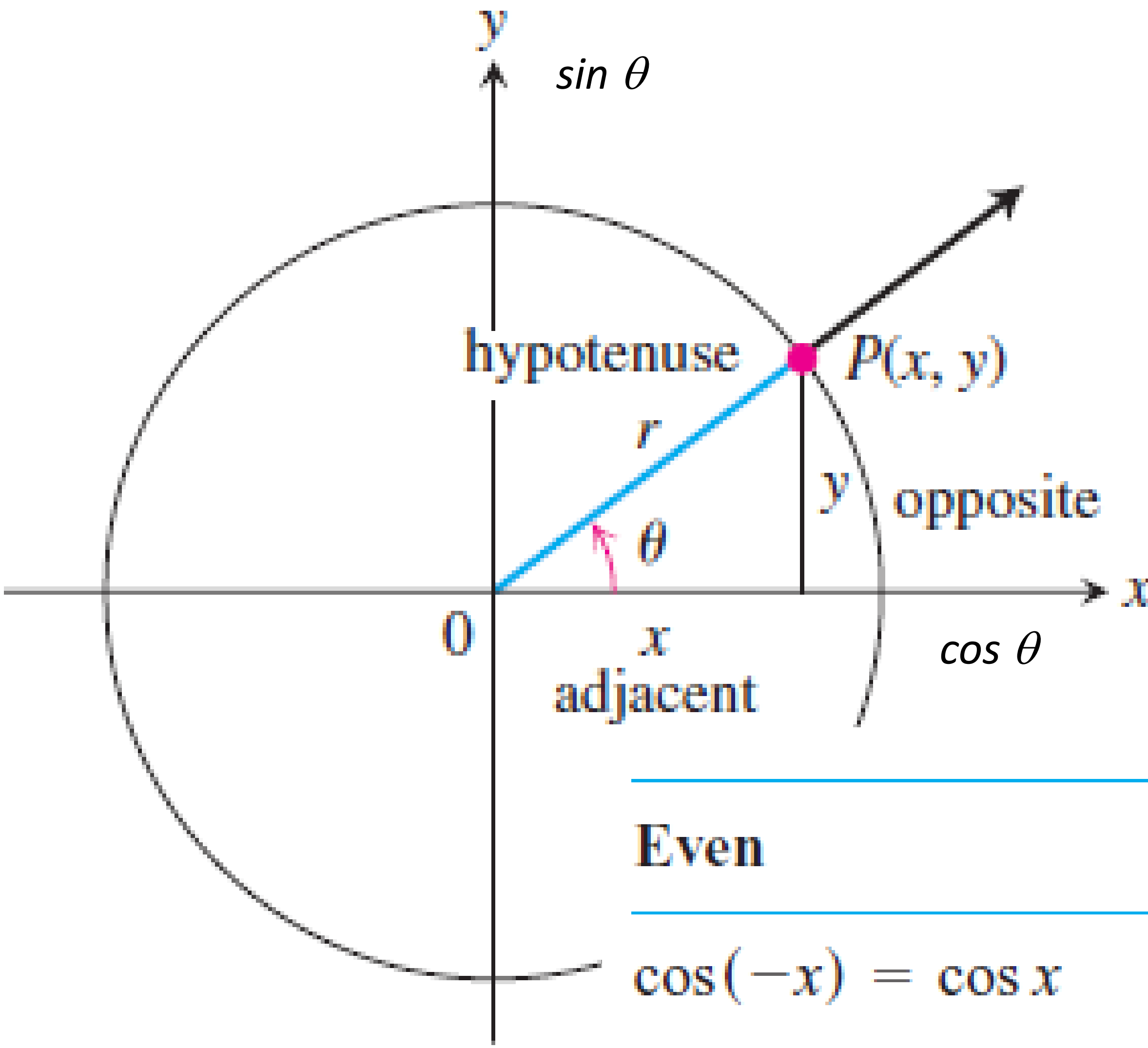
$$\sin\theta = \frac{\text{opp}}{\text{hyp}} \qquad \csc\theta = \frac{\text{hyp}}{\text{opp}}$$

$$\cos\theta = \frac{\text{adj}}{\text{hyp}} \qquad \sec\theta = \frac{\text{hyp}}{\text{adj}}$$

