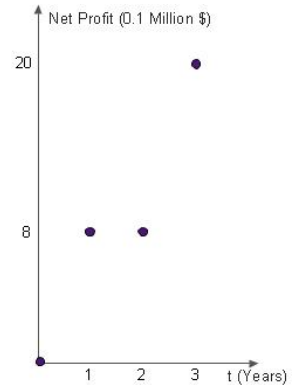


The books and GSM phones are not allowed. The lecture notes and the calculators can be used. All the answers must be clearly stated, otherwise no partial credit will be given. The duration is 110 minutes.

1. A company's annual net profits during 4 years can be seen on the graph. The profit estimation for the 4th year is required to prepare an investment plan. Find the equations to make a parabolic estimation of the net profit for the 4th year. (25 pts)

Hint: Write the equations to solve C, D and E in the parabola, $P(t) = C + Dt + Et^2$. **Note that you don't need to solve C, D and E, just write the set of equation. Leave them unsolved.**



2. a) Find an orthonormal vector set by using the vectors $[1 \ 1 \ -1]$ and $[1 \ -1 \ 1]$. (15 pts)
 b) How many orthonormal vectors can be added to this vector set, explain the reason. (5 pts)
3. Consider the equation set below:

$$\text{Eq1:} \quad x + 3y + 5z = 1$$

$$\text{Eq2:} \quad 3x + y - z = 5$$

$$\text{Eq3:} \quad x - z = 1.75$$

- a) Find all $b = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix}$ vectors, if the following equation can be written as the linear combination of Eq1, Eq2 and Eq3. (15 pts)

$$\text{Eq:} \quad b_1x + b_2y + b_3z = b_4$$

- b) Does the vector set found in (a) form a subspace in \mathbb{R}^4 (5 pts)?
 c) Choose a 'b' vector that makes the equation set unsolvable (Explain how is 'b' chosen). Then show that the equation set (Eq1, Eq2, Eq3 and Eq4) is unsolvable. (5)

4. Suppose each gibbonacci number is G_{k+2} is the average of two previous numbers G_{k+1} and G_k Then;

$$G_{k+2} = \frac{G_k + G_{k+1}}{2}, G_{k+1} = G_{k+1}$$

$$\begin{bmatrix} G_{k+2} \\ G_{k+1} \end{bmatrix} = \begin{bmatrix} & A \end{bmatrix} \begin{bmatrix} G_{k+1} \\ G_k \end{bmatrix}$$

- a) Determine the state transition matrix. (5 pts)
 b) Find the eigenvalues and eigenvectors of A. (10 pts)
 c) Indicate the characteristic equation (5 pts)
 d) Indicate the eigenvectors on the complex plane. Comment on the stability of the process. (10 pts)
 e) If $G_0=0$ and $G_1=1$ show that the gibbonacci numbers approach to 0.666. (15 pts)
 f) Determine the initial condition that results in zero steady state. (10 pts)