Spring 2014 MATH2056 Linear Algebra HW2

1.1.2. In each part, determine whether the equations form a linear system.

(a)
$$\begin{aligned} -2x + 4y + z &= 2 \\ 3x - \frac{2}{y} &= 0 \end{aligned}$$

(b)
$$\begin{aligned} x &= 4 \\ 2x &= 8 \end{aligned}$$

(c)
$$\begin{aligned} 4x - y + 2z &= -1 \\ -x + (\ln 2)y - 3z &= 0 \end{aligned}$$

(d)
$$\begin{aligned} 3z + x &= -4 \\ y + 5z &= 1 \\ 6x + 2z &= 3 \\ -x - y - z &= 4 \end{aligned}$$

1.1.12. In each part, find a system of linear equations corresponding to the given augmented matrix.

(a) $\begin{bmatrix} 2 & -1 \\ -4 & -6 \\ 1 & -1 \\ 3 & 0 \end{bmatrix}$

(b) $\begin{bmatrix} 0 & 3 & -1 & -1 & -1 \\ 5 & 2 & 0 & -3 & -6 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 2 & 3 & 4 \\ -4 & -3 & -2 & -1 \\ 5 & -6 & 1 & 1 \\ -8 & 0 & 0 & 3 \end{bmatrix}$

(d) $\begin{bmatrix} 3 & 0 & 1 & -4 & 3 \\ -4 & 0 & 4 & 1 & -3 \\ -1 & 3 & 0 & -2 & -9 \\ 0 & 0 & 0 & -1 & -2 \end{bmatrix}$

1.2.22. Solve the given homogeneous linear system by any method.

$$\begin{aligned} x_1 + 3x_2 \qquad + x_4 &= 0 \\ x_1 + 4x_2 + 2x_3 \qquad &= 0 \\ -2x_2 - 2x_3 - x_4 &= 0 \\ 2x_1 - 4x_2 + x_3 + x_4 &= 0 \\ x_1 - 2x_2 - x_3 + x_4 &= 0 \end{aligned}$$

1.2.26 Determine the values of a for which the system has no solutions, exactly one solution, or infinitely many solution.

$$\begin{aligned} x + 2y + \qquad z &= 2 \\ 2x - 2y + \qquad 3z &= 1 \\ x + 2y - (a^2 - 3)z &= a \end{aligned}$$

1.2.30. Solve the following systems, where *a*, *b*, and *c* are constants.

$$\begin{aligned} x_1 + x_2 + x_3 &= a \\ 2x_1 \qquad + 2x_3 &= b \\ 3x_2 + 3x_3 &= c \end{aligned}$$

1.3.6

Consider the matrices

$$A = \begin{bmatrix} 3 & 0 \\ -1 & 2 \\ 1 & 1 \end{bmatrix},\quad B = \begin{bmatrix} 4 & -1 \\ 0 & 2 \end{bmatrix},\quad C = \begin{bmatrix} 1 & 4 & 2 \\ 3 & 1 & 5 \end{bmatrix},\quad D = \begin{bmatrix} 1 & 5 & 2 \\ -1 & 0 & 1 \\ 3 & 2 & 4 \end{bmatrix},\quad E = \begin{bmatrix} 6 & 1 & 3 \\ -1 & 1 & 2 \\ 4 & 1 & 3 \end{bmatrix}$$

in each part compute the given expression (where possible).

(a) $\left(2D^T - E\right)A$

(b) $(4B)C + 2B$

(c) $(-AC)^T + 5D^T$

(d) $\left(BA^T - 2C\right)^T$

(e) $B^T\left(CC^T - A^TA\right)$

(f) $D^TE^T - (ED)^T$

1.3.24. Find the 4×4 matrix $A = [a_{ij}]$ whose entries satisfy the stated condition.

(a) $a_{ij} = i + j$

(b) $a_{ij} = i^{j-1}$

(c) $a_{ij} = \begin{cases} 1 & \text{if } |i-j| > 1 \\ -1 & \text{if } |i-j| \le 1 \end{cases}$

1.4.30. Assuming that all matrices are $n \times n$ and invertible, solve for *D*.

$$ABC^TDBA^TC = AB^T$$

1.4.53. (a) Show that if *A*, *B*, and $A + B$ are invertible matrices with the same size, then

$$A\left(A^{-1} + B^{-1}\right)B(A+B)^{-1} = I$$

(b) What does the result in part (a) tell you about the matrix $A^{-1} + B^{-1}$?

