Midterm Exam Math 255, by Dr. M. Sakalli, CSE, Marmara Univ. Nov. 24, 2009

Differential Equations. Duration: 1hr 45 minutes. Good luck.

Note 1: Any misconduct or any attempt to copy or cheating will lead your complete exclusion from the exam.

Note 2: All the questions appearing in this page were completely or partially presented (taught) in the class.

1. Prove that the difference of any two particular solutions to inhomogeneous DE is also a solution to its homogeneous version.

2. Reduction of order: You have a second order homogeneous DE,

   $y'' + p(x)\, y' + q(x)\, y = 0$,

   and suppose you are given the first (nontrivial) solution as, $y_1(x)$.

a) Show that the second solution (non-proportional to $y_1(x)$) can be related by a variable $u(x)$ to the first solution which will end up with an equation of reduced order and yielding the solution of $y_2(x)$ as a function of y_1.

b) This is an application of the question given above. DE you are given is

   $x^2 y'' - 3x y' + 4y = 0$, and the first solution is $y_1 = x^2$.

   i) Show that if $y_1 = x^2$ is a solution.
   ii) If it is, then find y_2 by reducing the order. If it is not suggest a solution. And then proceed.

3. Apply derivation operator on y to solve $(D-r_1)(D-r_2)\, y = 0$, hint reduce equation to the first order by substitution.

4. $y'' - 3y' + 2y = e^x$

a) Solve the reduced (homogeneous) version of this DE.
b) Solve particular solution of non-homogeneous DE by using the method of undetermined coefficients.
c) Solve the same particular solution by using ESL.

5. Find the particular solution of DE: $y^{(4)} + 5y'' + 2y = 2 - 5e^{(3x)}$ simply by using ESL.

   Hint: represent the equation in the form of operators and then apply ESL step by step.