

The books and GSM phones are not allowed. The lecture notes and the calculators can be used. All the answers must be clearly stated, otherwise no partial credit will be given. The duration is 120 minutes. **Solve each question on a separate page.**

1. Vector Spaces (20 pts)

1.1 a) Find the value of k if the intersection of the following volumes gives a surface in \mathbb{R}^4 .

$$\frac{x}{2} = -k^3y + \left(\frac{k^2}{2}\right)z - \frac{4}{k}t + b_1, \quad 16y = -x + 4z - k^2t + b_2 \quad (10 \text{ pts}).$$

- b) Determine the value of k if the intersection is a volume (5. Pts)
- c) Under which condition does the space found in 'b' satisfy the vector space rules (5 pts)?

2. Linear Transformations (20 pts)

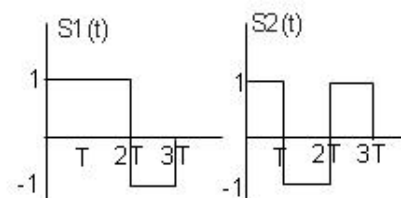
2.1 The transformation $(T: \mathbb{R}^n \rightarrow \mathbb{R}^n)$ transforms any nxn matrix into its eigenvalue matrix (Λ). Determine, if this transformation is linear (10 pts)

2.2 Show that there is a linear transformation $T: \mathbb{R}^2 \rightarrow \mathbb{R}^4$ such that: $T([-1 \ 1]^T) = [2 \ 3 \ 1 \ -2]^T$, $T([2 \ 3]^T) = [1 \ 2 \ 4 \ -1]^T$ (10 pts).

3. Orthogonality (25 pts)

3.1. Show that scalar multiplication (inner product) of two unit vectors gives the cosine of the angle between these vectors. (**Do not use any formula**) (10 pts)

3.2. a) Find an orthonormal set of signals by using $S_1(t)$ and $S_2(t)$. (10 pts)



- b) How many orthonormal functions can be added to the function set in (a). Explain the reason. (5 pts)

4. Eig.Value, Eig. vector problems, Power Series, Stability (55 pts)

4.1 Consider the diff. Equation: $V_c''(t) + \frac{R}{L}V_c'(t) + \frac{1}{LC}V_c(t) = 0$. Here $R=1000$ and the characteristic equation is: $\lambda^2 + 10\lambda + 9 = 0$. Find L and C in the equation and solve the diff. equation (10 pts). Propose a nonzero initial condition set that results in shortest time to reach the steady state (10 pts).

4.2 . Multinational companies in the America, Asya and Europe have assets of \$4 trillion. At the start, \$2trillion are in the Americas and \$2trillion is in the Europe. Each year $\frac{1}{2}$ of the American Money stays at home, and $\frac{1}{4}$ goes to each of Asia and Europe. For Asia and Europe, $\frac{1}{2}$ stays home and $\frac{1}{2}$ is sent to the Americas.

a) Which one of the following dfferential equation set is true? (A_k : Asset in American companies in year k, AS_k : Asset in Asian companies in year k, E_k : Asset in Europe companies in year k,) (10 pts)

$$\begin{bmatrix} A_{k+1} \\ AS_{k+1} \\ E_{k+1} \end{bmatrix} = \begin{bmatrix} .5 & -.25 & -.25 \\ .5 & -.5 & -.5 \\ .5 & -.5 & 0 \end{bmatrix} \begin{bmatrix} A_k \\ AS_k \\ E_{Gk} \end{bmatrix} \quad \begin{bmatrix} A_{k+1} \\ AS_{k+1} \\ E_{k+1} \end{bmatrix} = \begin{bmatrix} -.5 & .5 & .5 \\ .25 & -.5 & 0 \\ .25 & 0 & -.5 \end{bmatrix} \begin{bmatrix} A_k \\ AS_k \\ E_{Gk} \end{bmatrix} \quad \begin{bmatrix} A_{k+1} \\ AS_{k+1} \\ E_{k+1} \end{bmatrix} = \begin{bmatrix} .5 & .5 & .5 \\ .25 & .5 & 0 \\ .25 & 0 & .5 \end{bmatrix} \begin{bmatrix} A_k \\ AS_k \\ E_{Gk} \end{bmatrix}$$

- b) Find the distribution of the \$4 trillion dolar at year k. (15 pts)
- c) Find the limiting distribution of the \$4 trillion dolar as the world ends (steady state) ? (10 pts)