CHAPTER: 4
Activated Sludge Processes

Assist. Prof. Bilge Alpaslan Kocamemi
Marmara University
Department of Environmental Engineering
Istanbul, Turkey
Suspended Growth (Activated Sludge)
Treatment Process Configurations

Basic activated sludge process consists of the following 3 basic components:

1. A reactor in which the microorganism responsible for treatment are kept in suspension and aerated

2. Liquid-solids separation in a sedimentation tank

3. A recycle system for returning solids removed from the liquid-solids separation unit back to the reactor (to maintain a sufficient conc of biomass in the aeration tank)
Types of activated-sludge process

(i) Complete mix reactors

(ii) Plug flow reactors

(iii) Sequencing batch reactors
**Complete mix activated sludge processes**

- Tank contents are thoroughly mixed

\[
\begin{align*}
\text{substrate load} & \\
\text{MLVSS conc} & \\
\text{oxygen demand} & \quad \text{uniform throughout the tank}
\end{align*}
\]

- Substrate conc. in the effluent is same as the substrate conc. in the reactor

- Relatively simple to operate but to have low organic subs. conc. (i.e low F/M) that encourage the growth of filamentous bacteria causing sludge bulking
care should be taken to prevent short-circuiting of untreated or partially treated ww

(influent and effluent withdrawal points selection are important)

If shock loads or toxic discharges (large number of industrial connections) are a design consideration

a complete mix reactor can more easily withstand changing ww characteristics because the incoming ww is more or less uniformly dispersed with the reactor contents

Complete mix reactors are superior to plug flow reactors where wide fluctuations in flow rates occur
however, in actual practice, a true plug flow regime is essentially impossible to obtain because of longitudinal dispersion caused by aeration & mixing

by dividing the aeration tank into a series of reactors (staged reactor design) process approaches plug flow kinetics with improved treatment efficiency compared to a complete mix process
Plug-Flow Activated Sludge Processes

→ involves relatively long, narrow aeration basins, so that the concentration of soluble substances and colloidal and suspended solids varies along the reactor length.

→ all particles entering the reactor stay in the reactor an equal amount of time.

→ substrate conc is continuously varying of distance in the reactor.

  at the influent end → high readily degradable substrate
  at the effluent end → low readily degradable substrate

→ where loading is reasonable constant, plug flow systems produce a more mature sludge with excellent settling characteristics.

The true plug flow system is theoretically more efficient in the stabilization of most soluble wastes than in continuous flow stirred tank reactors.
In actual practice, a true plug flow regime is essentially impossible to obtain because of longitudinal dispersion caused by aeration & mixing.

By dividing the aeration tank into a series of reactors (staged reactor design), the process approaches plug flow kinetics with improved treatment efficiency compared to a complete mix process.
Biomass & Substrate Mass Balances in a Complete Mix Reactor

A) Wasting from the Sludge Return Line

To maintain sufficient biomass concentration in aeration tank
→ more concentrated sludge
→ requires small waste sludge pumps

To maintain a given SRT, excess activated sludge produced each day must be wasted
B) Wasting from the Aeration Tank

Withdrawal of mixed liquor (ww+biomass) directly from aeration tank

Less concentrated

Good method if the process includes phosphorus removal

At the bottom of secondary clarifiers anaerobic conditions may develop

Release of phosphorus
Solids Retention Time (SRT) → mean cell residence time, sludge age, $\Theta_c$

The sludge or biomass requires a certain amount of time to assimilate the substrate and reproduce.

If the sludge is not able to reproduce itself before being washed out of the system, failure will result.

The average period of time during which the sludge has remained in the aeration tank is

- SRT or $\Theta_c$

Residence time of sludge in the clarifier → does not contribute to the effective sludge age.

- no substrate in sec clarifier
- low DO conc
- metabolic activity of sludge is not significant
\[ SRT(\theta_c) = \frac{\text{mass of organisms in the aeration tank}}{\text{mass of organisms removed daily}} \]

- Wasting from RAS line or directly from aeration basin
- M.o. overflowing the secondary clarifier weir into effluent (i.e., m.o. escaping from secondary clarifier)

→ The SRT (solids retention time) is completely analogous to HRT (hydraulic retention time).

