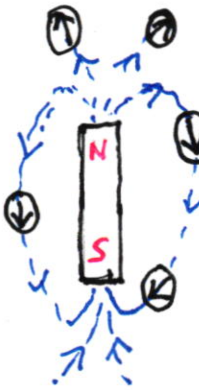
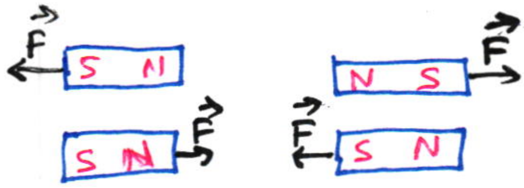
~ STATIC MAGNETIC FIELDS ~



A magnet is a source of magnetic field.
The compass needle lines up in the direction of magnetic field.

Note that, magnet is composed of two poles; North and South. and field lines are from North to South. Magnetic fields are stronger, close to the poles.

Like poles repel each other while unlike poles attract



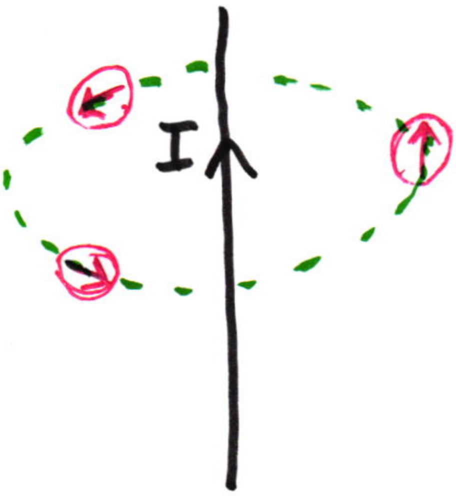
Recall that an electrical charge can be source of electric field (monopole). However there doesnot exist magnetic monopoles. If one breaks a magnet into two parts; two new magnets are obtained.

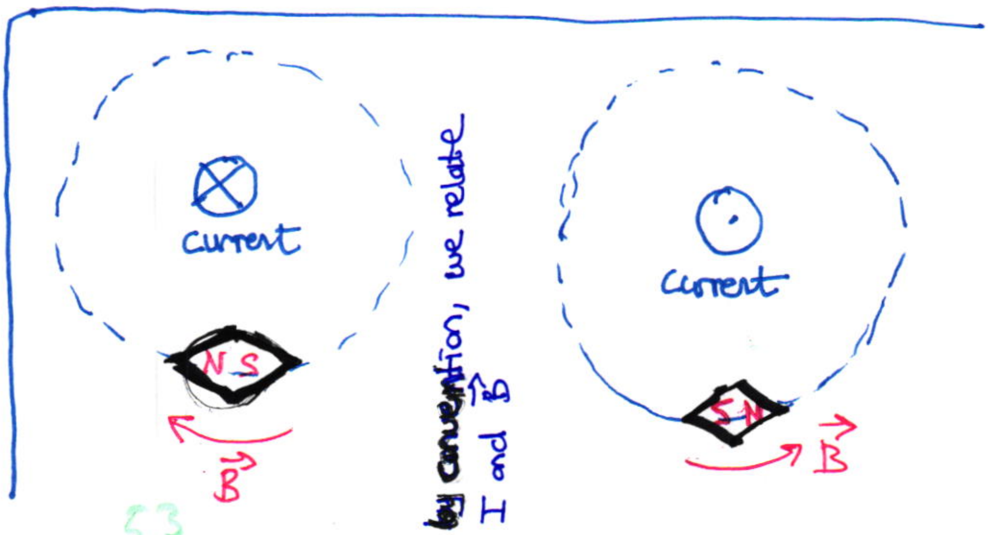
Remember that $\vec{E}$ was defined as the force per unit charge $\vec{E} = \frac{\vec{F_e}}{q}$

If no magnetic monopole exists, how we may define magnetic field. is a question.

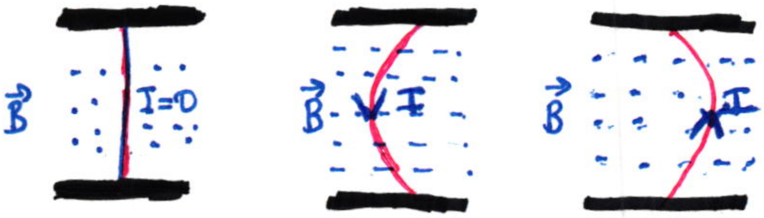
In 1819, Ørsted noticed that compass needle deflected when a current due to a switched on and off battery flowes through a wire. This observation shows relation between electricity and magnetism.

→ observation. By convention, we say current flow and magnetic field is related by Right Hand Rule.



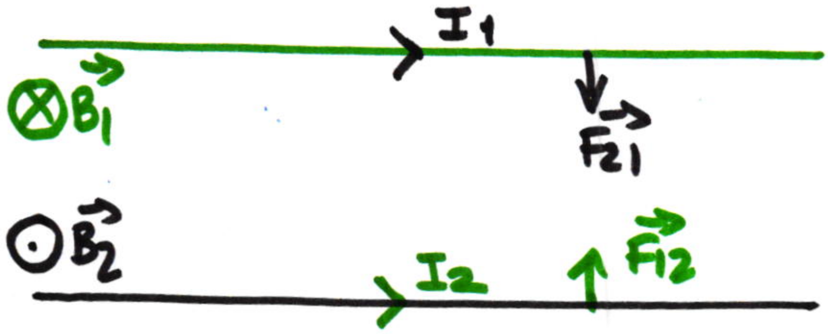


One other observation with magnetism is "a current carrying wire is affected by a magnetic field"! (Ampere)



Imagine the observation on the left side. It seems the direction of force is related to direction of both I and $\vec{B}$

So far, we are informed about observations that a current carrying wire creates magnetic field and a forse is experienced by a current carrying wire under magnetic field. Now, consider two parallel current carrying wires;



$\vec{a_F} = \vec{a_I} \times \vec{a_B}$ → unit vector of magnetic field affects on wire

$\vec{a_F}$: unit vector of force exerted

$\vec{a_I}$: unit vector of current (direction of flow)

If currents would ~~flow~~ be of the opposite direction, wires should repel each other. Is not it?

Experimentally, it is obsered that the force experienced by a particle with velocity vector $\vec{v}$ is perpendicular to the direction of $\vec{v}$. $\boxed{\text{That is } \vec{F_B} \perp \vec{v}}$

Also, magnitude of $\vec{F_B}$ is proportional to ~~the~~ the velocity.

$\boxed{\text{That is } |\vec{F_B}| \propto |\vec{v}|}$

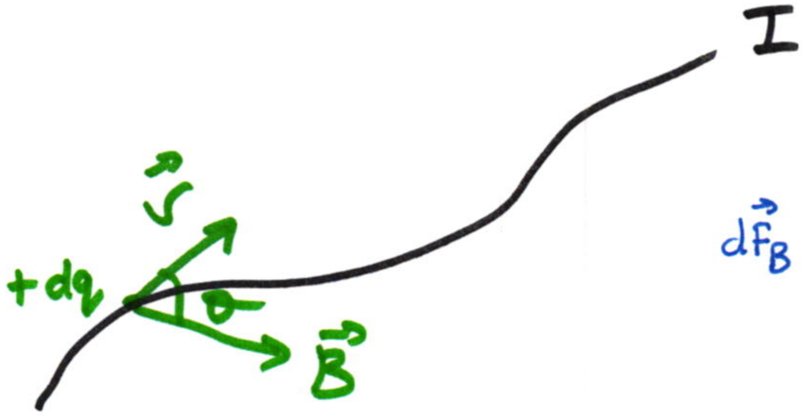
Besides, $|\vec{F_B}| \propto q$

Putting together all observations (both qualitative and quantative), the exerted magnetic force is formulated as;

$$\boxed{\vec{F_B} = q(\vec{v} \times \vec{B})}$$

$N : C . \frac{m}{sec} . B\text{unit}$

$B\text{unit}: \frac{N\,sec}{C.m} \triangleq \text{Tesla} \approx 10^{-4}\,\text{Gauss}$ (magnetic field of earth is approximately $\frac{1}{2}$ Gauss)



$d\vec{F}_B = dq\,(\vec{v}_d \times \vec{B})$, $I = dq/dt$

drift velocity

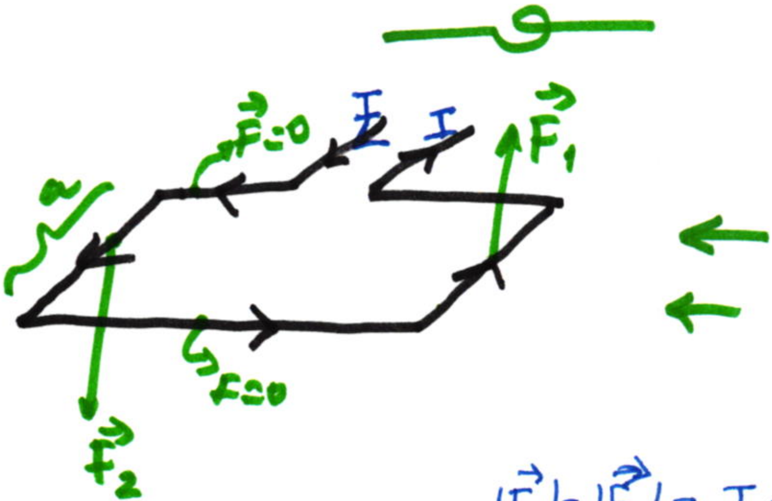
$\vec{v}_d \cdot dt = d\vec{l}$

Manipulating, one may obtain;

$$d\vec{F}_B = I\,(d\vec{l} \times \vec{B})$$

Integrating both sides

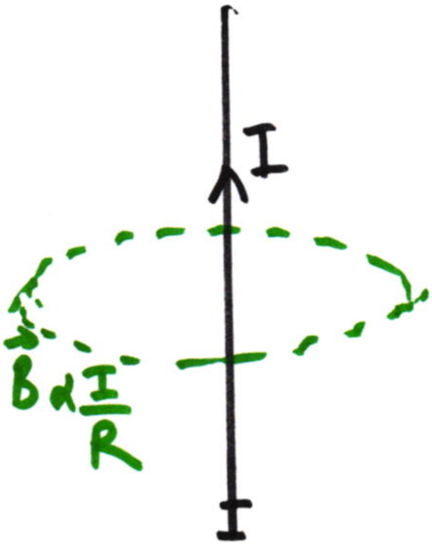
$$\vec{F}_B = I \int d\vec{l} \times \vec{B}$$



