Spring 2014 MATH2056 Linear Algebra HW2

*Exercises Set 2.1 Question 4 (2.1.4)

$$A = \begin{bmatrix} 2 & 3 & -1 & 1 \\ -3 & 2 & 0 & 3 \\ 3 & -2 & 1 & 0 \\ 3 & -2 & 1 & 4 \end{bmatrix}$$

Find

(a) M_{32} and C_{32}.

(b) M_{44} and C_{44}.

(c) M_{41} and C_{41}.

(d) M_{24} and C_{24}.

*Exercises Set 2.1 Question 14 (2.1.14)

Use the arrow technique to evaluate the determinant of the given matrix.

$$\begin{bmatrix} c & -4 & 3 \\ 2 & 1 & c^2 \\ 4 & c-1 & 2 \end{bmatrix}$$

*Exercises Set 2.1 Question 18 (2.1.18)

Find all values of λ for which det(A)=0

$$A = \begin{bmatrix} \lambda-4 & 4 & 0 \\ -1 & \lambda & 0 \\ 0 & 0 & \lambda-5 \end{bmatrix}$$

*Exercises Set 2.1 Question 26 (2.1.26)

Evaluate det (A) by a cofactor expansion along a row or column of your choice

$$A = \begin{bmatrix} 4 & 0 & 0 & 1 & 0 \\ 3 & 3 & 3 & -1 & 0 \\ 1 & 2 & 4 & 2 & 3 \\ 9 & 4 & 6 & 2 & 3 \\ 2 & 2 & 4 & 2 & 3 \end{bmatrix}$$

*Exercises Set 2.1 Question 41 (2.1.41)

Prove that the equation of the line through the distinct points (a_1,b_1) and (a_2,b_2) can be written as

$$\begin{vmatrix} x & y & 1 \\ a_1 & b_1 & 1 \\ a_2 & b_2 & 1 \end{vmatrix} = 0$$

*Exercises Set 2.2 Question 14 (2.2.14)

Evaluate the determinant of the given matrix by reducing the matrix to row echelon form.

$$\begin{bmatrix} 1 & -2 & 3 & 1 \\ 5 & -9 & 6 & 3 \\ -1 & 2 & -6 & -2 \\ 2 & 8 & 6 & 1 \end{bmatrix}$$

*Exercises Set 2.2 Question 31 (2.2.31)
Confirm the identities without evaluating the determinants directly.

$$\begin{vmatrix} a_1 & b_1 & a_1+b_1+c_1 \\ a_2 & b_2 & a_2+b_2+c_2 \\ a_3 & b_3 & a_3+b_3+c_3 \end{vmatrix} = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

*Exercises Set 2.3 Question 6 (2.3.6)
Verify that det(AB)=det(BA)and determine whether the equality det(A+B)=det(A)+det(B) holds.

$$A = \begin{bmatrix} -1 & 8 & 2 \\ 1 & 0 & -1 \\ -2 & 2 & 2 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 2 & -1 & -4 \\ 1 & 1 & 3 \\ 0 & 3 & -1 \end{bmatrix}$$

*Exercises Set 2.3 Question 18 (2.3.18)
Find the values of *k* for which *A* is invertible.

$$A = \begin{bmatrix} 1 & 2 & 0 \\ k & 1 & k \\ 0 & 2 & 1 \end{bmatrix}$$

*Exercises Set 2.3 Question 20 (2.3.20)
Decide whether the given matrix is invertible, and if so, use the adjoint method to find its inverse.

$$A = \begin{bmatrix} 2 & 0 & 3 \\ 0 & 3 & 2 \\ -2 & 0 & -4 \end{bmatrix}$$

*Exercises Set 2.3 Question 32 (2.3.32)
Let Ax=b be the system

$$\begin{aligned} 4x + y + z + w &= 6 \\ 3x + 7y - z + w &= 1 \\ 7x + 3y - 5z + 8w &= -3 \\ x + y + z + 2w &= 3 \end{aligned}$$

(a) Solve by Cramer's rule.
(b) Solve by Gauss–Jordan elimination.
(c) Which method involves fewer computations?

*Exercises Set 2.3 Question 36 (2.3.36)
In each part, find the determinant given that *A* is a 4x4 matrix for which det(A)=-2

(a) $\det(-A)$
(b) $\det(A^{-1})$
(c) $\det(2A^T)$
(d) $\det(A^3)$

*Exercises Set 2.3 Question 39 (2.3.39)

Show that if A is a square matrix then $\det(A^TA) = \det(AA^T)$.

*Exercises Set 3.1 Question 12 (3.1.12)

Find a nonzero vector **u** with initial point $P(-1, 3, -5)$ such that

(a) **u** has the same direction as $\mathbf{v} = (6, 7, -3)$.

(b) **u** is oppositely directed to $\mathbf{v} = (6, 7, -3)$.

*Exercises Set 3.1 Question 14 (3.1.14)

Let $\mathbf{u} = (-3, 1, 2)$, $\mathbf{v} = (4, 0, -8)$, and $\mathbf{w} = (6, -1, -4)$. Find the components of

(a) $\mathbf{v} - \mathbf{w}$

(b) $6\mathbf{u} + 2\mathbf{v}$

(c) $-\mathbf{v} + \mathbf{u}$

(d) $5(\mathbf{v} - 4\mathbf{u})$

(e) $-3(\mathbf{v} - 8\mathbf{w})$

(f) $(2\mathbf{u} - 7\mathbf{w}) - (8\mathbf{v} + \mathbf{u})$

*Exercises Set 3.1 Question 34 (3.1.34)

Let P be the point $(1, 3, 7)$. If the point $(4, 0, -6)$ is the midpoint of the line segment connecting P and Q, what is Q?

*Exercises Set 3.2 Question 2 (3.2.2)

Find the norm of **v**, a unit vector that has the same direction as **v**, and a unit vector that is oppositely directed to **v**.

(a) $\mathbf{v} = (-5, 12)$

(b) $\mathbf{v} = (1, -1, 2)$

(c) $\mathbf{v} = (-2, 3, 3, -1)$

*Exercises Set 3.2 Question 6 (3.2.6)

Evaluate the given expression with u=(-2,-1,4,5), v=(3,1,-5,7) and w=(-6,2,1,1)

(a) $\|\mathbf{u}\| - 2\|\mathbf{v}\| - 3\|\mathbf{w}\|$

(b) $\|\mathbf{u}\| + \|-2\mathbf{v}\| + \|-3\mathbf{w}\|$

(c) $\|\,\|\mathbf{u} - \mathbf{v}\|\mathbf{w}\|$

*Exercises Set 3.2 Question 12 (3.2.12)

Find the Euclidean distance between **u** and **v**.

(a) $\mathbf{u} = (1, 2, -3, 0)$, $\mathbf{v} = (5, 1, 2, -2)$

(b) $\mathbf{u} = (2, -1, -4, 1, 0, 6, -3, 1)$,
$\mathbf{v} = (-2, -1, 0, 3, 7, 2, -5, 1)$

(c) $\mathbf{u} = (0, 1, 1, 1, 2)$, $\mathbf{v} = (2, 1, 0, -1, 3)$

*Exercises Set 3.2 Question 20 (3.2.20)

Find a unit vector that is oppositely directed to the given vector.

(a) $(-12, -5)$

(b) $(3, -3, -3)$

(c) $(-6, 8)$

(d) $\left(-3, 1, \sqrt{6}, 3\right)$