However, HRT and SRT are very different:

(θ) HRT → is on the orders of hours
(θ_c) SRT → is on the orders of days

To achieve this, cells (microorganisms) are recycled from clarifier over and over again.
For the case of aeration tank with no clarifier and thus no sludge recycle

\[ \theta = \theta_c = \frac{VX}{QX} \]

→ SRT is the most critical parameter for activated – sludge design

It affects:

- treatment process performance
- aeration tank volume
- sludge production
- oxygen requirements
It is the residence time at which the cells are washed out or wasted from the system faster than they can reproduce.

To ensure adequate waste treatment, biological treatment processes are usually designed and operated with $\text{SRT} = 2-20 \, \text{SRT}_{\text{min}}$. 

$\text{SRT}_{\text{min}} (\Theta_{c\text{,min}}) \rightarrow \text{critical value}$
<table>
<thead>
<tr>
<th>Treatment goal</th>
<th>SRT Range, d</th>
<th>Factors affecting SRT</th>
</tr>
</thead>
<tbody>
<tr>
<td>Removal of soluble BOD in domestic wastewater</td>
<td>1-2</td>
<td>Temperature</td>
</tr>
<tr>
<td>Conversion of particulate organics in domestic wastewater</td>
<td>2-4</td>
<td>Temperature</td>
</tr>
<tr>
<td>Develop flocculent biomass for treating domestic wastewater</td>
<td>1-3</td>
<td>Temperature</td>
</tr>
<tr>
<td>Develop flocculent biomass for treating industrial wastewater</td>
<td>3-5</td>
<td>Temperature/ compounds</td>
</tr>
<tr>
<td>Provide complete nitrification</td>
<td>3-18</td>
<td>Temperature/ compounds</td>
</tr>
<tr>
<td>Biological phosphorus removal</td>
<td>2-4</td>
<td>Temperature</td>
</tr>
<tr>
<td>Stabilization of activated sludge</td>
<td>20-40</td>
<td>Temperature</td>
</tr>
<tr>
<td>Degradation of xenobiotic compounds</td>
<td>5-50</td>
<td>Temperature/ specific bacteria/ compounds</td>
</tr>
</tbody>
</table>

* Adapted from Grady et al. (1999)
A) Mass Balance for the system including wasting from the Sludge Return Line

\[
\begin{align*}
Q, S, X, R, S, Q, w, X, R, S, Q, e, X, e, S
\end{align*}
\]

Biomass Mass Balance

Accumulation = Inflow - Outflow + Generation

\[
\frac{dX}{dt} \forall = QX_0 - Q - Q_w X_e + Q_w X_R \int r_g' \forall
\]
\[
\frac{dX}{dt} = QX_0 - \left[ Q - Q_w \ X_e + Q_w X_R \right] r_g \forall
\]

Assumption: 1) the conc. of m.o in the influent is negligible
2) steady-state conditions prevail

\[
S = \text{effluent subst conc} = \frac{K_s \left( + k_d \theta_c \right)}{\theta_c \left( Y_k - k_d \right)} \left( Y_k = \mu_m \right)
\]
Substrate Mass Balance

Accumulation = Inflow - Outflow + Generation

\[ \frac{dS}{dt} \forall = QS_o - Q_e S + Q_w S - r_{su} \forall \]
\[
\frac{dS}{dt} = Q_{S_o} - Q_e S - Q_w S + r_{su} S
\]

Assumption: 1) the conc. of m.o in the influent is negligible
2) steady-state conditions prevail

\[
X = \frac{\theta_c}{\theta} \left[ \frac{Y \theta_o - S}{1 + k_d \theta_c} \right]
\]
B) Mass Balance for the system including wasting from the aeration tank

\[ Q, X_o, S_o \] -> \[ \text{influent} \] \[
\text{Aeration} \] \[
\text{Secondary Clarifier} \] \[
\text{Return Activated Sludge} \] \[
\text{WAS} \] \[
Q_w, X, S \] -> \[ \text{effluent} \] \[
(Q - Q_w), X_e, S \]

Biomass Mass Balance

Accumulation = Inflow - Outflow + Generation

\[ \frac{dX}{dt} \forall = QX_o - \left[ Q - Q_w \right] X_e + Q_w X + r_g \forall \]
\[ \frac{dX}{dt} \forall = QX_0 - Q - Q_w X_e + Q_w X - + r_g \forall \]

