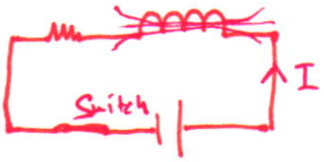
FARADAY'S LAW

Recall that, Oersted shown that an electric current flowing through the wire produces the magnetic field. Faraday asked the other way round, that is, whether if a magnetic field produces electricity or not.



Look at the inductor on the left. The magnetic field produced by inductor was not producing electricity, nowhere.

But once the switch was on and off, Faraday observed an (induced) current on a wire loop due to the changing magnetic field. He discovered Electromagnetic induction. He went on to show that a current is induced in a closed circuit if there is a magnetic flux through it that changes with time, whatever the origin of this flux. That could be a moving bar magnet. He eventually, summarized his results in the following statement (which is known as Faraday's law of induction)

"Whenever there is a change in the magnetic flux linked with a circuit an electromotive force (e.m.f) is induced, the magnitude of which is proportional to the rate of change of the flux through the circuit"

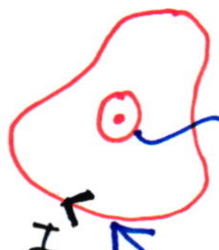
A second law of electromagnetic induction, known as Lenz Law, was formulated by Heinrich Lenz in 1835. It states that;

"The direction of the induced current is always such as to oppose the change producing it."

The two laws can be summarized as;

Induced e.m.f $= \oint_C \vec{E}\,d\vec{l} = -d\phi_B/dt = -d/dt \int_S \vec{B}\cdot d\vec{s}$

Note that the minus sign is due to the Lenz Law. For instance;



Let there be increasing flux (ϕ_B) out of the page. The induced current is such that the producing $\vec{B}$ field is in the opposite direction

Therefore induced current is clock-wise

open surface

If we desire to calculate total magnetic field flows through the surface, we simply sum the B fields perpendicular to the surface at each point.

For a differential surface ds, $d\phi$ (differential flux (which is scalar) is;

$$d\phi_B = \vec{B}.\vec{ds}.\ B.ds.\cos\theta \quad (*)$$

For whole surface;

$$\phi_B = \int_S \vec{B}\,\vec{ds}$$

S → open surface

ϕ_B → magnetic flux [Weber, Tesla.m^2]

→ Note that if it was a closed surface (that is a volume), integral clearly becomes 0 since no magnetic monopole exists ($\nabla.\vec{B}=0$)

In other words, even there exists no battery in the circuit, one may have current flowing by exposing a time-varying magnetic flux.

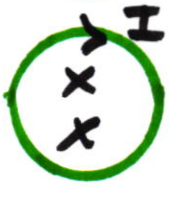
For a coil composed of N turns, the total e.m.f is;

Induced e.m.f $= -N\, d\phi_B/dt$

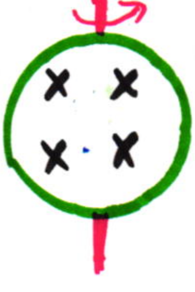
Now, look at equation (*). If one aims to vary ϕ by time, may change B, surface or angle between magnetic field and surface.

I)  $\vec{B}$ uniform into the page — decreases ↓ 

which induces current clock wise

II)  Area of the loop ($\vec{B}$ into the page) — decreases ↓ 

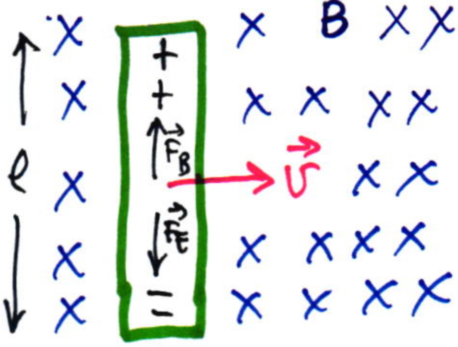
which induces current clock wise

III)  Loop is rotated so the angle between $\vec{B}$ and surface varies — θ increases ↑ 

which induces current clock wise

If you move a bar magnet towards a loop, magnetic field increases downwards. A current flows on loop such that killing this change as possible as it is. So the direction of the current is such that the producing magnetic field by the loop is upwards. Hence current flows as it is shown on the left.