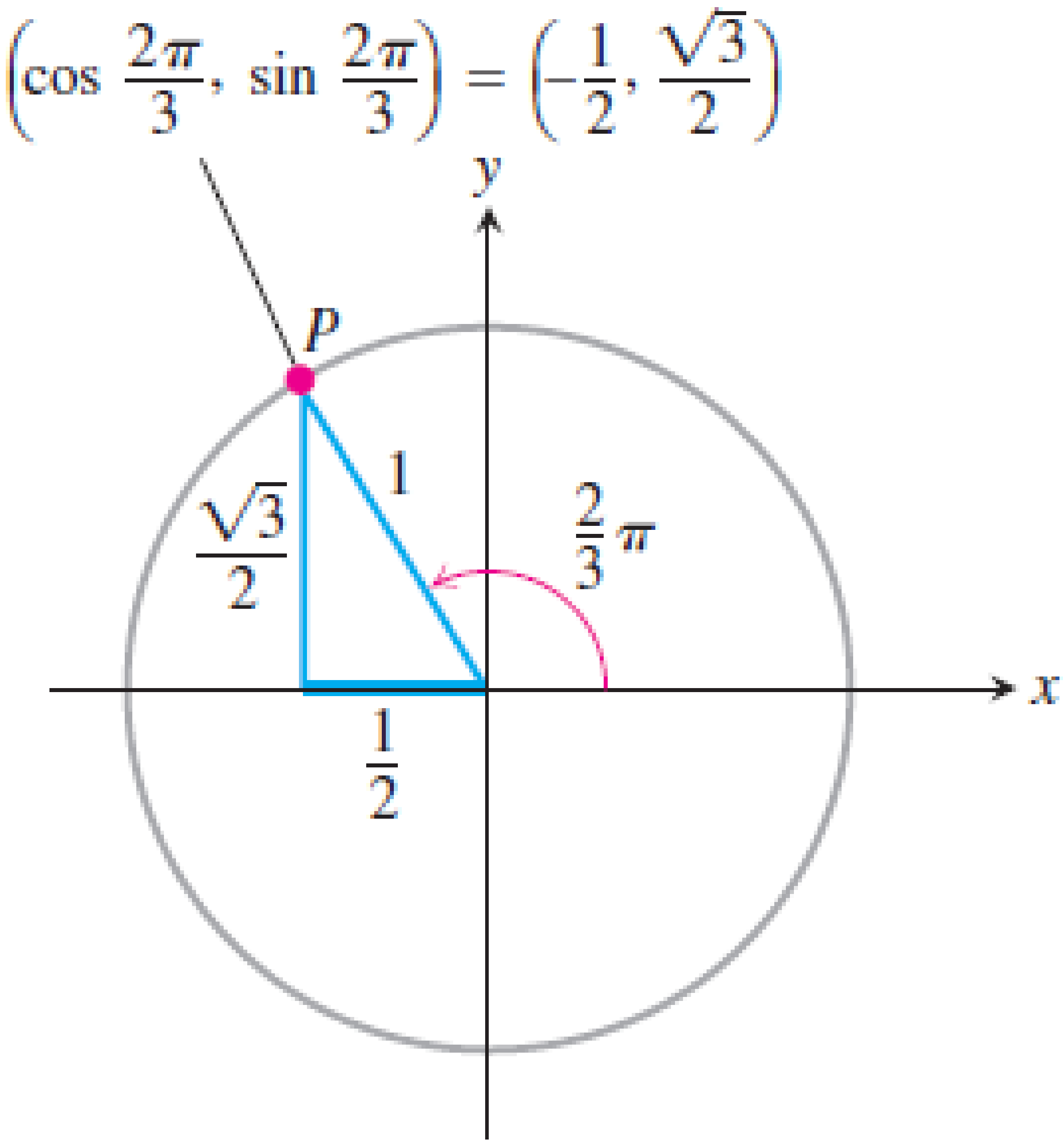
$$\tan\theta = \frac{\text{opp}}{\text{adj}} \qquad \cot\theta = \frac{\text{adj}}{\text{opp}}$$