Assumption: 1) the conc. of m.o in the influent is negligible
2) steady-state conditions prevail

\[ S = \text{eff. subst. conc.} = \frac{K_s (Y_k - k_d)}{\theta c} + k_d \theta c \]

\[ \frac{1}{\theta c \theta c} \]
Substrate Mass Balance

Accumulation = Inflow - Outflow + Generation

\[
\frac{dS}{dt} = QS_o - \left( Q_e S + Q_w S \right) + r_{su} \forall
\]
Accumulation = Inflow - Outflow + Generation

\[ \frac{dS}{dt} \bigodot = Q_S - P_e S + Q_w S + r_{su} \bigodot \]

Assumption: 1) the conc. of m.o in the influent is negligible
2) steady-state conditions prevail

\[ X = \frac{\theta_c}{\theta} \left[ \frac{-S}{1 + k_d \theta_c} \right] \]
Solids Production

Total MLVSS conc in the aeration tank, $X_T$  

$= \text{biomass conc} + \text{non–biodegradable VSS conc}$  

$X \quad X_i$  

nbVSS from cell debris  

nbVSS in the influent
Biomass conc. \( (X, g / m^3) = \frac{\theta_c}{\theta} \left[ \frac{Y \xi_o - S}{1 + k_d \theta_c} \right] \) (from substrate mass balance)

\[ nbVSS \text{ from cell debris (g/m}^3\text{)} = F_d k_d X \theta_c \]

\[ = F_d k_d \left[ \frac{\theta_c}{\theta} \left( \frac{Y(S_o - S)}{1 + k_d \theta_c} \right) \right] \theta_c \]

\[ nbVSS \text{ in the influent (g/m}^3\text{)} = \frac{QX_{o,i}}{\theta} \theta_c = \frac{X_{o,i} \theta_c}{\theta} \]

\[ X_T = \frac{\theta_c}{\theta} \left[ \frac{Y \xi_o - S}{1 + k_d \theta_c} \right] + F_d k_d \left[ \frac{\theta_c}{\theta} \left( \frac{Y \xi_o - S}{1 + k_d \theta_c} \right) \right] \theta_c + \frac{X_{o,i} \theta_c}{\theta} \]
\[
\text{Sludge age } = \theta_c = \frac{\text{mass of solids in the aeration tank}}{\text{mass of solids wasted per day}} = \frac{\forall X_T}{P_{X_T, VSS}}
\]

\[
P_{X_T, VSS} = \forall \left\{ \frac{\theta_c}{\theta} \left[ \frac{Y \xi_o - S}{1 + k_d \theta C} \right] + F_d k_d \left[ \frac{\theta_c}{\theta} \left( \frac{Y \xi_o - S}{1 + k_d \theta C} \right) \right] \theta_c + \frac{X_{o,i} \theta_c}{\theta} \right\}
\]

\[
P_{X_T, VSS} = \forall \left[ \frac{Y \xi_o - S}{\theta (1 + k_d \theta C)} \right] + F_d k_d \frac{Y \xi_o - S}{\theta (1 + k_d \theta C)} \theta_c + \frac{X_{o,i}}{\theta}
\]

\[
P_{X_T, VSS} = \frac{Q Y \xi_o - S}{\theta + k_d \theta C} + F_d k_d Y \frac{Q \xi_o - S \theta C}{\theta + k_d \theta C} + X_{o,i} Q
\]
\[ P_{XT, VSS} = \frac{QY (\xi_o - S)}{\left(1 + k_d \theta_c \right)} + F_d k_d Y \frac{Q (\xi_o - S) \theta_c}{\left(1 + k_d \theta_c \right)} + X_{o,i} Q \]

\[ P_{XT, TSS} = \frac{QY (\xi_o - S)}{VSS/TSS} + F_d k_d Y \frac{Q (\xi_o - S) \theta_c}{VSS/TSS} + X_{o,i} Q + Q(TSS_0 - VSS_0) \]

\[ \theta_C = \frac{VX_{TSS}}{P_{XT, TSS}} = \frac{VX_{VSS}}{P_{XT, VSS}} \]
more biomass decays
more cell debris accumulates

the difference bw MLVSS and biomass VSS conc increases

The Observed Yield ($Y_{\text{obs}}$)

$$Y_{\text{obs}} = \frac{\text{amount of solids production measured}}{\text{amount of substrate removal measured}}$$

$$Y_{\text{obs}} = \frac{QY (c_0 - S)}{1 + k_d \theta_c} + F_d k_d \frac{Y Q (c_0 - S \theta_c)}{1 + k_d \theta_c} + X_{o,i} Q$$

→ decreases as the $\theta_c$ ↑ due to biomass loss by more endogenous respiration

→ lower with increasing temp as a result of a higher endogenous resp. rate at higher temp.