Motional EMF (A moving Conductor in a Static Magnetic Field) $d\vec{B}/dt = 0$



Consider a conducting bar with a length ℓ moving through a uniform $\vec{B}$ (which is directed into the page). Positive charges experience a magnetic force $\vec{F}_B = q\vec{v}\times\vec{B}$ and moves upward for given geometry, therefore negative charges moves downward

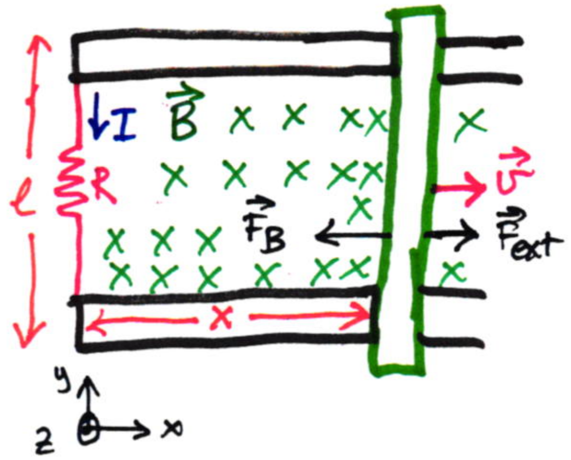
Charge move becomes in equilibrium when $\vec{F}_B = \vec{F}_E$ (+ - charge distribution gives rise to $\vec{E}$ field, hence $\vec{F}_e$ occurs)

Disregarding directions, if $\vec{F}_B = \vec{F}_e$, then $qvB = qE$ that is $E = vB$

Potential difference between end points of conductor then becomes $\Delta V = E\ell$
$= B\ell v$

In general, motional e.m.f around a closed loop (conducting) can be written as;

$$\varepsilon = \oint_C (\vec{v}\times\vec{B})\, d\vec{\ell}$$



A conducting bar slides in $\vec{a_x}$ direction through a uniform magnetic field $B_0(-\vec{a_z})$ on conducting rails that are at a distance ℓ apart and connected by a resistor R.
An external force is applied on conducting bar such that the velocity $\vec{v} = v_0\vec{a_x}$

The magnetic flux $\phi_B = \int \vec{B}\,d\vec{s} = B\ell x$ through the closed loop formed by the bar and the rail.

Induced e.m.f $\mathcal{E} = -d\phi_B/dt = -B\ell \underbrace{(dx/dt)}_{\text{speed}} = -B\ell v$

So the induced current is $I = -B\ell v/R$ (minus sign indicates direction, be careful)

Note that when integrating $\int \vec{B}\,d\vec{s}$, we have chosen $\vec{B} \parallel d\vec{s}$ (so the result is positive). That is the closed loop attached to $d\vec{s}$ is in clockwise direction

That is

$\otimes d\vec{s}$ $\quad d\vec{\ell}$

Hence current with minus sign is as to this rotation.
There fore $I = B\ell v/R\,(-\vec{a_y})$ as depicted in the figure.
↳ on the resistance

The magnetic force experienced by the bar as it moves to the right is

$$\vec{F_B} = I\int d\vec{\ell}\times\vec{B} = I\ell(\vec{a_y})\times(-B_0\vec{a_z}) = -I\ell\vec{a_z}B = -\frac{B^2\ell^2 v}{R}\vec{a_x}$$

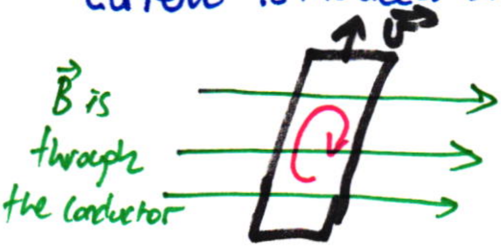
If the bar moves with a constant velocity, net force on bar is 0.
So the external force equals to magnetic force for a constant velocity. Hence;

$$-\vec{F_B} = \vec{F_{ext}} = \frac{B^2\ell^2 v}{R}\vec{a_x}$$

mechanical power dissipated by the bar $P_m = \vec{F_{ext}}\cdot\vec{v} = \dfrac{B^2\ell^2v^2}{R}$ is equal to electrical power dissipated by the resistor $P_e = \dfrac{\mathcal{E}^2}{R} = \dfrac{B^2\ell^2v^2}{R}$ which satisfies energy conservation.

Eddy Currents

If a solid conductor is used instead of a conducting loop, a circulating current is induced and is called as eddy current



The induced eddy currents generate a magnetic force that opposes the motion, making it more difficult to move the conductor across the magnetic field.

If one increases the resistance of conductor, counteract against motion decreases (a demo will be shown)

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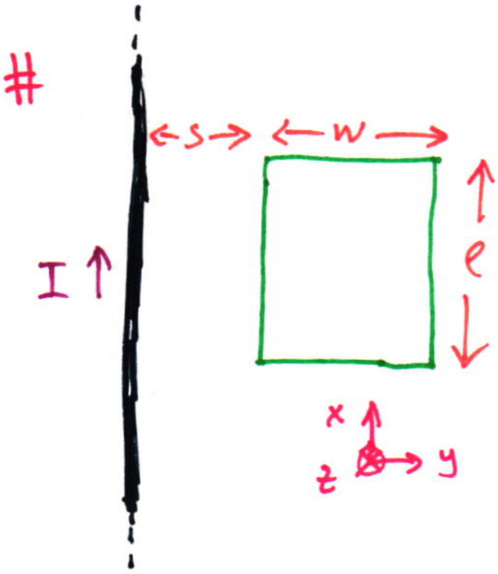


a) Determine the magnetic flux through the rectangular loop due to the current I ?