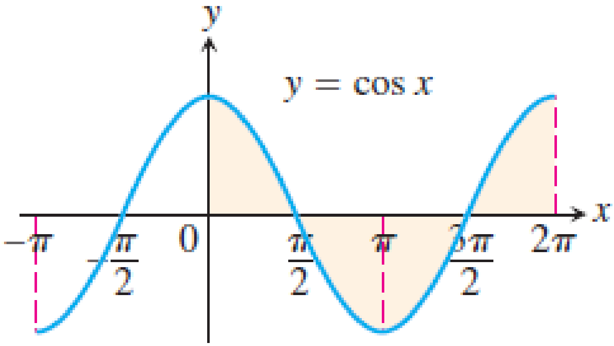

Degrees
Radians
45
$\sqrt{2}$
1
45
90
1
$\frac{\pi}{4}$
$\sqrt{2}$
1
$\frac{\pi}{4}$
$\frac{\pi}{2}$
1
30
2
$\sqrt{3}$
60
90
1
$\frac{\pi}{6}$
2
$\sqrt{3}$
$\frac{\pi}{3}$
$\frac{\pi}{2}$
1


y
Terminal ray
Positive measure
Initial ray
x



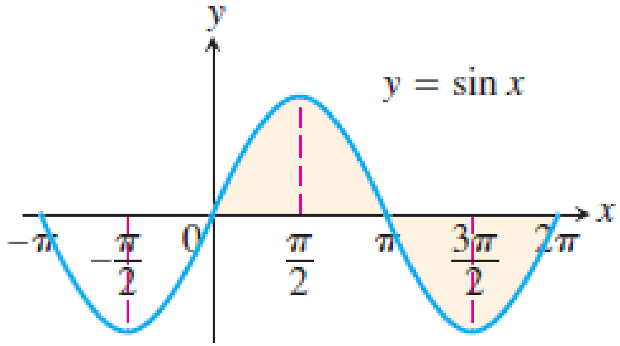
Even	**Odd**
$\cos(-x) = \cos x$	$\sin(-x) = -\sin x$
$\sec(-x) = \sec x$	$\tan(-x) = -\tan x$
	$\csc(-x) = -\csc x$
	$\cot(-x) = -\cot x$


$\left(\cos \frac{2\pi}{3}, \sin \frac{2\pi}{3}\right) = \left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$
y
P
1
$\frac{\sqrt{3}}{2}$
$\frac{2}{3}\pi$
x
$\frac{1}{2}$



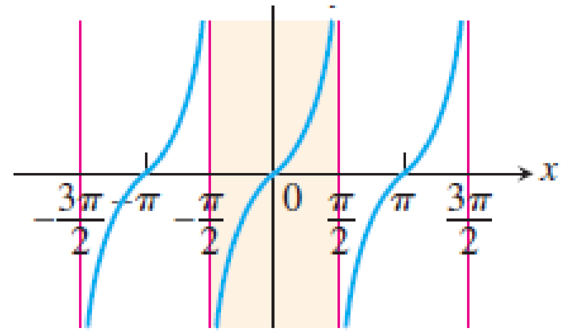
Domain: $-\infty < x < \infty$
Range: $-1 \le y \le 1$
Period: 2π

(a)



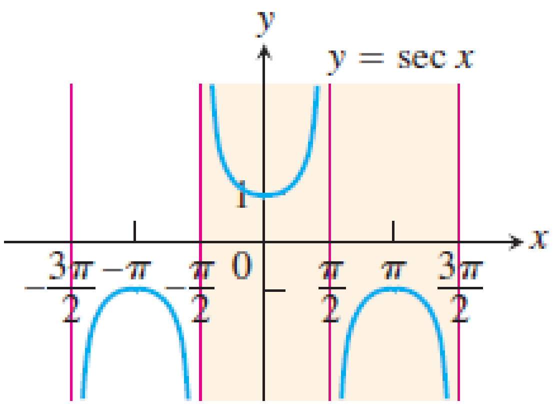
Domain: $-\infty < x < \infty$
Range: $-1 \le y \le 1$
Period: 2π

(b)



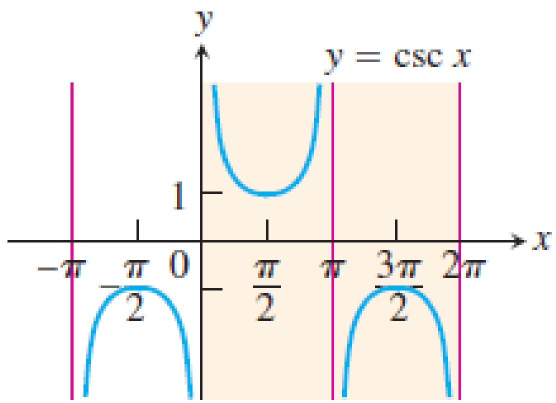
Domain: $x \neq \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \ldots$
Range: $-\infty < y < \infty$
Period: π

(c)