$\vec{B}$ Uniform magnetic field

$$|\vec{F}_1| = |\vec{F}_2| = I\,a\,|\vec{B}|$$

The main idea behind commutator motor



Biot and Savart proposed that a current carrier may divided into current elements $d\ell$ which can be integrated to determine $\vec{B}$ at a point. Note the similarity between $d\vec{E}$ and dq.

$$\vec{d\ell}\ (I),\ \vec{a_R} \quad R \quad d\vec{B} = C \cdot \frac{I}{R^2}\left(\vec{d\ell} \times \vec{a_R}\right)$$

$$C \rightarrow 10^{-7} = \frac{\mu_0}{4\pi} \rightarrow \text{magnetic constant } (4\pi 10^{-7})$$

This approach is called as "Biot-Savart" rule.

In a compact form, $\vec{B} = \int_{wire} \frac{\mu_0 I}{4\pi} \frac{\vec{d\ell} \times \vec{a_R}}{R^2}$

Some applications of Biot-Savart Law

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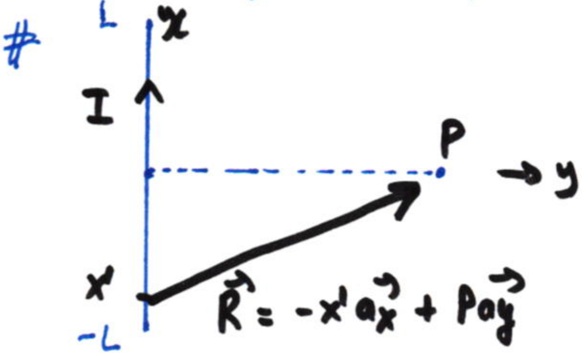


$$d\vec{B} = \frac{\mu_0 I}{4\pi} \frac{\vec{d\ell} \times \vec{a_R}}{R^2}$$

→ magnetic field contribution of each current carrying segment. We are to integrate them

Here;

$$\vec{B} = \frac{\mu_0 I}{4\pi} \int_{-L}^{L} \frac{dx' \vec{a_x} \times (-x' \vec{a_x} + P \vec{a_y})}{(x'^2 + \rho^2)^{3/2}}$$

$$\boxed{\begin{array}{l} \vec{a_x} \times \vec{a_x} \equiv 0 \\ \vec{a_x} \times \vec{a_y} \equiv \vec{a_z} \end{array}}$$

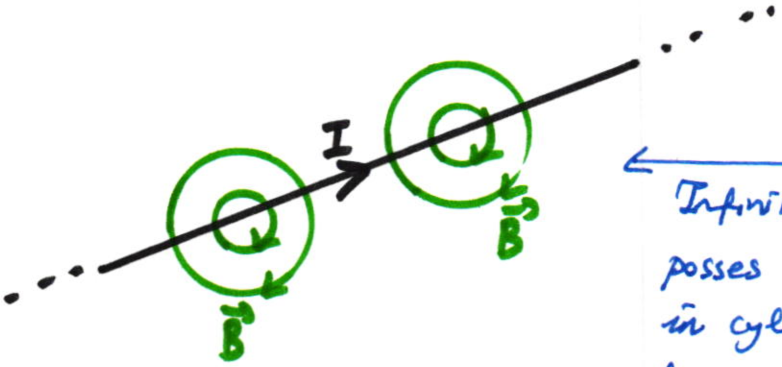
$$= \frac{\mu_0 I \cdot \rho}{4\pi} \boxed{\int_{-L}^{L} \frac{dx'}{(x^2 + \rho^2)^{3/2}}} \rightarrow \boxed{\frac{x}{\rho^2 \sqrt{x^2 + \rho^2}}}_{-L}^{L}$$

$$= \frac{\mu_0 I \rho}{4\pi} \frac{2L}{\rho^2 \sqrt{x^2 + \rho^2}} = \frac{\mu_0 I L}{2\pi \rho \sqrt{\rho^2 + L^2}} \text{ Tesla}$$

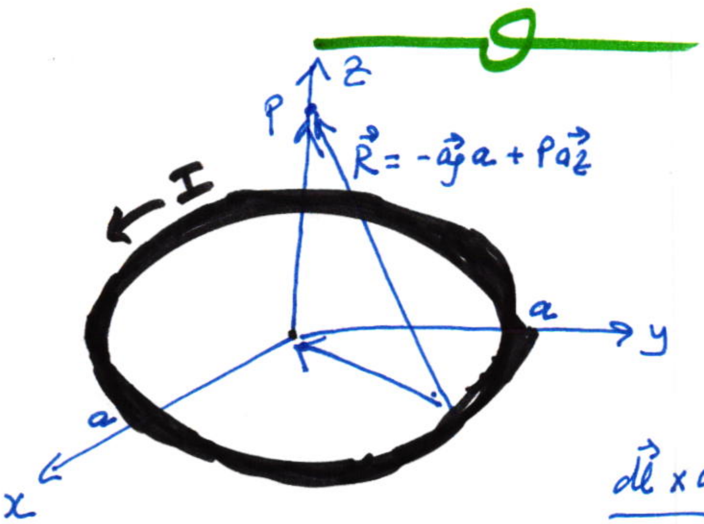
If line length goes to infinity $(L \to \infty)$

$$\lim_{L\to\infty}\left(\vec{B} = \frac{\mu_0 I L}{2\pi\rho\sqrt{\rho^2 + L^2}}\right) = \lim_{L\to\infty}\frac{\mu_0 I}{2\pi\rho}\sqrt{\frac{L^2}{\rho^2+L^2}} = \boxed{\frac{\mu_0 I}{2\pi\rho}}$$

imagine that result



Infinitely long wire posses a magnetic field in cylindrical form for which the direction can be determined via right-hand rule



A current loop, located on xy plane carries I ampere. $\vec{B}$ at P?

$\vec{dl} = a\,d\phi\,\vec{a_\phi}$

$$d\vec{B} = \frac{\mu_0}{4\pi} I \frac{\vec{dl} \times \vec{a_R}}{R^2}$$ → magnetic field of current segment

$$\frac{\vec{dl} \times \vec{a_R}}{R^2} = \frac{a\,d\phi\,\vec{a_\phi} \times (-\vec{a_\rho}\,a + \rho\vec{a_z})}{(a^2+\rho^2)^{3/2}}$$

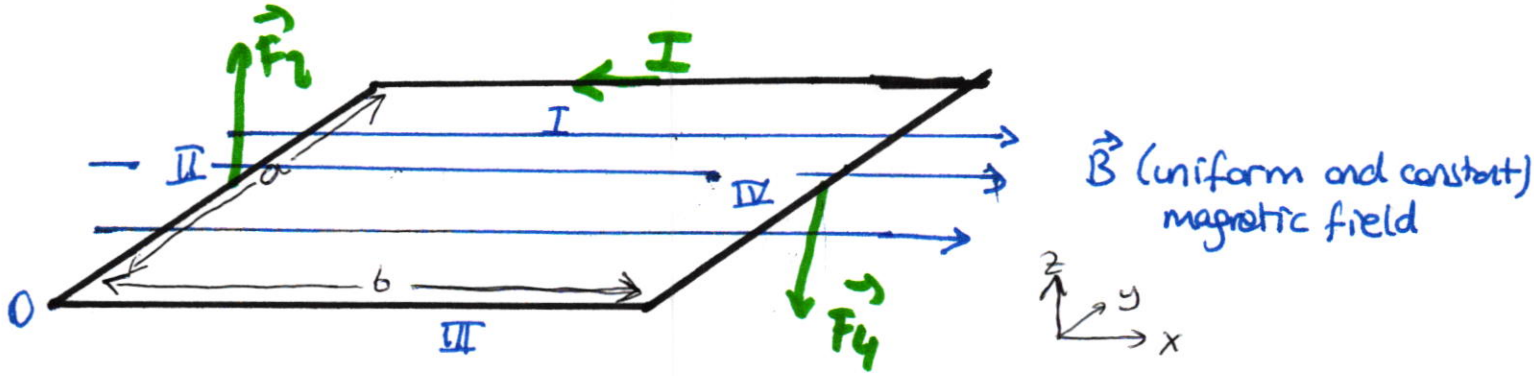
Now, be careful with $d\phi\vec{a_\phi} \times \rho\vec{a_z}$ term which gives $\rho\, d\phi\, \vec{a_\rho}$. Through entire loop, this term vanishes due to the circular symmetry. If you wish to see it mathematically, you may express $\vec{a_\rho}$ in cartesian coordinates as we did in electrostatic examples. Here $\vec{B}$ becomes;

$$\vec{B} = \frac{\mu_0 I}{4\pi}\int_0^{2\pi}\frac{a^2\, d\phi\, \vec{a_z}}{(a^2+\rho^2)^{3/2}} = \frac{\mu_0 I a^2}{2(a^2+\rho^2)^{3/2}}$$

Simply at origin $(\rho = 0)$, magnetic field becomes;

$$\vec{B} = \frac{\mu_0 I}{2a}$$

Magnetic Forces and Torques



Recall $\vec{F}_B = I \int d\vec{l} \times \vec{B}$. Let's investigate the force experienced by the current carrying rectangular loop.