→ higher when no primary treatment is used, as more nbVSS remains in the influent ww.
$$Y_{\text{obs}} = \frac{QY_0 - S}{1 + k_d \theta_c} + F_d k_d \frac{YQ_0 - S \theta_c}{1 + k_d \theta_c} + X_{o,i} Q$$

Biomass cell debris influent nbVSS conc. depends on ww characteristics & type of pretreatment $S \ll S_0$

$$\frac{X_{o,i}}{S_0} = 0.1 - 0.3 \text{ with primary treatment}$$

$$= 0.3 - 0.5 \text{ without primary treatment}$$

$$P_{X_T, VSS} = QY_{\text{obs}} \theta_0 - S$$
F/M (Food / Microorganism) Ratio

\[
\frac{F}{M} = \frac{\text{total applied substrate rate}}{\text{total microbial biomass}} = QS_0 \forall X
\]

- \(Q \rightarrow\) influent flowrate (m\(^3\)/d)
- \(S_o \rightarrow\) influent BOD or bsCOD conc (g/m\(^3\))
- \(X \rightarrow\) mixed liquor biomass conc. in the aeration tank (g/m\(^3\))
- \(\forall \rightarrow\) aeration tank volume (m\(^3\))

\[U = \frac{\text{specific substrate utilization rate}}{X} = \frac{\dot{Q}_o - S}{X} = \frac{S_o - S}{\theta X}\]

\[E = \text{process BOD or bsCOD removal eff \%} = \frac{S_o - S}{S_o} \times 100\]
\[
\frac{U}{E} = \frac{S_o - S}{\frac{\theta X}{100}} = \frac{S_o}{\theta X(100)} = \frac{QS_o}{\sqrt{X}100} \rightarrow \frac{U}{E} = \frac{F/M}{100} \rightarrow U = \frac{F/M}{100}
\]

F/M → 0.1 – 0.05 g BOD/ g VSS.d (for \( \Theta_C = 20 – 30 \) d)

→ 0.3 – 0.5 g BOD/g VSS.d (for \( \Theta_C = 5 – 7 \) d)
Return Sludge Pumping Rate

A) Wasting from the Secondary Clarifier

\[ Q, X_0, S_0 \rightarrow \text{influent} \]

\[ S, X, A \]

\[ Q + Q_R \rightarrow X \]

\[ \text{Secondary Clarifier} \]

\[ \text{Return Activated Sludge} \]

\[ Q_R, X_R, S \]

\[ (Q - Q_W), X_e, S \]

\[ \text{Waste Activated Sludge (WAS)} \]

\[ Q_w, X_R, S \]
Biomass Mass Balance around secondary clarifier

Accumulation = Inflow - Outflow + Generation

\[
\frac{dX}{dt} \forall = (Q + Q_r)X - Q_e X_e + Q_w X_r + Q_r X_R
\]

Assumption: 1) steady-state conditions prevail
2) solids in the effluent from the settling tank is negligible
\[
\frac{dx}{dt} = 0 = \frac{Q+Q_R}{\forall} X - \left[ \frac{Q_e X_e + Q_w X_R + Q_R X_R}{\forall} \right]
\]

\[
\frac{QX + Q_R X}{\forall} = \frac{Q_w X_R}{\forall} + \frac{Q_R X_R}{\forall}
\]

\[
\theta_c = \frac{\forall X}{Q - Q_w X_e + Q_w X_R} \approx \frac{\forall X}{Q_w X_R}
\]

(for wasting from RAS)

\[
\frac{Q_w X_R}{\forall} = \frac{X}{\theta_c}
\]

\[
\frac{QX + Q_R X}{\forall} = \frac{X}{\theta_c} + \frac{Q_R X_R}{\forall}
\]
\[
\frac{Q_X}{\forall} - \frac{X}{\theta_C} = \frac{Q_RX_R}{\forall} - \frac{Q_RX}{\forall}
\]

\[
\left(\frac{Q_X}{\forall} - \frac{X}{\theta_C}\right)\forall
\]

\[
\frac{Q_RX_R - X}{X_R - X} = Q_R
\]

\[
Q_R = \frac{XQ - \frac{X\forall}{\theta_C}}{X_R - X}
\]

\[
\rightarrow Q_R = \frac{X\left(Q - \frac{Q\theta}{\theta_C}\right)}{X_R - X}
\]

\[
R = \frac{Q_R}{Q} = \frac{1 - \left(\frac{1}{\theta_C}\right)}{X_R/X - 1}
\]
If you write **biomass mass balance around aeration tank**
(boundary cond : black dotted line)

