MATH2056 Homework 3

determine whether the given planes are parallel.

3.3.14. $x-4y-3z-2=0$ and $3x-12y-9z-7=0$

3.3.16. $(-4, 1, 2)\cdot(x, y, z)=0$ and $(8, -2, -4)\cdot(x, y, z)=0$

determine whether the given planes are perpendicular.

3.3.18. $x-2y+3z=4, \quad -2x+5y+4z=-1$

find $\|\text{proj}_{\mathbf{a}}\mathbf{u}\|$.

3.3.20. (a) $\mathbf{u}=(5, 6), \ \mathbf{a}=(2, -1)$

(b) $\mathbf{u}=(3, -2, 6), \ \mathbf{a}=(1, 2, -7)$

find the vector component of **u** along **a** and the vector component of **u** orthogonal to **a**.

3.3.22. $\mathbf{u}=(-1, -2), \ \mathbf{a}=(-2, 3)$

3.3.28. $\mathbf{u}=(5, 0, -3, 7), \mathbf{a}=(2, 1, -1, -1)$

find the distance between the point and the line.

3.3.30. $x-3y+2=0; \ (-1, 4)$

use the given equation of a line to find a point on the line and a vector parallel to the line.

3.4.6. $(x, y, z)=(4t, 7, 4+3t)$

3.4.8. $\mathbf{x}=(1-t)(0, -5, 1)$

find vector and parametric equations of the plane containing the given point and parallel vectors.

3.4.12. Point: $(0, 5, -4)$; vectors: $\mathbf{v}_1=(0, 0, -5)$ and $\mathbf{v}_2=(1, -3, -2)$

3.4.26. Consider the linear systems

$$\begin{bmatrix} 1 & -2 & -3 \\ 2 & 1 & 4 \\ 1 & -7 & 5 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

and

$$\begin{bmatrix} 1 & -2 & -3 \\ 2 & 1 & 4 \\ 1 & -7 & 5 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}=\begin{bmatrix} 2 \\ 7 \\ -1 \end{bmatrix}$$

(a) Find a general solution of the homogeneous system.

(b) Confirm that $x_1=1, x_2=1, x_3=1$ is a solution of the nonhomogeneous system.

(c) Use the results in parts (a) and (b) to find a general solution of the nonhomogeneous system.

(d) Check your result in part (c) by solving the nonhomogeneous system directly.

3.5.18 find the volume of the parallelepiped with sides **u**, **v**, and **w**.

$\mathbf{u}=(3, 1, 2), \mathbf{v}=(4, 5, 1), \mathbf{w}=(1, 2, 4)$

3.5.20. determine whether **u**, **v**, and **w** lie in the same plane when positioned so that their initial points coincide.

$\mathbf{u}=(5, -2, 1), \mathbf{v}=(4, -1, 1), \mathbf{w}=(1, -1, 0)$

3.5.36. It is a theorem of solid geometry that the volume of a tetrahedron is $\frac{1}{3}$(area of base) · (height). Use this result to prove that the volume of a tetrahedron whose sides are the vectors **a**, **b**, and **c** is $\frac{1}{6}\left|\mathbf{a}\cdot(\mathbf{b}\times\mathbf{c})\right|$ (see the accompanying figure).

3.5.37. Use the result of Exercise 26 to find the volume of the tetrahedron with vertices P, Q, R, S.

(a) $P(-1, 2, 0), Q(2, 1, -3), R(1, 1, 1), S(3, -2, 3)$

(b) $P(0, 0, 0), Q(1, 2, -1), R(3, 4, 0), S(-1, -3, 4)$

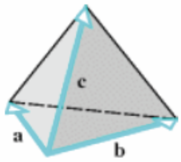

Figure Ex-36

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determine whether each set equipped with the given operations is a vector space. For those that are not vector spaces identify the vector space axioms that fail.

4.1.6. The set of all n-tuples of real numbers that have the form $(x, x, \dots, x)$ with the standard operations on R^n.

4.1.8. The set of all 2×2 invertible matrices with the standard matrix addition and scalar multiplication.

4.2.10. In each part express the vector as a linear combination of $\mathbf{p}_1 = 2 + x + 4x^2$, $\mathbf{p}_2 = 1 - x + 3x^2$, and $\mathbf{p}_3 = 3 + 2x + 5x^2$.

