The books, GSM phones and calculators are not allowed. The lecture notes can be used. ***All the answers must be clearly stated, otherwise no partial credit will be given****.The duration is 100 minutes.*

Name:

Student ID:

1. Suppose an m by n (m×n) matrix A has rank r. Pick three **nonzero , different** numbers from your student ID. These numbers will be used as m,n and r values (choose m, n and r properly). Complete the following sentences according to this information.
   a. The equation A*x*=b (always / sometimes but not always) has (a unique solution / many solutions / no solution). (5 pts)
   b. The column space of A is .... dimensional inside a .... dimensional space. The column space (contains / does not contain) all of the n dimensional vectors. (5 pts)
   c. The left null space of A has the dimension of ..... inside $\mathbb{R}^{\cdots\cdots}$. (5 pts)

2. a) Find all of the vectors in the space formed by the intersection of the following planes in $\mathbb{R}^3$, $x = 3z \text{ and } y = 3x + 2$ (20 pts).

   b) Does the set of vectors in the intersection form a vector space? Explain the reason (10 pts).

   c) Let the vectors in x=3z plane spans S1 space and S2 space is spanned by the vectors in y=3x+2 plane. S1 and S2 spaces are in $\mathbb{R}^3$. State the following equation:

   dim(S1) + dim(S2)=dim(S1∩S2) + dim(S1+S2). (15 pts)

3. Consider the A matrix given below;

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 2 & 0 \end{bmatrix}$$

   a. Is it possible to find a B matrix such that B×A gives identity matrix? If possible, find how many B matrices satisfy this condition. If not possible, explain the reason (10 pts). ***YOU DO NOT NEED TO CALCULATE THE 'B' MATRICES, JUST COMMENT ON THE POSSIBILITY.***
   b. Calculate the complete solution for $A^T x = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$. (10 pts)
   c. Find the row space and column space of the A matrix (10 pts).
   d. Consider C vectors satisfying C×A=0 equation. Find the basis of the space formed by C vectors. Is this space a vector space? Explain. (10 pts)

4. Consider the following equation set,

$$\begin{aligned} Eq1: &\quad x + \propto y + 5z + 3t = 1 \\ Eq2: &\quad 3x + 3y + \propto^2 z - 9t = 2 \\ Eq3: &\quad x + y + 3z + \propto t = 2/3 \end{aligned}$$

   a) Calculate ∝ if that equation set has infinite solutions spanning a ***2-dimensional*** space(15 pts).
   b) Calculate ∝ if that equation set has infinite solutions spanning a ***1-dimensional*** space(15 pts).