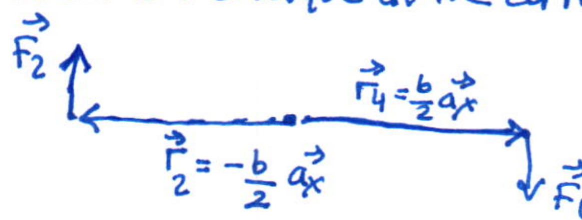
Since $\vec{dl}$ and $\vec{B}$ are perpendicular to each other on I & III, force will be 0. ($\vec{F_1} = \vec{F_3} = 0$). On segment 2 and segment 4;

$$\vec{F_2} = I \int d\vec{l} \times \vec{B} = I \int_0^a -\vec{a_y}\, dy \times B\vec{a_x} = IaB\vec{a_z}$$

$$\vec{F_4} = I \int d\vec{l} \times \vec{B} = I \int_0^a \vec{a_y}\, dy \times B\vec{a_x} = -IaB\vec{a_z}$$

So, the net force $\vec{F}_{net} = \vec{F_1} + \vec{F_2} + \vec{F_3} + \vec{F_4} = 0 + IaB\vec{a_z} + 0 + -IaB\vec{a_z} = 0$ on the loop. Let's examine the torque at the current position.

$\tau = \vec{r} \times \vec{F}$

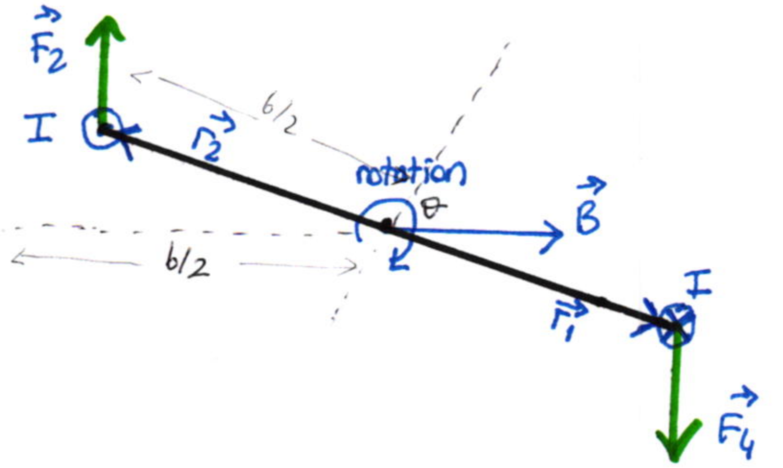


$$\tau = -\frac{b}{2}\vec{a_x} \times IaB\vec{a_z} + \frac{b}{2}\vec{a_x} \times -IaB\vec{a_z} = IabB\vec{a_y}$$ (with respect to the center of loop)

So we expect loop to rotate in clock wise direction w.r.t $\vec{a_y}$ direction with a strength $IabB$

rotation of loop with a torque $IabB$

Now, let's generalize our analysis.

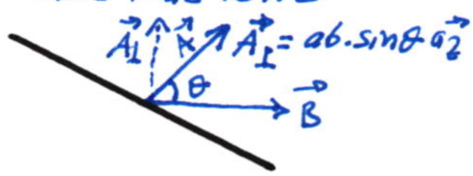


$$\vec{r_2} = \underbrace{\frac{b}{2}}_{\text{magnitude}} \underbrace{(-\sin\theta\,\vec{a_x} + \cos\theta\,\vec{a_z})}_{\text{direction}}$$

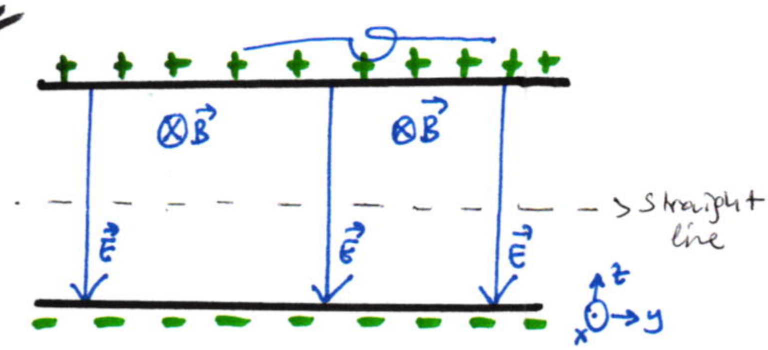
$$\vec{r_4} = \frac{b}{2}(\sin\theta\,\vec{a_x} - \cos\theta\,\vec{a_z})$$

$$\vec{\tau} = \vec{r_2}\times\vec{F_2} + \vec{r_4}\times\vec{F_4} = 2\vec{r_2}\times\vec{F_2} = 2\vec{r_4}\times\vec{F_4} = b(-\sin\theta\,\vec{a_x} + \cos\theta\,\vec{a_z})\times(IaB\,\vec{a_z})$$

$$\vec{\tau} = \underbrace{Iab\sin\theta\,\vec{a_y}}_{\text{note this term}}$$



example

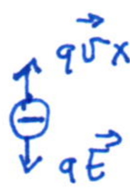


Uniform $\vec{E}$ in $-\vec{a_z}$ direction
Uniform $\vec{B}$ in $\vec{a_x}$ direction
If motion of electron does not change, find velocity in terms of $\vec{E}$ and $\vec{B}$

Lorentz force experienced by electron is

$$\vec{F} = q(\vec{E} + \vec{v}\times\vec{B})$$

If $|\vec{E}| = |\vec{v}\times\vec{B}| = vB$, particle will move on straight line ($\vec{a_y}$ direction)

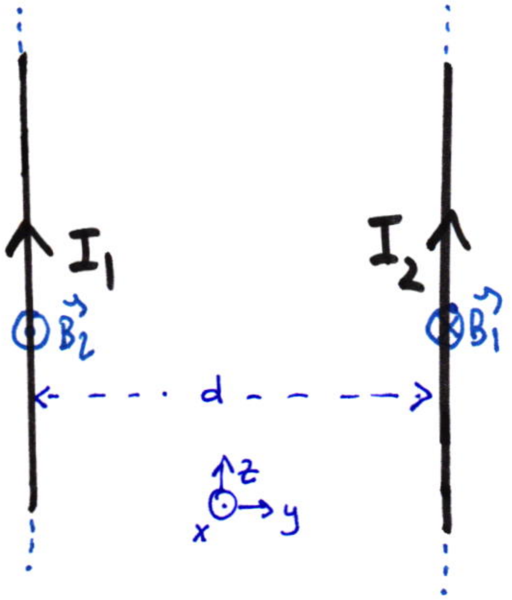


in equilibrium

$$q\vec{E} = q\vec{v}\times\vec{B}$$

Hence, with a velocity $v = \dfrac{E}{B}$, particles move in a straight line

example



An infinitely long wire carrying I_1 current will possess a magnetic field distribution all around as well as wire II. Wire II will experience a force due to the magnetic field created by wire I, similarly wire 1 will experience a force by the magnetic field produced by wire 2. Recall magnetic fields produced by wires on each other are;

$$\vec{B}_1 = \frac{I_1 \mu_0}{2\pi d}(-\vec{a_x})\,; \quad \vec{B}_2 = \frac{I_2 \mu_0}{2\pi d}\vec{a_x}$$

Recall that $\vec{F} = I \int d\vec{l} \times \vec{B}$. First let's see $\vec{F_{12}}$ (force per unit length of wire 2)

$$\vec{F_{12}} = I_2 \int dz \vec{a_z} \times \frac{I_1 \mu_0}{2\pi d}(-\vec{a_x}) = \frac{-I_1 I_2 \mu_0}{2\pi d}\vec{a_y} \underbrace{\int dz}_{\text{per unit length}} = -\frac{I_1 I_2 \mu_0 \vec{a_y}}{2\pi d} \quad [N/m]$$

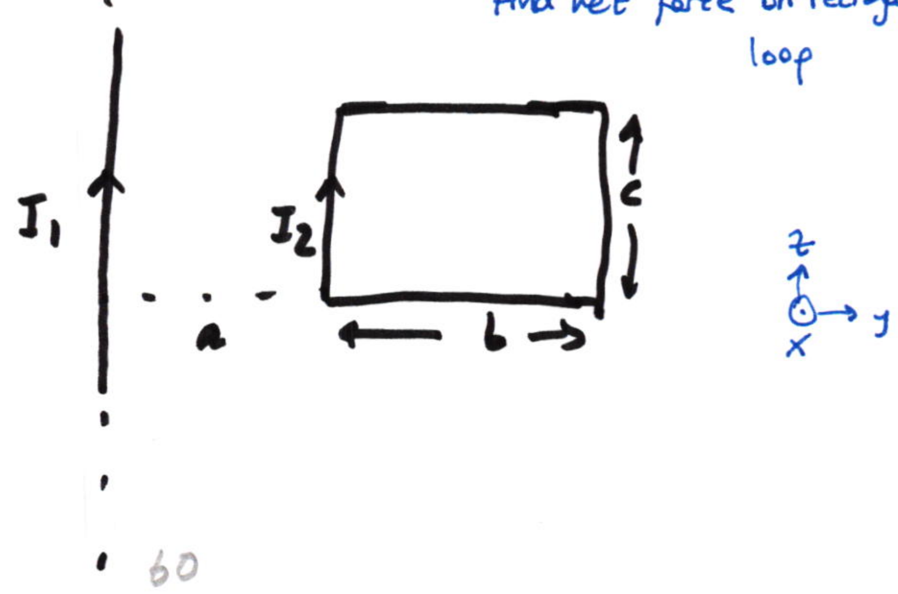
Similarly one may obtain

$$\vec{F_{21}} = \frac{I_1 I_2 \mu_0 \vec{a_y}}{2\pi d} \quad [N/m]$$

Note that $\vec{F_{12}} = -\vec{F_{21}}$ the forces of action and reaction holds (Newton's 3rd rule)