(a) $-9 - 7x - 15x^2$

(b) $6 + 11x + 6x^2$

(c) 0

(d) $7 + 8x + 9x^2$

4.2.12. Suppose that $\mathbf{v}_1 = (2, 1, 0, 3)$, $\mathbf{v}_2 = (3, -1, 5, 2)$, and $\mathbf{v}_3 = (-1, 0, 2, 1)$. Which of the following vectors are in span $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$?

(a) $(2, 3, -7, 3)$

(b) $(0, 0, 0, 0)$

(c) $(1, 1, 1, 1)$

(d) $(-4, 6, -13, 4)$

4.3.6. Assume that $\mathbf{v}_1$, $\mathbf{v}_2$, and $\mathbf{v}_3$ are vectors in R^3 that have their initial points at the origin. In each part, determine whether the three vectors lie on the same line.

(a) $\mathbf{v}_1 = (-1, 2, 3), \mathbf{v}_2 = (2, -4, -6), \mathbf{v}_3 = (-3, 6, 0)$

(b) $\mathbf{v}_1 = (2, -1, 4), \mathbf{v}_2 = (4, 2, 3), \mathbf{v}_3 = (2, 7, -6)$

(c) $\mathbf{v}_1 = (4, 6, 8), \mathbf{v}_2 = (2, 3, 4), \mathbf{v}_3 = (-2, -3, -4)$

4.3.8. (a) Show that the three vectors $\mathbf{v}_1 = (1, 2, 3, 4)$, $\mathbf{v}_2 = (0, 1, 0, -1)$, and $\mathbf{v}_3 = (1, 3, 3, 3)$ form a linearly dependent set in R^4.

(b) Express each vector in part (a) as a linear combination of the other two.

4.4.4. Which of the following form bases for P_2?

(a) $1 - 3x + 2x^2, \quad 1 + x + 4x^2, \quad 1 - 7x$

(b) $4 + 6x + x^2, \quad -1 + 4x + 2x^2, \quad 5 + 2x - x^2$

(c) $1 + x + x^2, \quad x + x^2, \quad x^2$

(d) $-4 + x + 3x^2, \quad 6 + 5x + 2x^2, \quad 8 + 4x + x^2$

4.4.5. Show that the following matrices form a basis for M_{22}.

$$\begin{bmatrix} 3 & 6 \\ 3 & -6 \end{bmatrix}, \quad \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}, \quad \begin{bmatrix} 0 & -8 \\ -12 & -4 \end{bmatrix}, \quad \begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix}$$

4.4.8. Find the coordinate vector of $\mathbf{w}$ relative to the basis $S = \{\mathbf{u}_1, \mathbf{u}_2\}$ of R^2.

(a) $\mathbf{u}_1 = (1, -1), \mathbf{u}_2 = (1, 1); \mathbf{w} = (1, 0)$

(b) $\mathbf{u}_1 = (1, -1), \mathbf{u}_2 = (1, 1); \mathbf{w} = (0, 1)$

(c) $\mathbf{u}_1 = (1, -1), \mathbf{u}_2 = (1, 1); \mathbf{w} = (1, 1)$

4.4.10. Find the coordinate vector of $\mathbf{p}$ relative to the basis $S = \{\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3\}$.

(a) $\mathbf{p} = 4 - 3x + x^2; \mathbf{p}_1 = 1, \mathbf{p}_2 = x, \mathbf{p}_3 = x^2$

(b) $\mathbf{p} = 2 - x + x^2; \mathbf{p}_1 = 1 + x, \mathbf{p}_2 = 1 + x^2, \mathbf{p}_3 = x + x^2$

4.5.4. Find a basis for the solution space of the homogeneous linear system, and find the dimension of that space.

$$\begin{aligned} x_1 - 3x_2 + x_3 &= 0 \\ 2x_1 - 6x_2 + 2x_3 &= 0 \\ 3x_1 - 9x_2 + 3x_3 &= 0 \end{aligned}$$

4.5.12. Find a standard basis vector for R^3 that can be added to the set $\{\mathbf{v}_1, \mathbf{v}_2\}$ to produce a basis for R^3.

(a) $\mathbf{v}_1 = (-1, 2, 3), \mathbf{v}_2 = (1, -2, -2)$

(b) $\mathbf{v}_1 = (1, -1, 0), \mathbf{v}_2 = (3, 1, -2)$