



**ENVE 302**

# **Environmental Engineering Unit Processes**

**CHAPTER: 3**

## **Bacterial Growth Kinetics**

**Assist. Prof. Bilge Alpaslan Kocamemi**

Marmara University

Department of Environmental Engineering

Istanbul, Turkey

# Microbial Growth Kinetics

Rate of growth of bacterial cells can be defined as

$$r_g = \mu \cdot X$$

rate of increase in bacterial pop. is proportional to the number of bacteria present

Autocatalytic rxn  
 $aA \rightarrow bB \quad r=k[B]$

$r_g$  = net biomass production rate (g VSS/m<sup>3</sup>.d)

$\mu$  = specific biomass growth rate  $\left( \frac{\text{g VSS}}{\text{g VSS.d}} \right)$

$X$  = biomass concentration  $\left( \frac{\text{g VSS}}{\text{m}^3} \right)$

Growth rate of bacteria  $\rightarrow$  can be defined by Monod eqn  $\frac{\text{g new cells}}{\text{g cells.d}}$

$$\mu = \mu_m \frac{S}{K_s + S}$$

where  $\mu$  = specific biomass growth rate

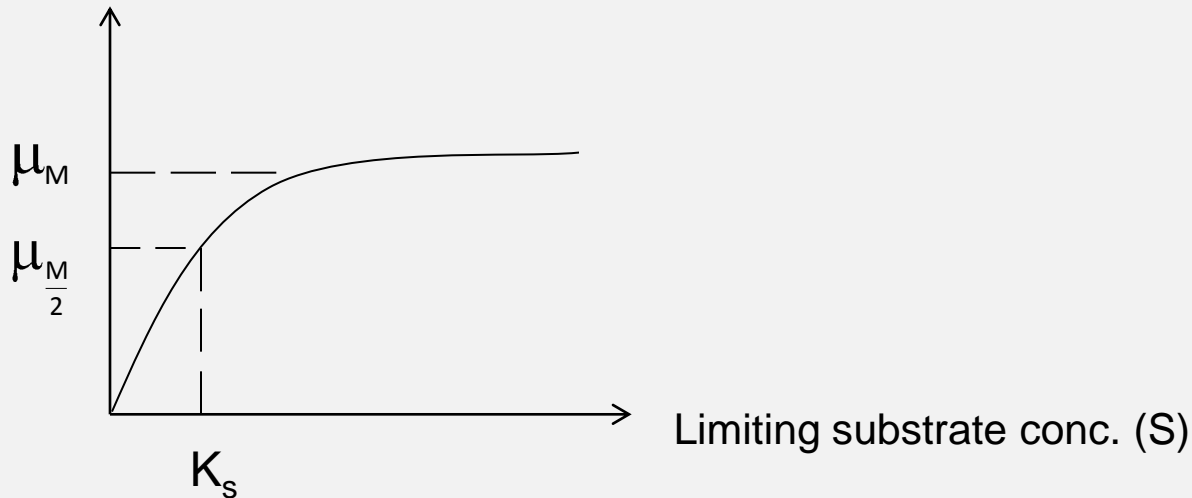
$\mu_{MF}$  max specific biomass growth rate (g VSS/g VSS.d)

The growth limiting substrate can be any of the essential (i.e e<sup>-</sup> donor, e<sup>-</sup> acceptor or nutrients) but often it is the e<sup>-</sup> donor that is limiting, as other requirements are usually available in excess.

←  $S$  = growth limiting substrate conc in solution (g/m<sup>3</sup>)

$K_s$  = half velocity constant (substrate conc at one half the max specific biomass growth rate (g/m<sup>3</sup>))

Specific growth rate ( $\mu$ )