exercise

Find net force on rectangular loop

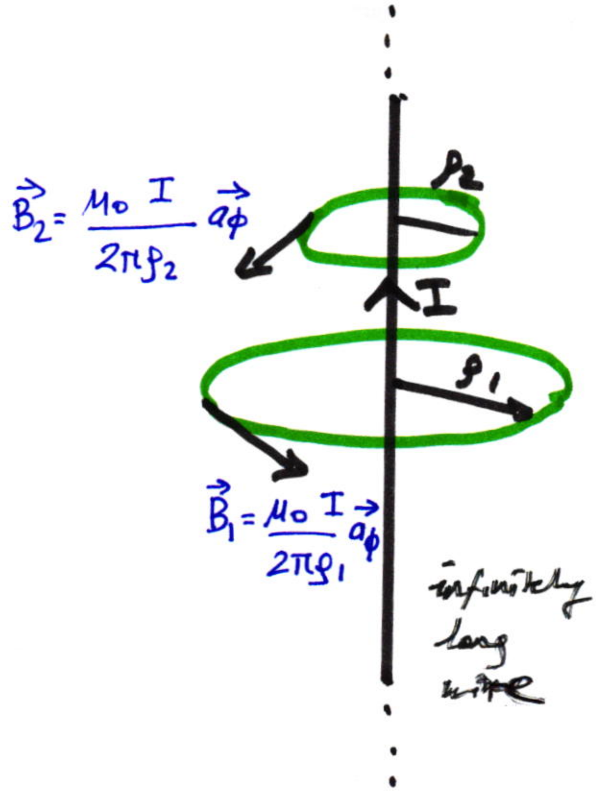


Ampere's Law

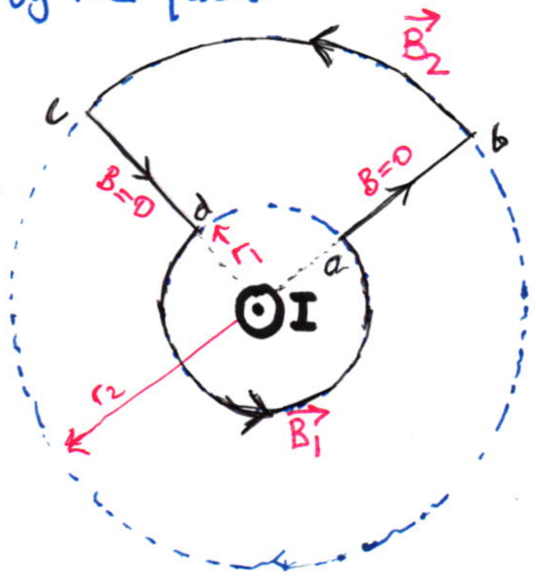
We readily know that the magnetic field due to an infinitely long, current carrying wire is;

$$\vec{B} = \mu_0 \frac{I}{2\pi\rho} \vec{a_\phi}$$

by Biot-Savart Law →



In order to utilize the circular symmetry, (on which the magnetic field is constant in magnitude), we applied Biot-Savart Law on circles. What Ampere recognized was, no matter which path you choose, there is a relation between the magnetic field on the path (closed) and the current enclosed by the path.



→ Here, abcda is a closed path. let's investigate the magnetic field on path - current enclosed by the path relation. As you see, path is not a circle as we used in Biot-Savart application. Note that, current flows perpendicular to the page.

$$\underset{\substack{abcda \\ \text{closed path}}}{\int} \vec{B}\,\vec{dl} = \underset{\substack{ab \\ \text{path}}}{\int} \vec{B}\,\vec{dl} + \underset{\substack{bc \\ \text{path}}}{\int} \vec{B}\,\vec{dl} + \underset{\substack{cd \\ \text{path}}}{\int} \vec{B}\,\vec{dl} + \underset{\substack{da \\ \text{path}}}{\int} \vec{B}\,\vec{dl}$$

As we know, with respect to Right hand rule, $B = 0$ on paths cd and ab

Besides, thanks to the Biot-Savart Law, we know that

$$\vec{B}_1 = \mu_0 \frac{I}{2\pi r_1} \vec{a_\phi}, \quad \vec{B}_2 = \mu_0 \frac{I}{2\pi r_2} \vec{a_\phi}$$

Hence;

$$\underset{\substack{abcda \\ \text{closedpath}}}{\int \vec{B}\,d\vec{l}} = \underbrace{B_2 \cdot r_2 \theta}_{\int_{bc} \vec{B}\,d\vec{l}} + \underbrace{B_1\, r_1 (2\pi - \theta)}_{\int_{da} \vec{B}\,d\vec{l}}$$

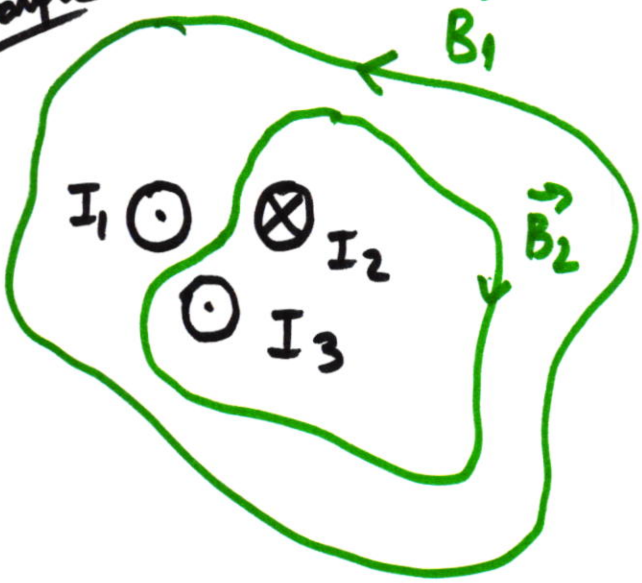
$$= \mu_0 \frac{I}{2\pi \cancel{r_2}} \cancel{r_2}\theta + \mu_0 \frac{I}{2\pi \cancel{r_1}} \cancel{r_1}(2\pi - \theta)$$

cancel cancel

$$\boxed{\int \vec{B}\,d\vec{l} = \mu_0 I}$$

↳ This result is valid for any amount of current and any chosen path. ~~That~~ is known as <u>Ampere's Law</u>.

example

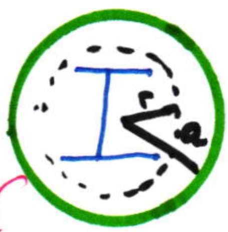


Note the direction of currents and magnetic fields.

$$\int \vec{B}_1\,d\vec{l} = (I_1 - I_2 + I_3)\mu_0$$

$$\int \vec{B}_2\,d\vec{l} = (I_2 - I_3)\mu_0$$

example



current I flows through the surface (circular) with radius a

$$\int \vec{B}\,d\vec{l} = \mu_0 \frac{I\pi r^2}{\pi a^2}$$

→ current enclosed

$$r<a \Rightarrow \vec{B} = \frac{I r \mu_0}{2\pi a^2} \vec{a_\phi}$$

→ will be explained during class

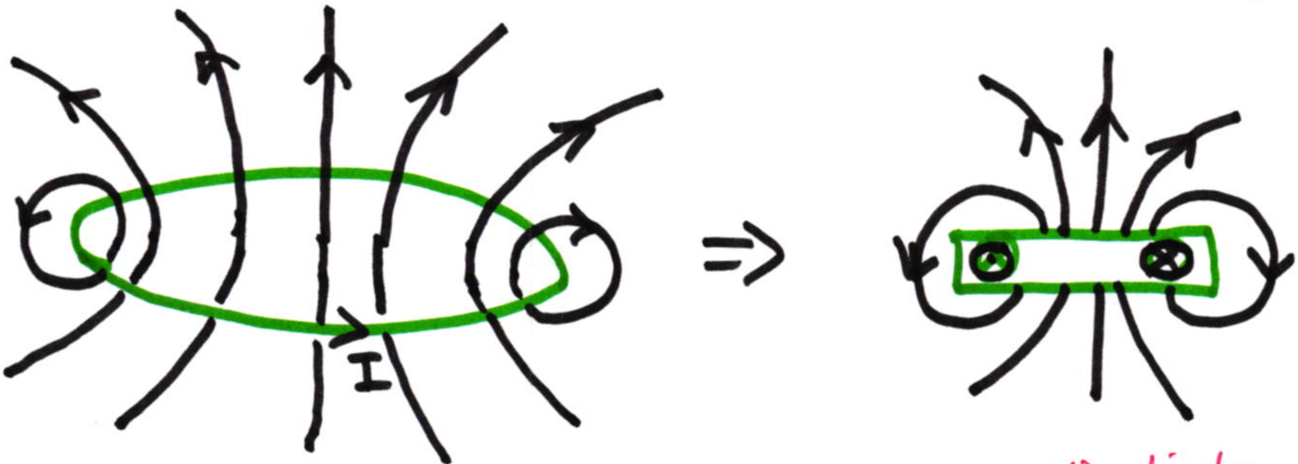
$$r>a \Rightarrow \vec{B} = \mu_0 \frac{I}{2\pi r} \vec{a_\phi}$$

Utilizing Ampere's Law to determine magnetic field requires certain symmetry conditions (remember Gauss' Law in electrostatics). No symmetrical systems can be analyzed by Biot-Savart Law.