1.5. 8. Find an elementary matrix E that satisfies the equation.

(a) $EB = D$

(b) $ED = B$

(c) $EB = F$

(d) $EF = B$

$$A = \begin{bmatrix} 3 & 4 & 1 \\ 2 & -7 & -1 \\ 8 & 1 & 5 \end{bmatrix} B = \begin{bmatrix} 8 & 1 & 5 \\ 2 & -7 & -1 \\ 3 & 4 & 1 \end{bmatrix} C = \begin{bmatrix} 3 & 4 & 1 \\ 2 & -7 & -1 \\ 2 & -7 & 3 \end{bmatrix} D = \begin{bmatrix} 8 & 1 & 5 \\ -6 & 21 & 3 \\ 3 & 4 & 1 \end{bmatrix} F = \begin{bmatrix} 8 & 1 & 5 \\ 8 & 1 & 1 \\ 3 & 4 & 1 \end{bmatrix}$$

1.5.22. Use the inversion algorithm to find the inverse of the given matrix, if the inverse exists.

$$\begin{bmatrix} -8 & 17 & 2 & \frac{1}{3} \\ 4 & 0 & \frac{2}{5} & -9 \\ 0 & 0 & 0 & 0 \\ -1 & 13 & 4 & 2 \end{bmatrix}$$

1.5.36. Write the *inverse* of the given matrix as a product of elementary matrices.

$$\begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

1.6.16. Determine conditions on the b_i's, if any, in order to guarantee that the linear system is consistent.

$$\begin{aligned} x_1 - 2x_2 - x_3 &= b_1 \\ -4x_1 + 5x_2 + 2x_3 &= b_2 \\ -4x_1 + 7x_2 + 4x_3 &= b_3 \end{aligned}$$

1.6.18. Consider the matrices

$$A = \begin{bmatrix} 2 & 1 & 2 \\ 2 & 2 & -2 \\ 3 & 1 & 1 \end{bmatrix} \text{ and } \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

(a) Show that the equation $A\mathbf{x} = \mathbf{x}$ can be rewritten as $(A - I)\mathbf{x} = \mathbf{0}$ and use this result to solve $A\mathbf{x} = \mathbf{x}$ for $\mathbf{x}$.

(b) Solve $A\mathbf{x} = 4\mathbf{x}$.

1.7.24. find all values of the unknown constant(s) in order for A to be symmetric.

$$A = \begin{bmatrix} 2 & a - 2b + 2c & 2a + b + c \\ 3 & 5 & a + c \\ 0 & -2 & 7 \end{bmatrix}$$

1.7.26. Find all values of x in order for A to be invertible.

$$A=\begin{bmatrix} x-\frac{1}{2} & 0 & 0 \\ x & x-\frac{1}{3} & 0 \\ x^2 & x^3 & x-\frac{1}{4} \end{bmatrix}$$

1.7.37. A square matrix A is called ***skew-symmetric*** if $A^T=-A$.

Prove:

(a) If A is an invertible skew-symmetric matrix, then A^{-1} is skew-symmetric.

(b) If A and B are skew-symmetric matrices, then so are A^T, $A+B$, $A-B$, and kA for any scalar k.

(c) Every square matrix A can be expressed as the sum of a symmetric matrix and a skew-symmetric matrix. [*Hint:* Note the identity $A=\frac{1}{2}\left(A+A^T\right)+\frac{1}{2}\left(A-A^T\right)$.]

1.7.40. Find all values of a, b, c, and d for which A is skew-symmetric.

$$A=\begin{bmatrix} 0 & 2a-3b+c & 3a-5b+5c \\ -2 & 0 & 5a-8b+6c \\ -3 & -5 & d \end{bmatrix}$$

1.8.4. The accompanying figure shows a network of one-way streets with traffic flowing in the directions indicated. The flow rates along the streets are measured as the average number of vehicles per hour.

(a) Set up a linear system whose solution provides the unknown flow rates.

(b) Solve the system for the unknown flow rates.

(c) Is it possible to close the road from A to B for construction and keep traffic flowing on the other streets? Explain.