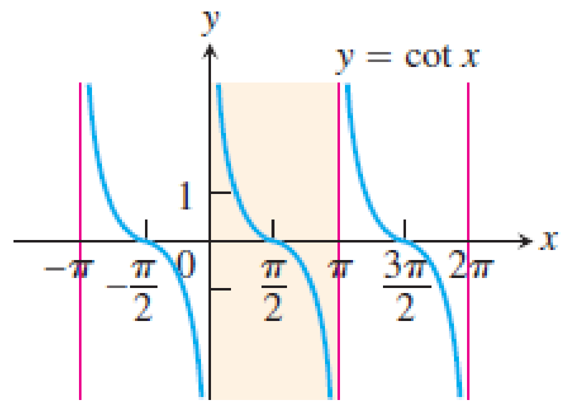
Domain: $x \neq \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \ldots$
Range: $y \le -1$ and $y \ge 1$
Period: 2π

(d)



Domain: $x \neq 0, \pm\pi, \pm 2\pi, \ldots$
Range: $y \le -1$ and $y \ge 1$
Period: 2π

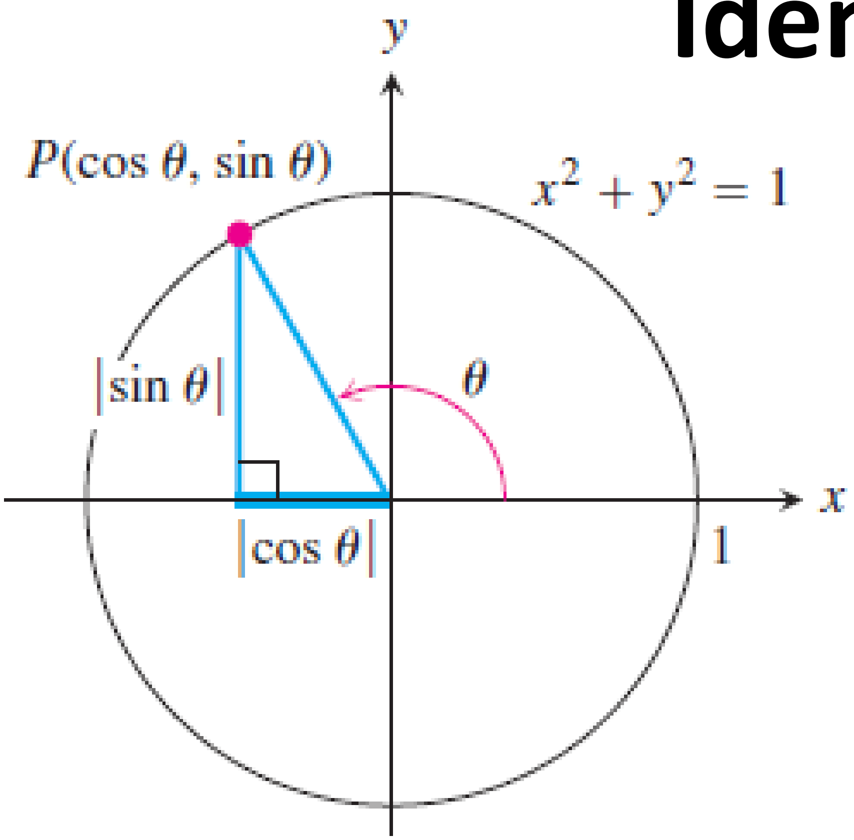
(e)



Domain: $x \neq 0, \pm\pi, \pm 2\pi, \ldots$
Range: $-\infty < y < \infty$
Period: π

(f)