Biot-Savart Law $\vec{B} = \frac{\mu_0 I}{4\pi} \int \frac{d\vec{l} \times \vec{a_R}}{R^2}$ any current source

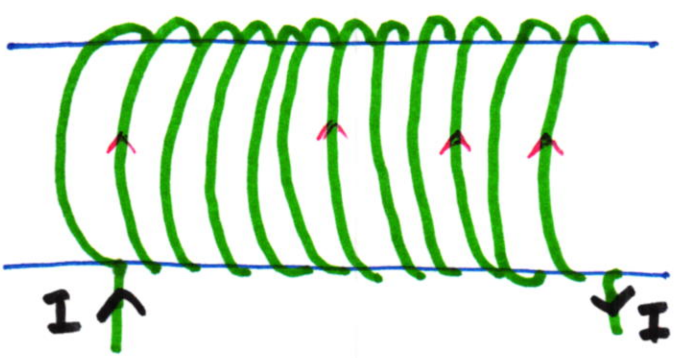
Amperes Law $\int \vec{B} \, d\vec{l} = \mu_0 I_{enclosed}$ current source with symmetry

Now, recall the magnetic field distribution of a current carrying loop.

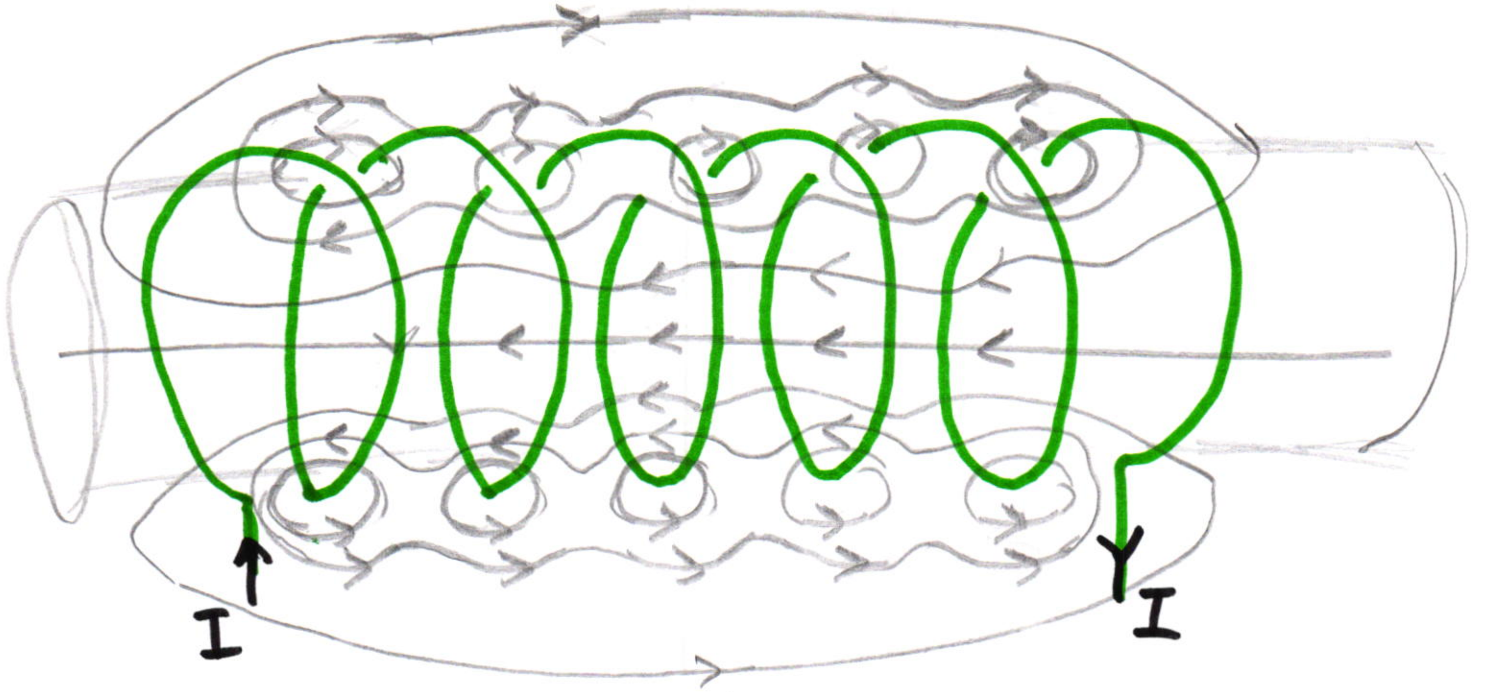


magnetic dipole

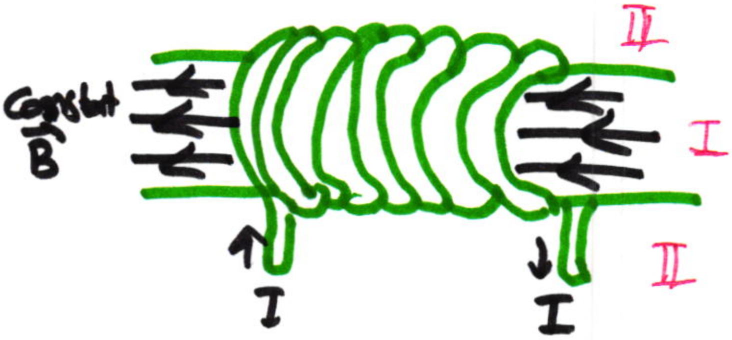
Now, imagine you wrap the wire twice, trice, or many turns. How would the magnetic field line distribution becomes (if the turns are closed to each other and tight).



$\vec{B} = ?$ Imagine it



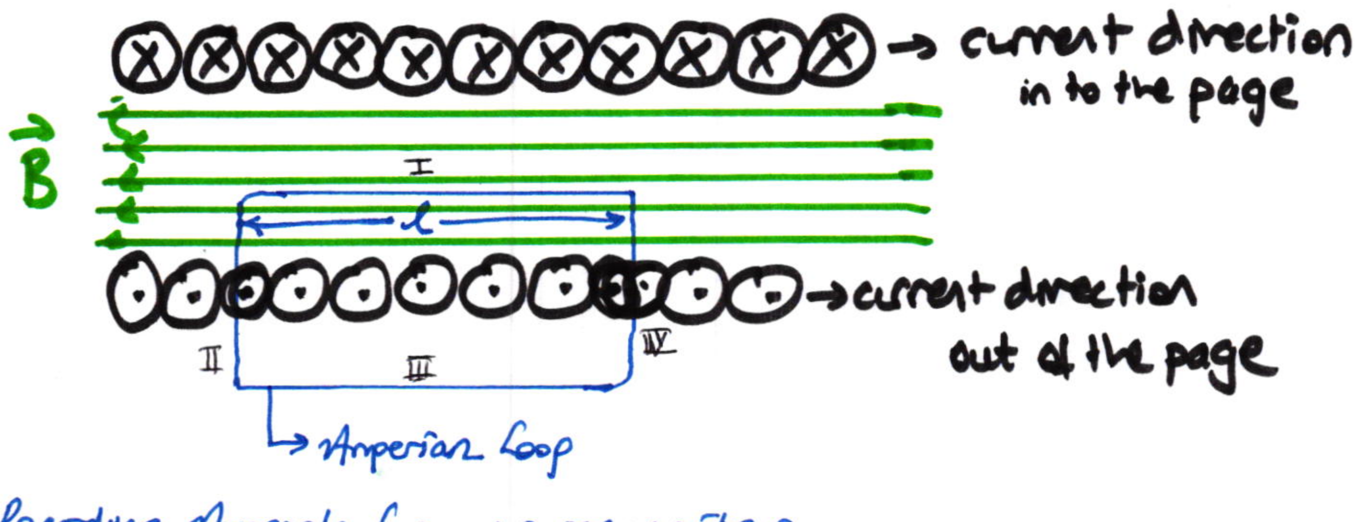
If you wrap lines tightly with N turns, you have a Solenoid.



In region I, magnetic field is almost CONSTANT.

In region II and III, (in the outer region of solenoid) magnetic field is almost 0

To determine $\vec{B}$, we may utilize Amperes Law.



Regarding Ampere's Law, we may write;

$$\int \vec{B}\, d\vec{l} = \mu_0 I_{enclosed}$$

Clearly, on path II and path IV, magnetic field is 0. Besides, as demonstrated (a demonstration on solenoids will be shown during lecture) on path III, namely outer of the solenoid, magnetic field is 0.

Hence;

$$\int \vec{B}\,d\vec{\ell} = \underbrace{B}_{0}(\text{path III} + \text{path IV} + \text{path II}) + B.\ell = \mu_0 \underbrace{I_{enclosed}}_{I.N}$$

Here, N is the number of turns in path I. So, to generalize lets define number of turns per length "n" = " N/ℓ "

Simply;

$$B = \mu_0 \cdot n \cdot I \rightarrow \text{magnetic field in a solenoid.}$$

↳ flowing current

→ number of turns per unit length

Direction of $\vec{B}$ can be determined by right hand rule considering the direction of current flow.

—o—

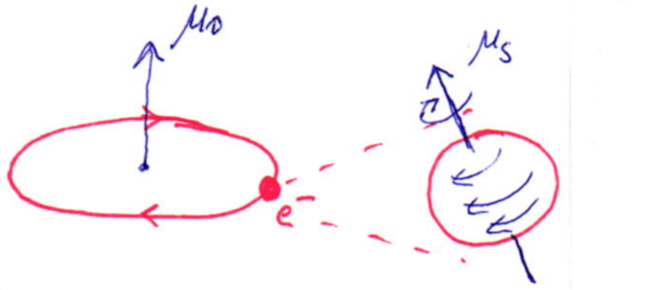
Remember what Ampere's Law says: $\int \vec{B}\,d\vec{\ell} = \mu_0 I_{enclosed}$, also regarding the example that the uniform current carrying cable in which we met a new concept current density $\vec{J}$, we may relate $\vec{B}$ and $\vec{J}$ in differential form. That is;

$$\int \vec{B}\,d\vec{\ell} \underset{\uparrow}{=} \int \underbrace{\nabla \times \vec{B}\,d\vec{s}} = \mu_0 I = \mu_0 \int \vec{J}\,d\vec{s}$$

Stokes theorem

$$\boxed{\nabla \times \vec{B} = \mu_0 \vec{J}}$$

Behaviour of Magnetic Materials



Orbital magnetic moment

Spin magnetic moment

Magnetization of a material is due to the atomic-scale current rotations. Both (1) Orbital motion of protons in nucleus and electrons around nucleus (2) spin motion of electrons contribute to the magnetization.