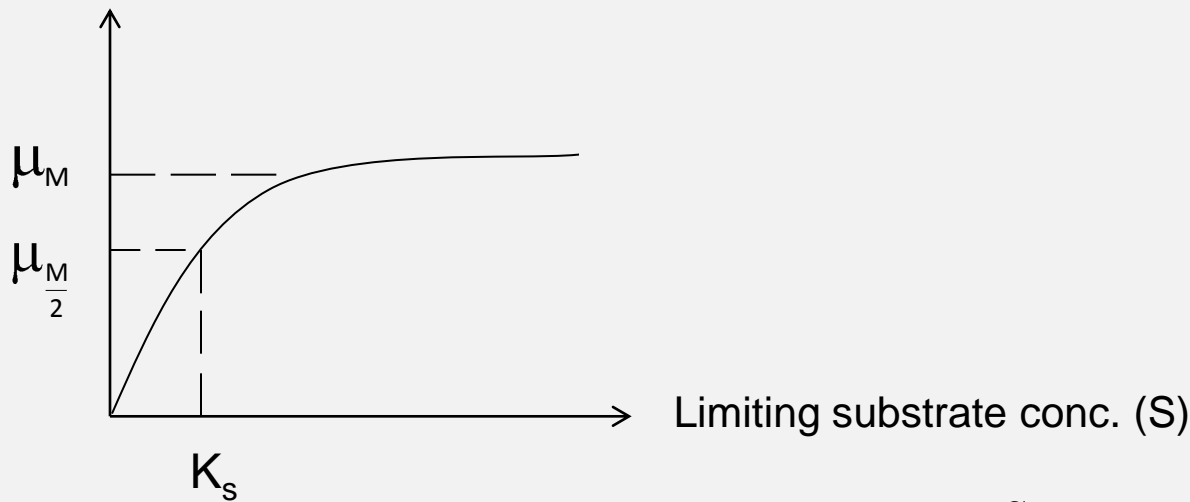
**For high substrate conc ( $S \gg K_s$ )**  $\rightarrow \mu = \mu_m \frac{S}{K_s + S}$

$\downarrow$   
negligible with respect to  $S$

$\rightarrow \mu = \mu_M$

Bacteria grow at their max. rates } Growth rate is independent from substrate conc.  $\rightarrow$  Zero order kinetics<sup>4</sup>

Specific growth rate ( $\mu$ )



**For low substrate conc ( $S \ll K_s$ )**  $\rightarrow \mu = \mu_m \frac{S}{K_s + S}$

$\downarrow$   
 Negligible with respect to  $K_s$

$\rightarrow \mu = (\mu_M/K_s)S \rightarrow$  First order kinetics

$r_g = \mu \cdot X$

Growth rate is dependent on subs conc

$$\left. \begin{array}{l} r_g = \mu \cdot X \\ \mu = \mu_M \frac{S}{K_s + S} \end{array} \right\} \frac{r_g}{X} = \mu_m \frac{S}{K_s + S} \rightarrow r_g = \frac{\mu_m X S}{K_s + S}$$

$$Y = \text{biomass yield coeff} = \frac{\text{g biomass produced}}{\text{g substrate utilized}}$$

$$r_g = \text{biomass production rate (g VSS/m}^3\text{d)}$$

$$r_{su} = \text{substrate utilization rate (g subst/m}^3\text{d)}$$

$$Y = \frac{r_g}{-r_{su}}$$

$$r_g = -Yr_{su}$$

(-) → substrate is decreasing with time due to substrate utilization

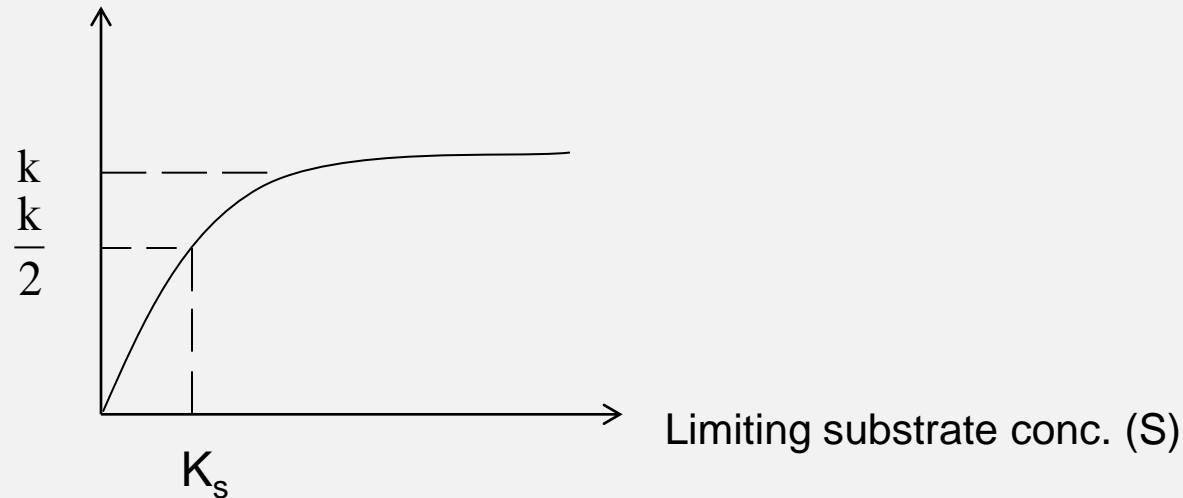
$$r_g = \frac{\mu_m X S}{K_s + S}$$

$$r_g = -Yr_{su}$$

$$r_{su} = \frac{-\mu_m}{Y} \frac{XS}{K_s + S}$$

**k = max specific substrate utilization rate**

Substrate utilization rate (mg/Ld)