b) Determine the induced e.m.f in the loop and direction of induced current if $I = \sin(2\pi t)$

↳ time [second]

a) The magnetic field produced by $I \rightarrow \oint_C \vec{B}\, d\vec{l} = \mu_0 I \Rightarrow \vec{B} = \mu_0 \dfrac{I}{2\pi \rho} \vec{a_\phi}$

Magnetic flux ϕ_M through loop is $\int \vec{B} d\vec{s}$

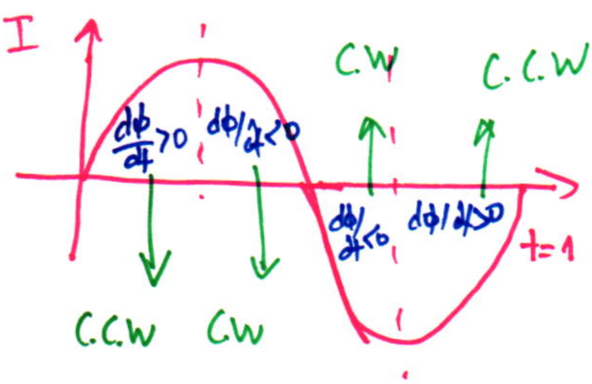
$$\Rightarrow \phi_M = \int_{x=x'}^{x'+l} \int_{y=s}^{s+w} \mu_0 \frac{I}{2\pi y} \vec{a_z} \cdot dx\, dy\, \vec{a_z}$$

$$= \frac{\mu_0 l . I}{2\pi} \int_{y=s}^{s+w} \frac{dy}{y} = \frac{\mu_0 l I}{2\pi} \ln\left(\frac{s+w}{w}\right) \quad [T.m^2 \text{ or } Weber]$$

b) $$\text{Induced e.m.f} = \mathcal{E} = \frac{-d\phi_M}{dt} = \frac{-d}{dt}\left(\frac{\mu_0 l I}{2\pi} \ln\left(\frac{s+w}{w}\right)\right)\Bigg|_{I = \sin(2\pi t)}$$

$$= -\mu_0 l \ln\left(\frac{s+w}{w}\right) \cos(2\pi t) \; [\text{Volt}]$$

w.r.t Lenz's law, current flows so that to decrease magnetic flux change



direction of induced current

C.C.W : counter clock-wise

C.W : clock-wise

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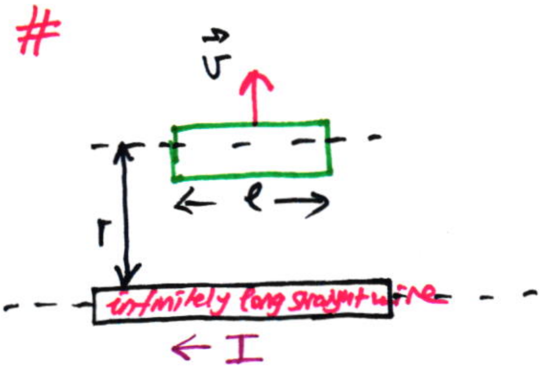


Determine the generated e.m.f between the ends of conducting rod, moving with a constant velocity $\vec{v}$.

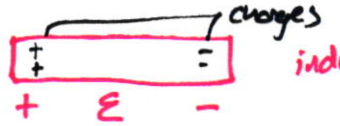
$$\mathcal{E} = \int(\vec{v}\times\vec{B})\,d\vec{\ell} \ ; \quad \vec{B} = \frac{I}{2\pi r}\,\vec{a_\phi} \quad (\vec{a_\phi} \text{ is into the page})$$

$$\Rightarrow \mathcal{E} = B v \ell = \frac{\mu_0 I}{2\pi r} v \ell$$

Note that sign is positive. That is $\vec{v}\times\vec{B} \,//\, d\vec{\ell}$ is chosen

Then, $d\vec{\ell}$ is directed to the left ($\leftarrow$)

Then



induced emf on rod

The Displacement Current

Recall that $\oint \vec{B} d\vec{l} = \mu I_{enclosed}$ (where $\mu = \mu_0 \cdot \mu_r$) according to Ampere's Law

Also, Faraday Law states $\oint_C \vec{E} d\vec{l} = -\frac{d}{dt} \int_S \vec{B} d\vec{s}$ which states that contour integral of $\vec{E}$ is proportional to a changing $\vec{B}$ by time. If a time-varying $\vec{E}$ field (dE/dt) produces magnetic field, the Ampere's Law is not valid as we mentioned before.