μ_s (spin magnetic moment): Atoms with an even number of electrons usually exist with electrons in pairs, in which spinning is in opposite direction. Hence, magnetic moments cancel each other. If the number of electrons is odd; there exist a non-zero spin moment.

μ_o (orbital magnetic moment)

by convention, it doesn't have to be

$$\vec{\mu_o} = \boxed{\frac{1}{2}} \oint_S \vec{r} \times \underbrace{\vec{J}\, \vec{ds}}_{I\vec{dl}}$$

$$= \frac{I}{2} \int_{\phi=0}^{2\pi} \vec{r} \times \vec{dl} \qquad \vec{r} = r \cdot \vec{a_\rho} \quad \vec{dl} = r \cdot d\phi \cdot \vec{a_\phi}$$

$$= \frac{I}{2} \int_0^{2\pi} r^2 \, d\phi \, \vec{a_z}$$

area of circular loop

$$\boxed{\vec{\mu_o} = I \pi r^2 \vec{a_z} = I\, S\, \vec{a_z}}$$

Note that, current's in the opposite direction of electron motion. That is

Similarly, magnetic moment of a Solenoid is

$\mu_{solenoid} = N \cdot I \cdot S$ (N: number of turns)

Magnetization vector is vectorial sum of dipole moments (μ_o) in a differential volume. That is

$$\vec{M} = \frac{\sum_{i=1}^{n\Delta v} \vec{\mu_o}^{\,i}}{\Delta v}$$

↳ magnetization vector.

In the presence of a material;

$\vec{B} = \mu_0 \vec{H} + \mu_0 \vec{M}$ relation is valid.

$\vec{B}$: total magnetic field vector in material

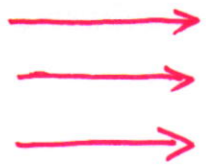
$\vec{H}$: magnetic field intensity in the absence of material [A/m]

In most cases, $\vec{M} = \chi_m \vec{H}$ where χ_m is the magnetic susceptibility of the material and a dimensionless quantity. χ_m relates $\vec{M}$ and $\vec{H}$ in a linear fashion and temperature-dependent for diamagnetic and paramagnetic materials. So, one may write;

$$\vec{B} = \mu_0 \vec{H} + \mu_0 \chi_m \vec{H} = \overbrace{\underbrace{\mu_0 (1+\chi_m)}_{\text{magnetic permeability}}}^{\mu_0 \cdot \mu_r = \mu} \vec{H} = \mu_r \vec{H}$$

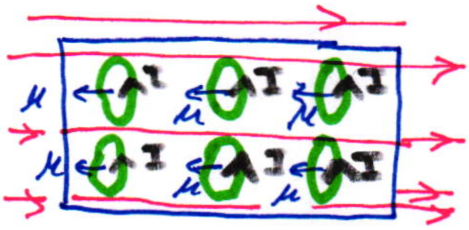
It is convinient to define $\mu_r \rightarrow$ magnetic permeability / magnetic constant $= \mu/\mu_0$ as the relative permeability.

Diamagnetic substances: These materials are those in which individual atoms do not possess net magnetic moment (electrons are paired). In an applied external field (magnetic), atoms have magnetic dipoles such that decreasing the external field inside the material.



External field $\vec{B_0} = \mu_0 \vec{H}$

Let's locate a diamagnetic material $\rightarrow$ inside B_0 field



magnetic dipoles allign such that decreasing $\vec{B_0}$ inside material

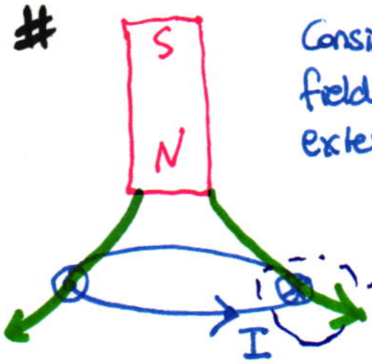
So the total field in substance is

$$\vec{B} = \vec{B_0} + \chi_m \vec{M} = \mu_0 (1+\chi_m) \vec{H}$$

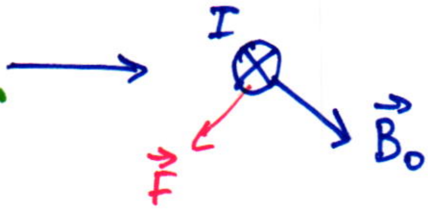
Since $\vec{B} < \vec{B_0}$, χ_m for diamagnetic materials are less than 0

	$\chi_m \times 10^5$
Water	-0.91
Copper	-1.0
Silver	-2.6

#



Consider a diamagnetic particle. It will possess a magnetic field (magnetic moment μ) in the opposite direction of external field.



So, the particle will be repelled by the external field $\vec{B}_0$.
(For example, you may repell water by a bar-magnet)

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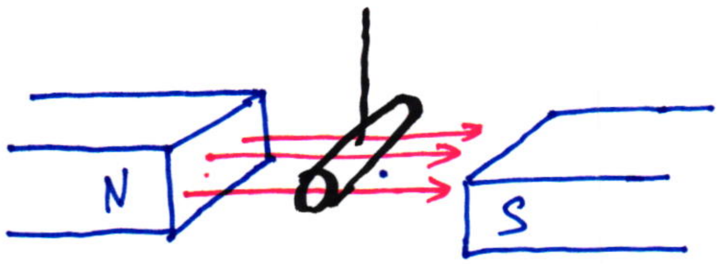
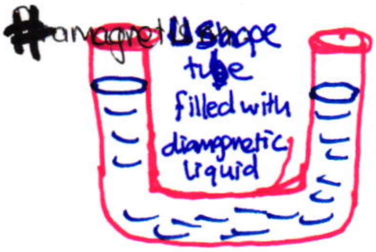
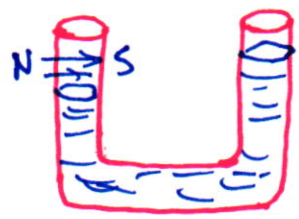


If you locate a glass rod in a magnetic field, it will rotate such that the magnetic field inside is at the least

#

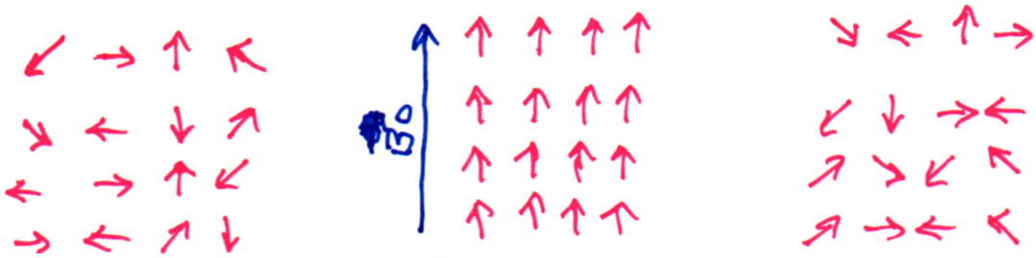


If one arm experiences magnetic field, liquid is depressed



Once the external field in all examples is removed, the net field in a diamagnetic substance becomes 0 again (In fact, any magnetic moment becomes 0 as well)

Paramagnetism: Paramagnetic substances are composed of unpaired electrons, so the net magnetic moment of a paramagnetic atom is not 0 as in diamagnetic case. Nevertheless, in the absence of external magnetic field, electrons so aligned that the net magnetic moment is 0. If an external field is applied, electrons are so aligned to cooperate with external field which increases $\vec{B}$ inside the material.



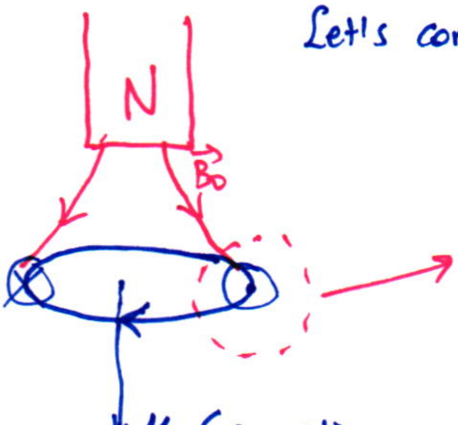
paramagnetic alignment in rest

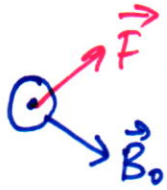
$\vec{B}_0$ is applied

$\vec{B}_0$ is removed

Therefore, a paramagnetic material is not a permanent magnet.
If the applied magnetic field (external), paramagnetic substance is attracted (by the magnet for example)

Let's consider a paramagnetic particle





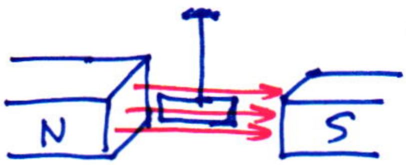
So the paramagnet is attracted by magnet

μ (magnetic moment of particle supports the magnetic field $\vec{B_0}$ (external)

Hence, magnetic field inside a paramagnetic substance is bigger than external field.

$$\vec{B} = \vec{B_0} + \chi \vec{M} \quad (\chi > 0)$$

#



When a paramagnetic rod is applied a uniform magnetic field, it alligns itself in the direction of external field.

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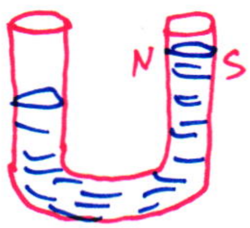
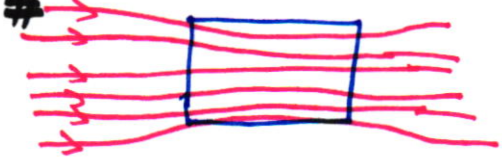


A paramagnetic liquid in U tube is elevated in the magnetic field applied arm.