Accumulation = Inflow - Outflow

\[
\frac{dx}{dt} \forall = QX_0 + Q_R X_R \geq Q + Q_R X
\]

**Assumptions:**
1) steady-state conditions prevail
2) \(X_0\) is negligible
3) new cell growth is negligible

\[
0 = \frac{Q_R X_R}{\forall} - \frac{QX}{\forall} - \frac{Q_R X}{\forall}
\]

\[
\frac{QX}{\forall} = \frac{Q_R X_R - X}{\forall}
\]

\[
R = \frac{Q_R}{Q} = \frac{X}{X_R - X}
\]
B) Wasting from the Aeration Tank

\[ \text{Accumulation} = \text{Inflow} - \text{Outflow} + \text{Generation} \]

\[ \frac{dX}{dt} = (Q + Q_R - Q_w)X - (Q_e X_e + Q_R X_R) \]

**Assumptions:**
1) Solids in the eff from the settling tank is negligible
2) Steady-state conditions prevail
\[
\frac{dx}{dt} = 0 = \frac{Q + Q_R - Q_w}{\forall} X - \frac{Q_e X e + Q_R X R}{\forall}
\]

\[
0 = \frac{Q X + Q_R X - Q_w X - Q_R X_R}{\forall}
\]

\[
\frac{Q_w X}{\forall} = \frac{Q X + Q_R}{\forall} X - X_R
\]

\[
\frac{X}{\theta_C} = \frac{Q X + Q_R}{\forall} X - X_R
\]

\[
Q_R = \left( \frac{\forall X}{\theta_C} - Q X \right) / \left( X - X_R \right)
\]

\[
Q_R = \frac{Q X - X}{\theta_C} / \frac{X_R - X}{\forall}
\]

Recycle ratio:

\[
R = \frac{Q_R}{Q} = \frac{1 - \theta / \theta_C}{X_R / X - 1}
\]
Determination of Biomass Conc. in the Return Sludge ($X_R$)

SVI (Sludge Volume Index) $\rightarrow$ settleability test method often used to control the rate of return sludge pumping

SVI $\rightarrow$ Volume occupied by 1g of sludge after 30 min of settling

- Mixed-liquor sample is placed in a 1 to 2-L cylinder
- MLSS conc. of the sample is determined
- Settled volume after 30 min is measured

$$SVI = \frac{\text{settled volume of sludge} \text{ ml/L}}{\text{suspended solids} \text{ mg/L}} = \frac{\text{ml}}{\text{g}}$$

SVI $\approx$ 100 ml/g $\rightarrow$ considered a good settling sludge

SVI $>$ 150 ml/g $\rightarrow$ associated with filamentous growth $\rightarrow$ sludge bulking problem
Example: A mixed liquor sample with a 3000mg/L TSS conc settles to a volume of 300mL in 30 min in a 1 L cylinder SVI=?

\[
SVI = \frac{300 \text{ ml/L}}{3000 \text{ mg/L}} = 0.1 \frac{\text{ml}}{\text{mg}} \frac{1000 \text{ mg}}{\text{g}} = 100 \frac{\text{ml}}{\text{g}}
\]

\[
\text{SVI} = \frac{\text{ml}}{\text{g}} \Rightarrow \frac{1}{\text{SVI}} = \frac{\text{g}}{\text{ml}} = X_R
\]

\[
X_R = \frac{10^6}{\text{SVI}} \Rightarrow X_R = \frac{10^6}{100} = 10000 \text{ mg/L}
\]

\[
R = \frac{Q_R}{Q} = \frac{X}{X_R - X} = \frac{3000}{10000 - 3000} = 0.43
\]

\[
\rightarrow X_R = g \frac{1000 \text{ ml}}{\text{IL}} \frac{1000 \text{ mg}}{1 \text{g}}
\]

\[
\rightarrow X_R = \frac{10^6}{\text{SVI}}
\]

In ATV approach

\[
X_R = \frac{10^6 2^{1/3}}{\text{SVI}}
\]
Table 8-5
Summary of equations used in the analysis of suspended growth processes