Identities



$$\cos^2\theta + \sin^2\theta = 1$$

$$1 + \tan^2\theta = \sec^2\theta.$$

$$1 + \cot^2\theta = \csc^2\theta.$$

Addition Formulas

$$\cos(A + B) = \cos A \cos B - \sin A \sin B$$

$$\sin(A + B) = \sin A \cos B + \cos A \sin B$$

Double-Angle Formulas

$$\cos 2\theta = \cos^2 \theta - \sin^2 \theta$$

$$\sin 2\theta = 2 \sin \theta \cos \theta$$

Half-Angle Formulas

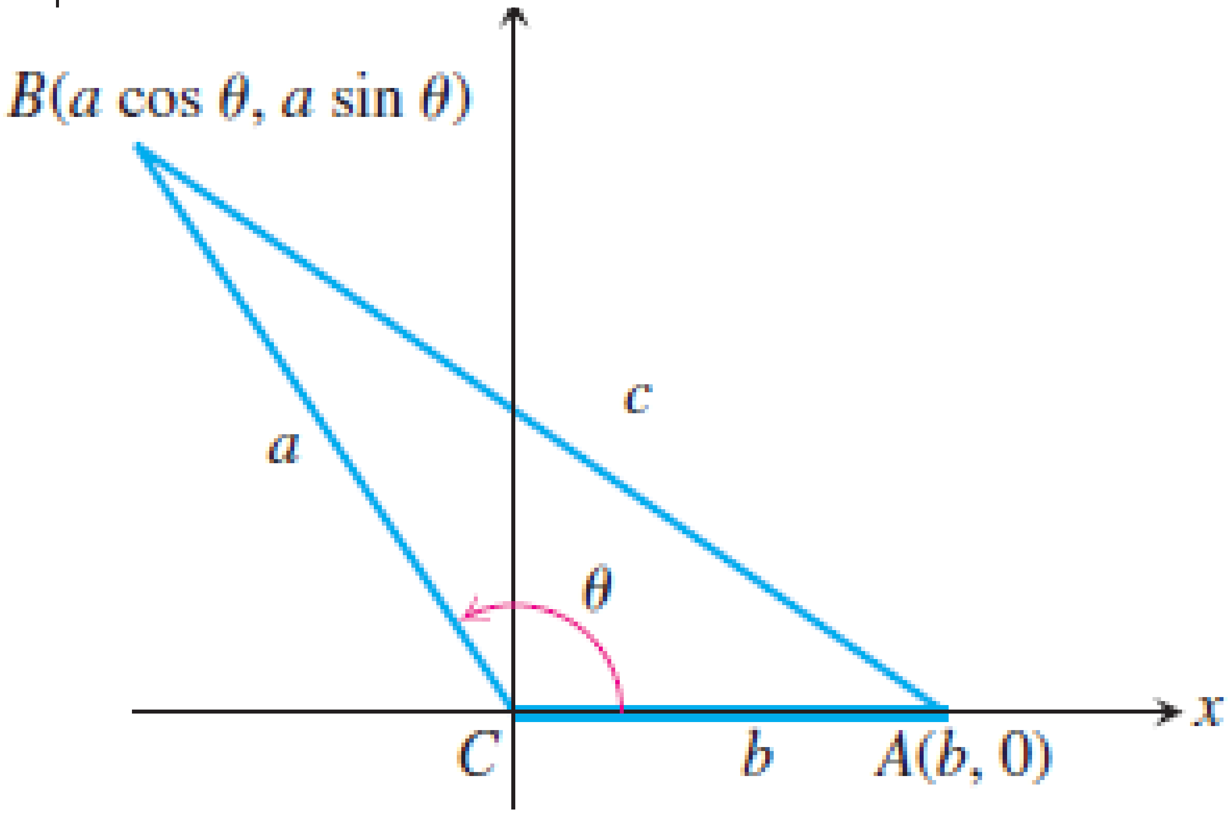
$$\cos^2 \theta = \frac{1 + \cos 2\theta}{2}$$

$$\sin^2 \theta = \frac{1 - \cos 2\theta}{2}$$

The Law of Cosines

If a, b, and c are sides of a triangle ABC and if θ is the angle opposite c, then

$$c^2 = a^2 + b^2 - 2ab\cos\theta.$$



Transformations of Trigonometric Graphs

The rules for shifting, stretching, compressing, and reflecting the graph of a function apply to the trigonometric functions. The following diagram will remind you of the controlling parameters.

Vertical stretch or compression; reflection about x-axis if negative

Vertical shift

$$y = af(b(x + c)) + d$$

Horizontal stretch or compression; reflection about y-axis if negative

Horizontal shift

$$f(x) = A \sin\left[\frac{2\pi}{B}(x - C)\right] + D,$$

where $|A|$ is the *amplitude*, $|B|$ is the *period*, C is the *horizontal shift*, and D is the *vertical shift* (Figure 1.76).