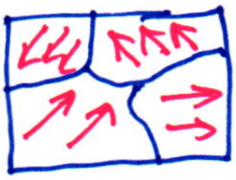
#



Field lines prefer to pass through paramagnetic material instead of air.

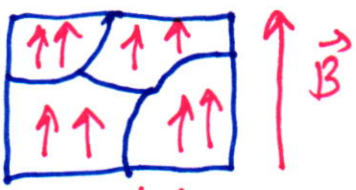
Note that due to the thermal agitation, magnetization of a paramagnetic material decreases by increasing temperature ($\vec{B} \propto T^{-1}$)

Ferromagnetism: Occurs in substances in which atoms have permanent magnetic dipole moments. Even if the external field is removed, dipoles are still in the alignment as they are exerted. In rest, dipoles are so aligned that creating domains.



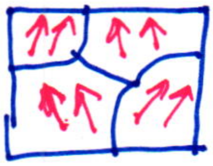
Domains in a ferromagnetic material in the absence of external field.

If an external field is applied, domains tend to align in the same direction



external field is applied

If the external field is removed, domains protect alignment as possible as they can



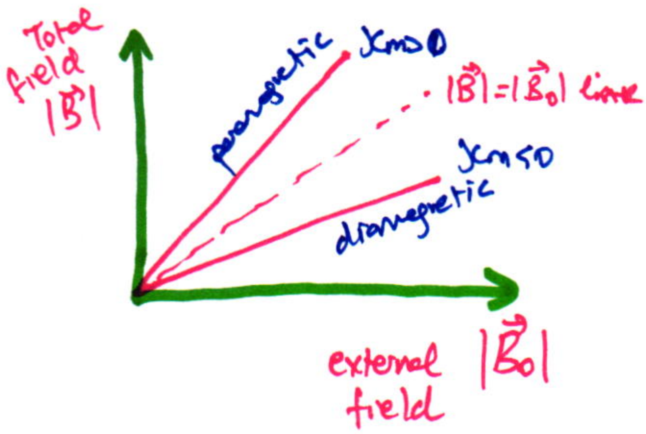
So, the substance becomes a permanent magnet!

The lower the temperature, the higher success of magnetization occurs. (due to the thermal agitation).

If one aims to remove the magnetization of ferromagnetic material, may exert force on it (bang with a hammer) or may increase the temperature {at curie temperature} (ferromagnet becomes paramagnet)

$\vec{B}$ inside material is much higher than external field ($\chi_m \gg 1$)

In diamagnetic and paramagnetic materials, total magnetic field and external magnetic field was related in a linear fashion by χ_m

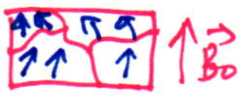


But the story for ferromagnetic materials is considerably different than that of paramagnetic and diamagnetic materials.

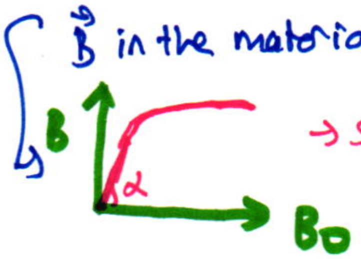
Let's look at the story of a ferromagnetic material in the sense of magnetization, step by step.

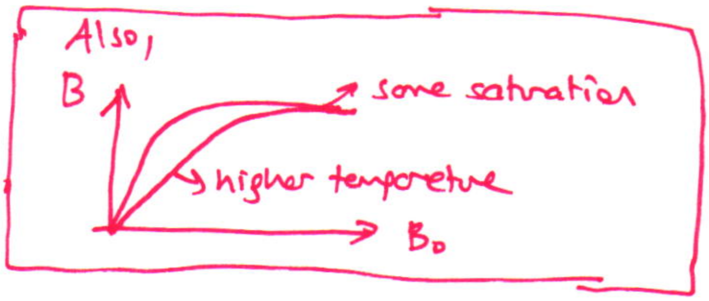
At first, dipole domains responce to the external field quickly and provides a great magnetic field in the same direction. So, $\vec{B}_{tot} \gg \vec{B}_0$

for instance $\tan\alpha \approx 10^3$, great magnetization

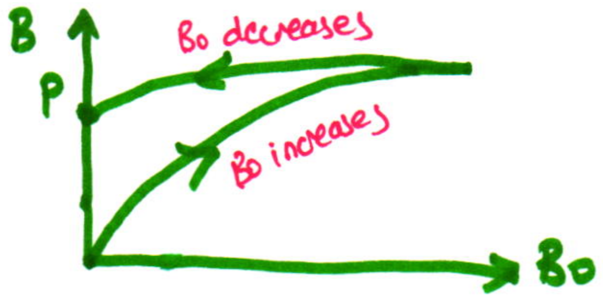


Then, material saturates and increasing $\vec{B_0}$ may not be adequate to increase $\vec{B}$ in the material



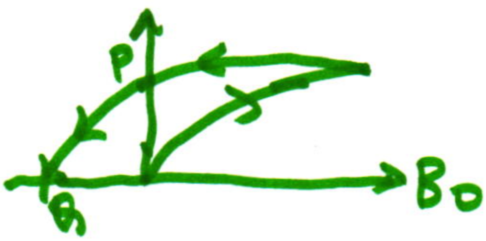


Now, if one decrease Bo down to 0, B inside material would not follow the same path back(and forth). Due to the magnetized domains →

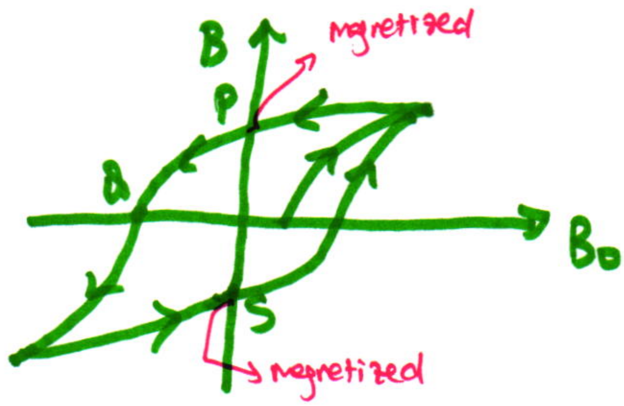


That is, although no Bo exists now, the material is magnetic at a level of point P. This is because some domains remain in alignment with that of the first applied $\vec{B_0}$.

Now if one increases Bo in reverse direction ($B_0 < 0$), domains will align in opposite direction with respect to the first case. So →

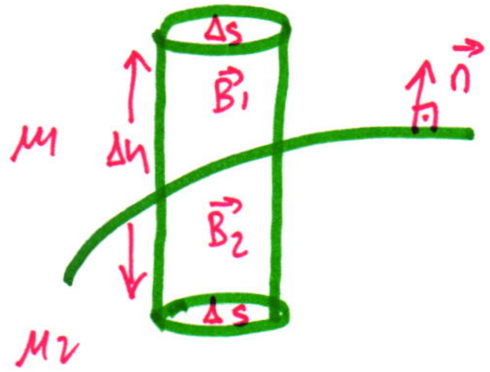


Similarly →



Dependence of magnetization on its history - Hysteresis

Boundary Conditions for Magnetostatic Fields



$$\int \vec{B} d\vec{s} = B_{1n} \cdot \Delta s - B_{2n} \cdot \Delta s \quad \text{as } \Delta h \to 0$$

Recall $\nabla \cdot \vec{B} = 0$ and $\int \nabla \cdot \vec{B} dv = \int \vec{B} d\vec{s}$)

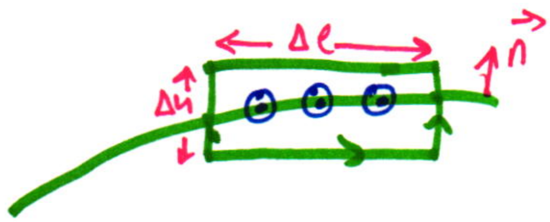
Hence

$$B_{1n} \Delta s - B_{2n} \Delta s = 0 \to \boxed{B_{1n} = B_{2n}}$$

In terms of magnetic field intensity $\vec{H}$,

$$\boxed{\mu_1 H_1 = \mu_2 H_2}$$

In vector form $\boxed{\vec{n} \cdot (\vec{B}_1 - \vec{B}_2) = 0}$



$$\oint_c \vec{H} d\vec{l} = H_{1t} \Delta l - H_{2t} \Delta l \quad \text{as } \Delta h \to 0$$
$$= \int \nabla \times \vec{H} d\vec{s} = \int \vec{J} d\vec{s} = I$$

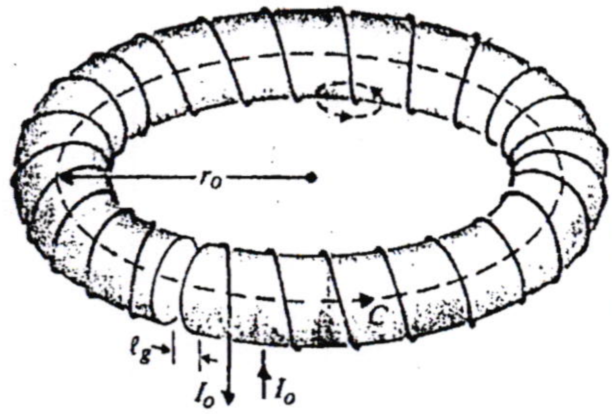
$$\Rightarrow H_{1t} \Delta l - H_{2t} \Delta l = J \Delta l$$

$$\Rightarrow \boxed{H_{1t} - H_{2t} = J}$$

In vector form, $\boxed{\vec{n} \times (\vec{H}_1 - \vec{H}_2) = \vec{J}_{surface}}$

Derivations are not performed in detail because you are familiar with the followed approach due to the same concepts in electrostatics.