<table>
<thead>
<tr>
<th>Equation</th>
<th>Eq. No.</th>
<th>Definition of terms</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_T = k_{20}t^{(T-20)}$</td>
<td>2-25</td>
<td>$k = $ maximum rate of substrate utilization, $T^{-1}$</td>
</tr>
<tr>
<td>$r_w = \frac{kXS}{K_s + S}$</td>
<td>7-12</td>
<td>$k = $ maximum rate of substrate utilization, $T^{-1}$</td>
</tr>
<tr>
<td>$\mu_m = kY$</td>
<td>7-13</td>
<td>$k_{dn} = $ endogenous decay coefficient, $T^{-1}$</td>
</tr>
<tr>
<td>$r_w = \frac{\mu_mX}{\gamma(K_s + S)}$</td>
<td>7-15</td>
<td>$k_{dn} = $ endogenous decay coefficient for nitrifying organisms, $T^{-1}$</td>
</tr>
<tr>
<td>$r_g = \frac{YkXS}{K_s + S} - k_{d}X$</td>
<td>7-22</td>
<td>$k_T = $ reaction rate coefficient at temperature ($T$)</td>
</tr>
<tr>
<td>$\mu = \frac{r_g}{X}$</td>
<td>7-23</td>
<td>$k_{DO} = $ reaction rate coefficient at 20°C</td>
</tr>
<tr>
<td>$SRT = \frac{VX}{(Q - Q_w)X_e + Q_wX_R}$</td>
<td>7-35</td>
<td>$K_v = $ half-velocity constant, $ML^{-3}$</td>
</tr>
<tr>
<td>$SRT = \frac{1}{\mu}$</td>
<td>7-37</td>
<td>$K_P = $ half-velocity constant for nitrate limited reaction, $ML^{-3}$</td>
</tr>
<tr>
<td>$S = \frac{K_R[1 + (k_{d})SRT]}{SRT(Y_k - k_{d}) - 1}$</td>
<td>7-39</td>
<td>$k_{org} = $ volumetric organic loading rate, $ML^{-3}T^{-1}$</td>
</tr>
<tr>
<td>$X = \left[\frac{SRT}{s}\right] \left[\frac{Y(S_o - S)}{1 + (k_{d})SRT}\right]$</td>
<td>7-43</td>
<td>$\mu = $ specific growth rate, $T^{-1}$</td>
</tr>
<tr>
<td>$(X_{VSS})/V = (P_{X,VSS})SRT$</td>
<td>7-54</td>
<td>$\mu_m = $ maximum specific growth rate, $T^{-1}$</td>
</tr>
<tr>
<td>$(X_{TS})/V = (P_{X,TS})SRT$</td>
<td>7-55</td>
<td>$\mu_n = $ specific growth rate for nitrification, $T^{-1}$</td>
</tr>
<tr>
<td>$R_o = Q(S_o - S) - 1.42 P_{X,bio}$</td>
<td>7-59</td>
<td>$\mu_{n,m} = $ maximum specific growth rate of nitrifying bacteria, $T^{-1}$</td>
</tr>
<tr>
<td>$F/M = \frac{QS_e}{VX}$</td>
<td>7-60</td>
<td>$N = $ nitrogen concentration, $ML^{-3}$</td>
</tr>
<tr>
<td>$L_{org} = \frac{(Q)(S_e)}{(V)}$</td>
<td>7-67</td>
<td>$\eta = $ ratio of substrate utilization rate with nitrate versus oxygen as the electron acceptor</td>
</tr>
<tr>
<td>$SF = SRT_{dos}/SRT_{min}$</td>
<td>7-71</td>
<td>$P_x = $ solids, $MT^{-1}$</td>
</tr>
<tr>
<td>$\mu_n = \left(\frac{N}{K_n + N}\right) \left(\frac{DO}{K_{do} + DO}\right) - k_{dn}$</td>
<td>7-93</td>
<td>$Q = $ flowrate, $L^3T^{-1}$</td>
</tr>
<tr>
<td>$r_{sv} = \left(\frac{kXS}{K_s + S}\right) \left(\frac{NO_3}{K_{s,NO_3} + NO_3}\right) \left(\frac{K_{o'}}{K_{o} + DO}\right)$</td>
<td>7-110</td>
<td>$Q_w = $ waste sludge flowrate $L^3T^{-1}$</td>
</tr>
</tbody>
</table>

Note: Expressions for units are $M = $ mass, $L = $ length, and $T = $ time.