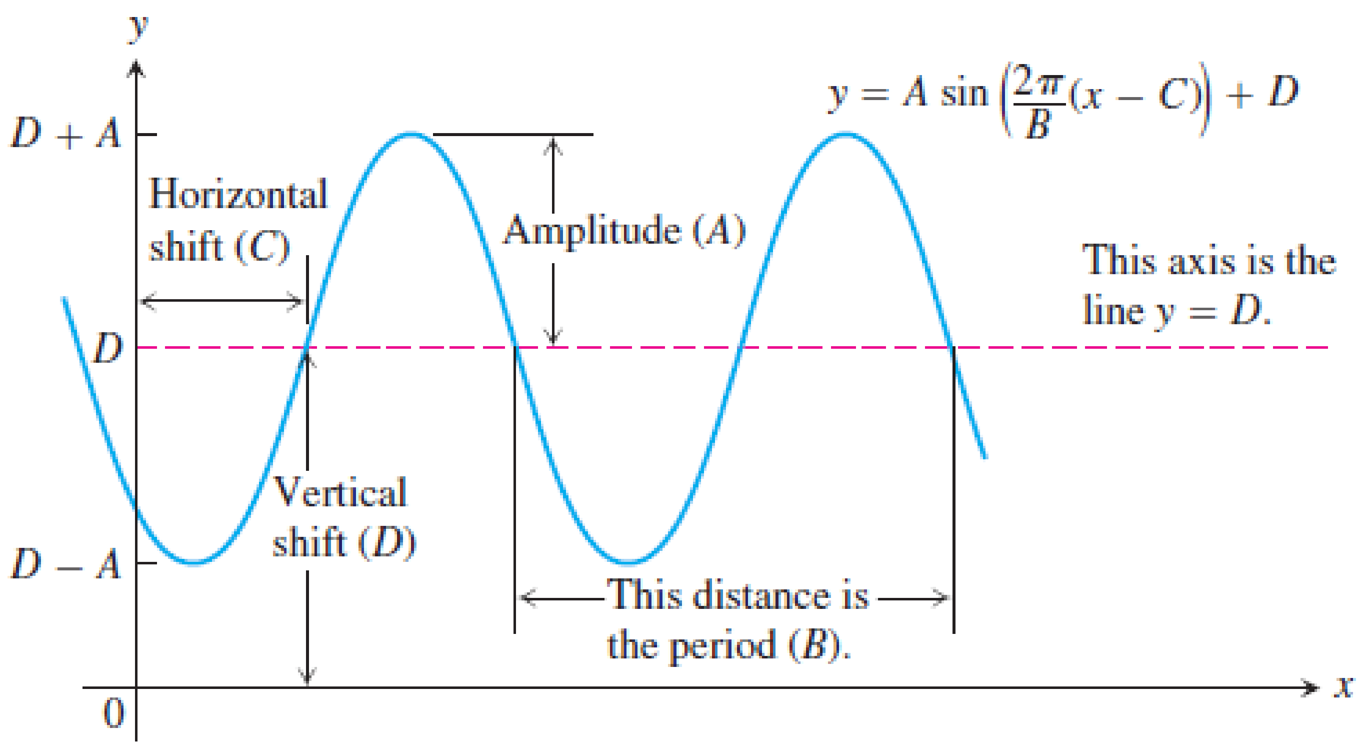


FIGURE 1.76 The general sine curve $y = A \sin[(2\pi/B)(x - C)] + D$, shown for A, B, C, and D positive (Example 2).

65. Temperature in Fairbanks, Alaska Find the **(a)** amplitude, **(b)** period, **(c)** horizontal shift, and **(d)** vertical shift of the general sine function

$$f(x) = 37 \sin\left(\frac{2\pi}{365}(x - 101)\right) + 25.$$

66. Temperature in Fairbanks, Alaska Use the equation in Exercise 65 to approximate the answers to the following questions about the temperature in Fairbanks, Alaska, shown in Figure 1.77. Assume that the year has 365 days.

a. What are the highest and lowest mean daily temperatures shown?

b. What is the average of the highest and lowest mean daily temperatures shown? Why is this average the vertical shift of the function?

65. (a) amplitude $= |A| = 37$

(c) right horizontal shift $= C = 101$

(b) period $= |B| = 365$

(d) upward vertical shift $= D = 25$

66. (a) It is highest when the value of the sine is 1 at $f(101) = 37\sin(0) + 25 = 62^\circ$ F.

The lowest mean daily temp is $37(-1) + 25 = -12^\circ$ F.

(b) The average of the highest and lowest mean daily temperatures $= \frac{62^\circ + (-12)^\circ}{2} = 25^\circ$ F.

The average of the sine function is its horizontal axis, $y = 25$.

Chapter

2

LIMITS AND CONTINUITY