

CSE2023 Discrete Computational Structure Fall 2014 Homework 2

1. Let $A=\{0,1\}$
 - a. How many strings of length three are there in A^* ?
 - b. How many strings of length five where at least two 1 next to each other are there in A^* ?
 - c. Compute the number of strings of length four in the set corresponding to regular expression $(01)^*1$?
 - d. Compute the number of strings of length five in the set corresponding to regular expression $(01)^*1$?
2. A firm who designs an advertising campaign has the possibility to work with 6 magazines, 3 newspapers, 2 TV stations, and 4 radio stations. In how many ways can six advertisements be run if...
 - a. Without any restrictions?
 - b. All six are to be in magazines?
 - c. Two are to be in magazines, two are to be in newspaper, one is to be on TV and one is to be on radio?
 - d. At least three are be in magazines?
3. Five fair coins are tossed and the results are recorded.
 - a. How many different sequences of heads and tails are possible?
 - b. What is the probability of getting exactly two tails?
 - c. What is the probability of getting at most 7 tails (7 included)?
4. **Suppose that three balls are selected at random from seven red balls and five black balls. Compute the probability that**
 - (a) **all three balls are red.**
 - (b) **at least two balls are black.**
 - (c) **at most two balls are black.**
 - (d) **at least one ball is red.**
5. For each of these relations on the set $\{1, 2, 3, 4\}$, decide whether it is reflexive, whether it is symmetric, whether it is antisymmetric, and whether it is transitive.
 - a. $\{(2, 2), (2, 3), (2, 4), (3, 2), (3, 3), (3, 4)\}$
 - b. $\{(1, 1), (1, 2), (2, 1), (2, 2), (3, 3), (4, 4)\}$
 - c. $\{(2, 4), (4, 2)\}$
 - d. $\{(1, 2), (2, 3), (3, 4)\}$
 - e. $\{(1, 1), (2, 2), (3, 3), (4, 4)\}$
 - f. $\{(1, 3), (1, 4), (2, 3), (2, 4), (3, 1), (3, 4)\}$
6. Determine whether the relation R on the set of all real numbers is reflexive, symmetric, antisymmetric, and/or transitive, where $(x, y) \in R$ if and only if
 - a. $x + y = 0$
 - b. $x = \pm y$
 - c. $x - y$ is a rational number.
 - d. $x = 2y$
 - e. $xy \geq 0$
 - f. $xy = 0$
 - g. $x = 1$
 - h. $x = 1$ or $y = 1$

7. The intersection graph of a collection of sets A_1, A_2, \dots, A_n is the graph that has a vertex for each of these sets and has an edge connecting the vertices representing two sets if these sets have a nonempty intersection. Construct the intersection graph of these collections of sets.

$$A_1 = \{0, 2, 4, 6, 8\}, A_2 = \{0, 1, 2, 3, 4\}, A_3 = \{1, 3, 5, 7, 9\}, A_4 = \{5, 6, 7, 8, 9\}, A_5 = \{0, 1, 8, 9\}$$

8. Determine whether the function $f : Z \times Z \rightarrow Z$ is onto or 1-1 or both if

- $f(m, n) = m + n$.
- $f(m, n) = m^2 + n^2$.
- $f(m, n) = m$.
- $f(m, n) = |n|$.
- $f(m, n) = m - n$.

9. Let $f(x) = ax + b$ and $g(x) = cx + d$, where a, b, c , and d are constants. Determine necessary and sufficient conditions on the constants a, b, c , and d so that $f \circ g = g \circ f$.

10. R be the relation whose di-graph is given in figure.

- Construct the adjacency matrix of relation R
- Define reachable relation set of R
- Find a closed path starting at vertex d .
- Find the cycles starting at vertex c .
- Find all paths of length 3.