MAGNETIC CIRCUITS



Assume that N turns of wire are wound around a toroidal core of a ferromagnetic material with permeability μ. The core has a mean radius r_0, a circular cross section of radius $a (a << r_0)$, and a narrow air gap of length l_g as shown in Figure. A steady current I_0 flows in the wire. Determine (a) the magnetic flux density. B_f, in the ferromagnetic core; (b) the magnetic field intensity, H_f in the core; and (c) the magnetic field intensity, H_g in the air gap.

Ampere's Law yields that $\oint_C \vec{H} d\vec{l} = N I_0$. Path C can be divided in core and air gap ($2\pi r_0 - l_g$ and l_g, respectively). Then,

$H_g l_g + H_f (2\pi r_0 - l_g) = N I_0$. Besides, $\underset{(B_f)}{B_{1n}} = \underset{(B_g)}{B_{2n}}$ at core-airgap interface ($\mu H_f = \mu_0 H_g$)

Combining above expressions;

$$\frac{B_f l_g}{\mu_0} + \frac{B_f (2\pi r_0 - l_g)}{\mu} = N I_0 \Rightarrow \boxed{B_f = \frac{N I_0}{\frac{l_g}{\mu_0} + \frac{r_0 2\pi - l_g}{\mu}} = B_g}$$

Hence, $\boxed{H_f = \frac{B_f}{\mu} = \frac{N I_0}{\mu_r l_g + \frac{2\pi r_0 - l_g}{\cancel{\mu_0}}}}$, $\boxed{H_g = \frac{B_g}{\mu_0} = \frac{N I_0}{\frac{l_g}{\cancel{\mu_0}} + \frac{2\pi r_0 - l_g}{\mu_r}}}$

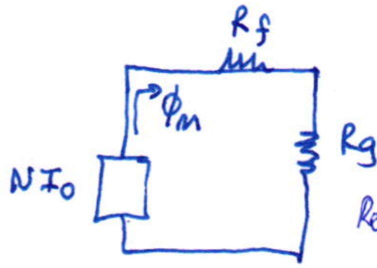
relative permeability of the core

$\left(\mu_r = \frac{\mu}{\mu_0}\right)$

Note that total magnetic flux through the cross-section (with an area of S) is;

$$\phi_M = \int_S \vec{B} d\vec{s} = B.S$$

with a unit weber or Tesla. m^2. Analogous to dc circuit analysis;



$$R_f = \frac{2\pi r_0 - l_g}{\mu . S}, \quad R_g = \frac{l_g}{\mu_0 . S}, \quad \mathcal{F}_m = N I_0$$

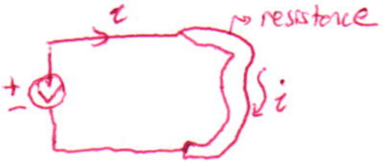
magnetomotive force

Reluctance $R_m = R_f + R_g$

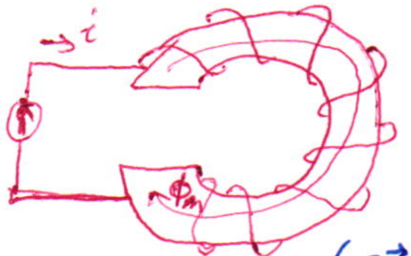
$$\boxed{\phi_m = \frac{\mathcal{F}_m}{R_m}} \left(\text{analogous to } I = \frac{V}{R} \text{ Ohm's Law}\right)$$

Hopkinson's Law

Electrical Circuit Analogy

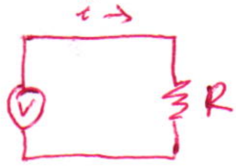


equivalent circuit



charge is conserved

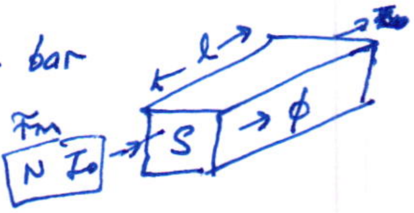
flux is conserved $\left(\oint \vec{B}\,d\vec{s} = \phi_m\right)$



$v = iR$

$\mathcal{F}_m = \phi_m \mathcal{R}$

Reluctance of a bar



As l increases, ϕ decreases

As S decreases, ϕ decreases

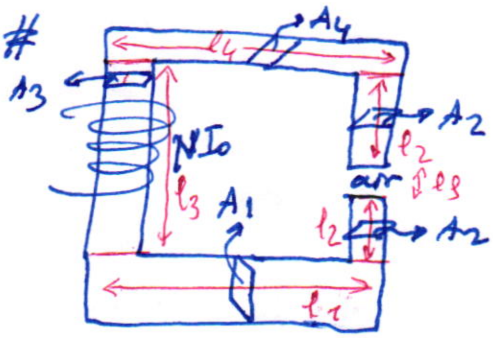
$\phi = BS$, $B \propto 1/L$ $\left(\oint \vec{H}\,d\vec{l} = NI_0\right)$

$H = \frac{NI_0}{L}$

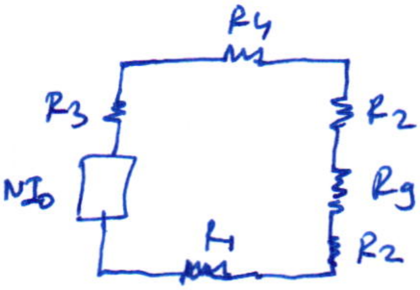
$\mathcal{F}_m = NI_0$

$\phi = \frac{\mu N I_0}{l} S \Rightarrow \mathcal{R} = \frac{\mathcal{F}_m}{\phi} = \frac{l}{\mu S}$

#



$\approx$



$R_3 = \frac{l_3}{\mu A_3}$, $R_2 = \frac{l_2}{\mu A_2}$, $R_1 = \frac{l_1}{\mu A_1}$,

$R_4 = \frac{l_4}{\mu A_4}$, $R_g = \frac{l_g}{\mu_0 A_2}$

Electrical		magnetic
voltage v	→	magnetomotive force $\mathcal{F}_m = N \cdot I_0$
current i	→	magnetic flux ϕ_m
resistance R	→	reluctance R_m
conductivity $1/\rho$	→	permeability μ
current density J	→	magnetic flux density B
electric field E	→	magnetic field intensity H

Similar to Kirchhoff's law, one may write, for any closed path in a magnetic circuit;

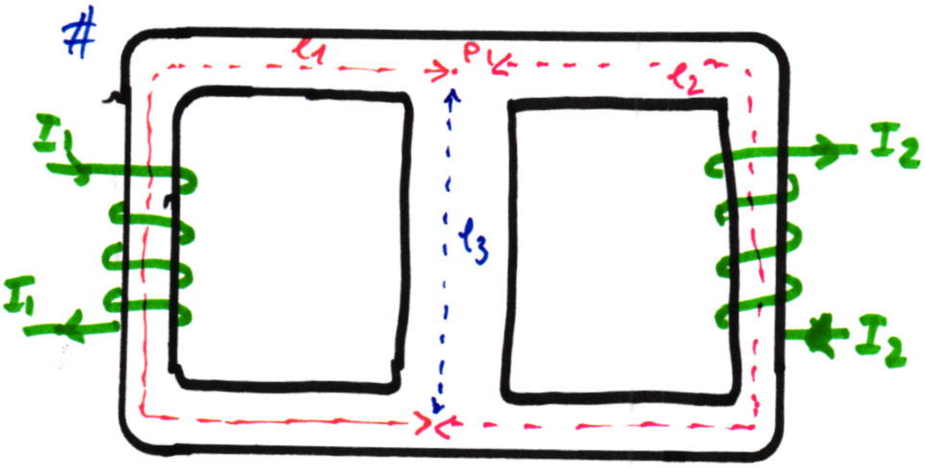
$$\boxed{\sum_j N_j I_j = \sum_k R_k \phi_k}$$

which states that around a closed path in a magnetic circuit the algebraic sum of ampere-turns (NI) is equal to the algebraic sum of the products of the reluctances and fluxes (ϕR)

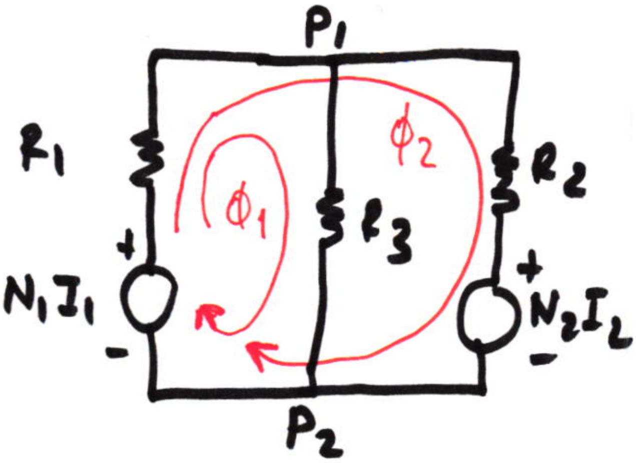
Besides, for a junction $\nabla \cdot \vec{B} = 0$ that is $\int \vec{B} d\vec{s} = \boxed{0 = \sum_j \phi_j}$;

which states that the algebraic sum of all the magnetic fluxes flowing out of a junction is 0.

#



The sectional area of core is S_c. Determine the magnetic flux in the center leg?



Loop 1: $N_1 I_1 = (R_1 + R_3)\phi_1 + R_1 \phi_2$

Loop 2: $N_1 I_1 - N_2 I_2 = R_1 \phi_1 + (R_1 + R_2)\phi_2$

$$\Rightarrow \phi_1 = \frac{R_2 N_1 I_1 - R_1 N_2 I_2}{R_1 R_2 + R_1 R_3 + R_2 R_3}$$

Note that;

$$R_1 = \frac{l_1}{\mu S_c}, \quad R_2 = \frac{l_2}{\mu S_c}, \quad R_3 = \frac{l_3}{\mu S_c}$$