Now, let's analyze a capacitor. During the charge of capacitor, $\vec{E}$ field increases between plates as the amount of charges increases on the plates. Besides, the current on the wire create a magnetic field.

If Surface is S_1, than $\oint_C \vec{B} d\vec{l} = \mu_0 I$

If Surface is S_2, than $\oint_C \vec{B} d\vec{l} = 0$ (No current flows between plates)

So, there is a lack at Ampere's Law. To amend this lack, Maxwell suggested to add a term to the Ampere's Law, the displacement current I_d

The displacement current I_d involves time-varying electric flux and dielectric properties of the medium, that is;

$$I_d = \varepsilon \frac{d\phi_E}{dt} \quad \text{where } \varepsilon = \varepsilon_0 \varepsilon_r \text{ and } \phi_E = \int_S \vec{E} d\vec{s}$$

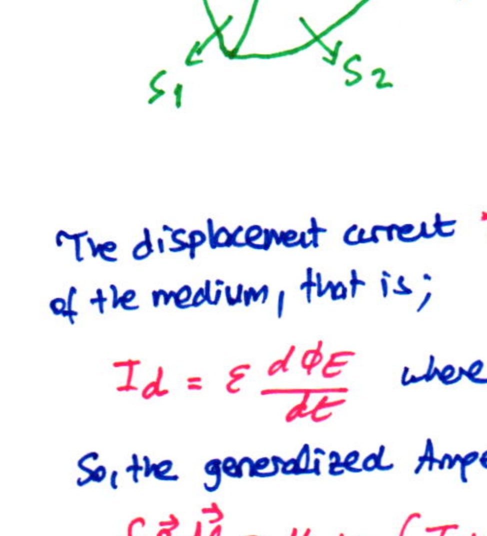
So, the generalized Ampere's Law reads;

$$\oint_C \vec{B} d\vec{l} = \mu_0 \mu_r \left(I + \varepsilon_0 \varepsilon_r \frac{d\phi_E}{dt} \right) = \mu \left(I + I_d \right)$$

→ conduction current (I)

↳ displacement current (I_d)

Now, let insert Amperian loop inside the capacitor;



The electric flux ϕ_E passing through S_2 is;

$$\int_{S_2} \vec{E} d\vec{s} = \phi_E = EA = Q / \varepsilon_0 \varepsilon_r$$

A ↓ area of capacitor plates

$\varepsilon_0 \varepsilon_r$ ↳ dielectric value in between plates of capacitor

Hence;

$$\boxed{\varepsilon_0 \varepsilon_r \frac{d\phi_E}{dt}} = \boxed{\frac{\varepsilon_0 \varepsilon_r}{\varepsilon_0 \varepsilon_r} \frac{dQ}{dt}}$$

which is equal to conduction current on the wire

which is defined as "displacement current" I_d.

This equivalence yields that the conduction current I, passes through S_1 is equal to the displacement current I_d, passes through S_2

MAXWELL'S EQUATIONS

LAW	Equation in Integral form	Equation in Diff. form	Physical Interpretation
Gauss Law for $\vec{E}$	$\int \vec{E} d\vec{s} = Q/\varepsilon$	$\nabla \cdot \vec{D} = \rho$ (charge density)	Electric flux through a closed surface is proportional to the enclosed charge
Faraday's Law	$\oint_C \vec{E} d\vec{l} = -d\phi_B/dt$	$\nabla \times \vec{E} + \frac{d\vec{B}}{dt} = 0$	Changing magnetic flux produces electric field
Gauss's Law for $\vec{B}$	$\oint_S \vec{B} d\vec{s} = 0$	$\nabla \cdot \vec{B} = 0$	The total magnetic flux through a closed surface is 0, no magnetic monopole (source) exists.
Ampere-Maxwell Law	$\oint_C \vec{B} d\vec{l} = \mu \left(I + \varepsilon \frac{d\phi_E}{dt}\right)$;	$\nabla \times \vec{H} = \vec{J} + \frac{d\vec{D}}{dt}$;	Electric current and changing electric flux produces a magnetic field

In equations above, $\varepsilon = \varepsilon_0 \varepsilon_r$, $\mu = \mu_0 \mu_r$ where ε_r and μ_r are relative dielectric permittivity and relative magnetic permeability of the medium, considered. Also, $\vec{J}$ is the conduction current density. Note that $\vec{D} = \varepsilon \vec{E}$ & $\vec{B} = \mu \vec{H}$

(Maxwell's Equations will be explained during class in detail)

Since $\nabla \cdot (\nabla \times)$ of a vector field is zero, one may conclude $\nabla \cdot \vec{J} + \frac{d}{dt} \nabla \cdot \vec{D} = 0$. which gives $\nabla \cdot \vec{J} + d\rho/dt = 0$. This implies the conservation of charges.