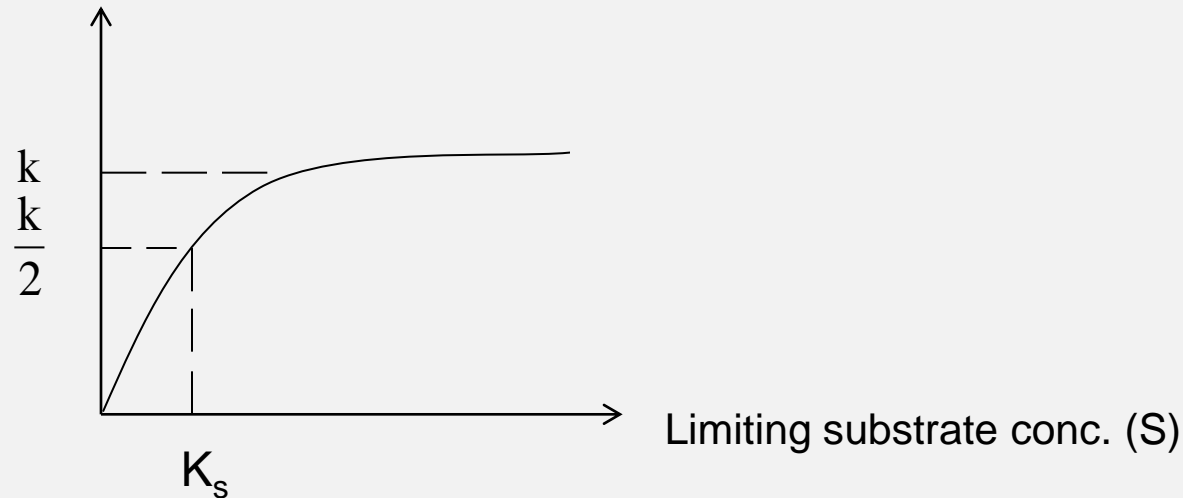


**For high substrate conc ( $S \gg K_s$ )**  $\rightarrow r_{su} = -\frac{kXS}{K_s + S} = -k.X$

$\downarrow$   
Negligible with respect to S

Subst. utilization  $\rightarrow$  zero order  $\rightarrow$  independent from subst conc.

Substrate utilization rate (mg/Ld)



**For low substrate conc ( $S \ll K_s$ )**  $\rightarrow r_{su} = -\frac{kXS}{K_s + S} = \frac{-kXS}{K_s}$

$\downarrow$   
negligible with respect to  $K_s$

Subst.utilization  $\rightarrow$  1<sup>st</sup> order  $\rightarrow$  subst utilization rate  $\rightarrow$  dependent on subst. conc.



Rate of growth & rate of substrate utilization expressions must be corrected to account for

1.the energy required for cell maintenance

2.cell death

→ m.o in all growth phases require energy for cell maintenance

→ in bacterial systems used for ww treatment not all the cells in the system are in log growth phase

**Endogenous decay coeff rate (time<sup>-1</sup>)** → accounts for the cell mass due to oxidation of internal storage products for energy for cell maintenance, cell death

The decrease in cell mass caused by energy requirement for cell maintenance and cell death is proportional to the concentration of m.o present

The endogenous decay term can be formulated as follows:

$$r_d = -k_d X \quad K_d = \text{endogenous decay coeff.} \left( \frac{\text{gVSS}}{\text{gVSSd}} \right)$$

$$r_g = \frac{\mu_m X S}{K_s + S}$$

$$r_d = -k_d X$$

$$r'_g = \frac{\mu_m X S}{K_s + S} - k_d X$$

↓  
net rate of bacterial growth (g VSS/m<sup>3</sup>d)

$$r_g = -Y r_{su} \rightarrow r'_g = -Y r_{su} - k_d X$$

$$r_g' = \frac{\mu_m X S}{K_S + S} - k_d X \quad \frac{r_g'}{X} = \frac{\mu_m X S}{K_S + S} - k_d X \Rightarrow \mu' = \frac{\mu_m S}{K_S + S} - k_d$$

$$r_g = \mu X$$

not specific growth rate, time<sup>-1</sup>

$$Y = -\frac{r_g}{r_{su}} \Rightarrow Y_{obs} = -\frac{r_g'}{r_{su}}$$

# Effect of Temperature

Effect of temperature on the rxn rate of a biological process is expressed as follows

$$k_T = k_{20} \phi^{(T-20)}$$

$k_T$  = rxn rate coeff at temp T

$k_{20}$  = rxn rate coeff at 20°C

$\phi$  = temp-activity coeff (1.02-1.25)

