Final Project HW, Math 255, by Dr. M. Sakalli, CSE, Marmara Univ. **14^{th} Jan, 2013**

COMPULSORY submission. The deadline is Friday the 18^{th} and. After exam until late hours, submissions will be accepted or Friday afternoon. Late submission will be accepted, arrange a meeting time by email please.
Submission in person by hand, you will be asked questions, you must submit your programs in your laptop. You will be graded at the time of your submission.

Four Questions, last two are in matlab.

Questions:

1. You are given a DE of $y''(t) + 5y'(t) + 6y(t) = u(t)$. Here u(t) is the input forcing the system. Assume that initial values are all zero. And $u(t) = \delta(t)$ is

   $\delta(t) = 1/(2\tau)$, at $-\tau < t < \tau$, $t = 0$

   $\delta(t) = 0$, $t \neq 0$

   a. Determine $\mathcal{U}(s)$ and $\mathcal{Y}(s)$. Assume that initial values are all zero.
   b. The transfer function in s is $\mathcal{Y}(s)/\mathcal{U}(s)$. Determine transfer function h(t).
   http://lpsa.swarthmore.edu/Transient/TransInputs/TransImpulse.html

2. Read this or check the link given below.

A powerful application of DEs: State space representation of DEs.

$[q]'(t) = [A][q](t) + [B]u(t)$ (1) [A] is a square and [B] is column matrix.

$[y](t) = [C][q](t) + [D]u(t)$ (2) [C], [D] are row and column matrices.

The first one is state space representation of a system replacing an nth order DE, with the 1st order systems coupled equations ([A] and [B]), the second equation is the observation equation, where the q(t) is **transitioned by** [C] and **additively shifted** by [D]u(t). Yu must see that the output differential equation q(s) is not directly observed. In other words, these two equations are in terms of systems,

input:u(t)→[h(t), black box]→output:(y(t)), where, h(t) is transfer funct of the systm, and in s domain

input:u(s)→[h(s), black box]→output:(y(s)).

That is H(s) = Y(s)/U(s).

a. Formulize $\mathcal{Q}(s)$ by writing (1) in the form of $\mathcal{Q}(s) = \Phi(s)\,\mathcal{B}(s)\,\mathcal{U}(s)$. Here $\Phi(s)$ is called **state transition** matrix. *******
b. Obtain $\mathcal{Y}(s)$, and formulize $\mathcal{H}(s)$

c. Now you are given a DE of $q'''(t) + 2q''(t) + 4q'(t) + 3q(t) = u(t)$. U(t) is the input forcing the system. Assume that initial values are all zero. Convert this eqn into a state space representation in the form of equation (1), where [A] is the system coefficients of the first order DEs, and [B] is the input forcing the equation. ***

d. Evaluate the state transition matrix $\mathcal{Q}(s)$ obtained in (a), write it in the form of, $\mathcal{Q}(s) = \Phi(s)\ \mathcal{B}\mathcal{U}(s)$.
e. Evaluate $\mathcal{Y}(s)$ implicitly as obtained in (b) as given at eqn(2). And determine $\mathcal{H}(s)$ by avoiding unnecessary computing for C=[5 1 0], and D= transpose([0 0 0]).
http://lpsa.swarthmore.edu/Representations/SysRepTransformations/TF2SS.html

3. You have a NL pendulum system of $\Theta''(t) + c\,\Theta' + k\sin(\Theta) = u(t)$.
   a. Linearize this equation. Covert it into a first order DEs of systems. Write a matlab program for this.
   b. Modify your matlab code for original NL equation.
   c. And run both matlab programs for various values of c, $k \geq 0$, in particular when there is no friction, by using
      [t, x]=ode45(@yourpendulumfnct, range, IVs), ie range=[0, 20];
      IVs =[1, 0], returning x in the form first order variations wrt t.
   d. Subplot, t vs x(:,1), t vs x(:,2), and phase curves for x(:,1) vx (x,2).

4. $t^3y''' + 3t^2y'' + 4ty' - 6y = u(t)$, $t>0$, $u(t) = t^4\log(t)$. Same questions raised in 3rd, with two additional subplots of t vs x(:,3), and x(:,1)vs x(:,3)..