T = temperature, °C

# Total Volatile Suspended Solids and Active Biomass

X in kinetic expressions used to describe biological kinetics

} Active biomass

VSS in a reactor consist of

→ nonbiodegradable VSS (nbVSS) in the influent ww fed to the biological reactor

→ active biomass

→ cell debris following endogenous decay

During cell death → cell lysis occurs

- release of cellular material into liquid for consumption by other bacteria
- a portion of cell mass (cell wall) is not dissolved and remains as nonbiodegradable material
- cell debris = 10-15 % original cell weight
- cell debris is also measured as VSS contributes to total VSS conc measured in the reactor mixed liquor

The rate of production of cell debris  $\propto$  endogenous decay rate

$$r_{xd} = F_d(k_d)X$$

$$r_{xd} = \text{rate of cell debris production} \left( \frac{\text{g VSS}}{\text{m}^3 \text{d}} \right)$$

$F_d$  = fraction of biomass that remains as cell debris 14  
(0.10 - 0.15 g VSS/gVSS)

Total VSS production rate in the aeration tank can be defined as follows

$$r_{\text{XT,VSS}} = \underbrace{-Yr_{\text{su}} - k_d X}_{\text{active biomass from soluble biodegradable COD}} + \underbrace{f_d \mu_d X}_{\text{nbVSS from cell debris}} + \underbrace{\frac{QX_{0,i}}{V}}_{\text{nbVSS in influent}}$$

$r_{\text{XT,VSS}}$  = total VSS production rate  $\text{g/m}^3\text{d}$

$Q$  = influent flowrate,  $\text{m}^3 / \text{d}$

$X_{0,i}$  = influent bVSS conc.,  $\text{g/m}^3$

$V$  = volume of reactor

$$Y_{\text{obs}} = \frac{\text{g VSS produced}}{\text{g substrate utilized}} = \frac{-r_{\text{XT,VSS}}}{r_{\text{su}}}$$

**Example:** For an industrial ww activated sludge process

influent ww  $\rightarrow$  bsCOD=300 g/m<sup>3</sup>

nbVSS=50 g/m<sup>3</sup>

Q=1000 m<sup>3</sup>/d

in the reactor  $\rightarrow$  X=2000 mg/L

bsCOD=15 g/m<sup>3</sup>

$\nabla$  =105 m<sup>3</sup>

cell debris fraction ( $F_d$ )=0.10

Determine ;

1.net biomass yield

2.observed solids yield

3.Biomass fraction in mass



**Table 7-9**

Typical kinetic coefficient for the activated sludge process for the **removal of organic matter** from domestic wastewater at 20°C

Coefficient	Unit	Value	
		Range	Typical
<b>k</b>	g bsCOD / g VSS.d	2-10	5
<b>K<sub>s</sub></b>	mg/L BOD	25-100	60
	mg/L bsCOD	10-60	40
<b>Y</b>	mg VSS/ mg BOD	0,4-0,8	0,6
	mg VSS/ mg bsCOD	0,3-0,6	0,4
<b>k<sub>a</sub></b>	g VSS/ g VSS.d	0,06-0,15	0,10

Ref: Metcalf & Eddy

**Table 8-10**

Activated-sludge kinetic coefficients for **heterotrophic bacteria** at 20 C\*

Coefficient	Unit	Range	Typical value
$\mu_m$	$g\ VSS/g\ VSS.d$	3,0-13,2	6,00
$K_s$	$g\ bCOD / m^3$	5,0-40,0	20,0
$Y$	$g\ VSS / g\ bCOD$	0,30-0,50	0,40
$k_d$	$g\ VSS / g\ VSS.d$	0,06-0,20	0,12
$f_d$	Unitless	0,08-0,20	0,15
$\Theta$ values			
$\mu_m$	Unitless	1,03-1,08	1,07
$k_d$	Unitless	1,03-1,08	1,04
$K_s$	Unitless	1,00	1,00

\* Adapted from Henze et al. (1987a); Barker and Dold (1997); and Grady et al. (1999)