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(Please, use both sides of this sheet)

1.1.2 a) Not a linear system, because second equation is not a linear equation

b), c), d) are linear systems

1.1.12 a)
$$2x=-1$$
$$-4x=-6$$
$$x=-1$$
$$3x=0$$

b)
$$3x-y-z=-1$$
$$5w+2x-3z=-6$$

c)
$$x+2y+3z=4$$
$$-4x-3y-2z=-1$$
$$5x-6y+z=1$$
$$-8x=3$$

d)
$$3w+y-4z=3$$
$$-4w+4y+z=-3$$
$$-w+3x-2z=-9$$
$$-z=-2$$

1.2.22

$$\begin{bmatrix} 1 & 3 & 0 & 1 & 0 \\ 1 & 4 & 2 & 0 & 0 \\ 0 & -2 & -2 & -1 & 0 \\ 2 & -4 & 1 & 1 & 0 \\ 1 & -2 & -1 & 1 & 0 \end{bmatrix} \xrightarrow[R_5=R_5-R_1]{R_2=R_2-R_1,\ R_4=R_4-R_1} \begin{bmatrix} 1 & 3 & 0 & 1 & 0 \\ 0 & 1 & 2 & -1 & 0 \\ 0 & -2 & -2 & -1 & 0 \\ 0 & -10 & 1 & -1 & 0 \\ 0 & -5 & -1 & 0 & 0 \end{bmatrix} \xrightarrow[R_5=R_5+5R_2]{R_3=R_3+2R_2,\ R_4=R_4+10R_2} \begin{bmatrix} 1 & 3 & 0 & 1 & 0 \\ 0 & 1 & 2 & -1 & 0 \\ 0 & 0 & 2 & -3 & 0 \\ 0 & 0 & 21 & -11 & 0 \\ 0 & 0 & 9 & -5 & 0 \end{bmatrix}$$

$$\xrightarrow{R_3=\frac{1}{2}R_3} \begin{bmatrix} 1 & 3 & 0 & 1 & 0 \\ 0 & 1 & 2 & -1 & 0 \\ 0 & 0 & 1 & -3/2 & 0 \\ 0 & 0 & 21 & -11 & 0 \\ 0 & 0 & 9 & -5 & 0 \end{bmatrix} \xrightarrow[R_5=R_5-9R_3]{R_4=R_4-21R_3} \begin{bmatrix} 1 & 3 & 0 & 1 & 0 \\ 0 & 1 & 2 & -1 & 0 \\ 0 & 0 & 1 & -3/2 & 0 \\ 0 & 0 & 0 & 41/2 & 0 \\ 0 & 0 & 0 & 17/2 & 0 \end{bmatrix} \xrightarrow{R_4=\frac{2}{41}\cdot R_4}$$

$$\begin{bmatrix} 1 & 3 & 0 & 1 & 0 \\ 0 & 1 & 2 & -1 & 0 \\ 0 & 0 & 1 & 3/2 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 17/2 & 0 \end{bmatrix} \xrightarrow{R_5=R_5-\frac{17}{2}R_4} \begin{bmatrix} 1 & 3 & 0 & 1 & 0 \\ 0 & 1 & 2 & -1 & 0 \\ 0 & 0 & 1 & -3/2 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \Rightarrow \begin{array}{l} x_1+3x_2+x_4=0 \\ x_2+2x_3-x_4=0 \\ x_3-3/2\,x_4=0 \\ x_4=0 \end{array}$$

Using back-substitution, we have unique solution
$$x_1=x_2=x_3=x_4=0$$

1.2.26

$$\begin{bmatrix} 1 & 2 & 1 & 2 \\ 2 & -2 & 3 & 1 \\ 1 & 2 & -(a^2-3) & a \end{bmatrix} \xrightarrow[R_3=R_3-R_1]{R_2=R_2-2R_1} \begin{bmatrix} 1 & 2 & 1 & 2 \\ 0 & -6 & 1 & -3 \\ 0 & 0 & -a^2+2 & a-2 \end{bmatrix} \xrightarrow{R_2=-\frac{1}{6}R_2}$$

$$\begin{bmatrix} 1 & 2 & 1 & 2 \\ 0 & 1 & -1/6 & 1/2 \\ 0 & 0 & -a^2+2 & a-2 \end{bmatrix} \Rightarrow \quad (-a^2+2)z = a-2$$

$$z = \frac{a-2}{-a^2+2}$$

* System has no solution when $a=\sqrt{2}$ or $a=-\sqrt{2}$ (z be undefined)
* For all remaining value, system has unique solution
* There is no value of a for which system has infinitely many solution

1.2.30

$$\begin{bmatrix} 1 & 1 & 1 & a \\ 2 & 0 & 2 & b \\ 0 & 3 & 3 & c \end{bmatrix} \xrightarrow{R_2=R_2-2R_1} \begin{bmatrix} 1 & 1 & 1 & a \\ 0 & -2 & 0 & -2a+b \\ 0 & 3 & 3 & c \end{bmatrix} \xrightarrow{R_2=-\frac{1}{2}R_2}$$

$$\begin{bmatrix} 1 & 1 & 1 & a \\ 0 & 1 & 0 & a-b/2 \\ 0 & 3 & 3 & c \end{bmatrix} \xrightarrow{R_3=R_3-3R_2} \begin{bmatrix} 1 & 1 & 1 & a \\ 0 & 1 & 0 & a-b/2 \\ 0 & 0 & 3 & -3a+3/2\,b+c \end{bmatrix} \xrightarrow{R_3=1/3R_3}$$

$$\begin{bmatrix} 1 & 1 & 1 & a \\ 0 & 1 & 0 & a-b/2 \\ 0 & 0 & 1 & -a+b/2+c/3 \end{bmatrix} \xrightarrow{R_1=R_1-R_3} \begin{bmatrix} 1 & 1 & 0 & 2a-b/2-c/3 \\ 0 & 1 & 0 & a-b/2 \\ 0 & 0 & 1 & -a+b/2+c/3 \end{bmatrix} \xrightarrow{R_1=R_1-R_2}$$

$$\begin{bmatrix} 1 & 0 & 0 & a-c/3 \\ 0 & 1 & 0 & a-b/2 \\ 0 & 0 & 1 & -a+b/2+c/3 \end{bmatrix}$$

Has a unique solution

$x_1 = a - c/3$

$x_2 = a - b/2$

$x_3 = -a + b/2 + c/3$

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1.3.6 a) $D^T = \begin{bmatrix} 1 & -1 & 3 \\ 5 & 0 & 2 \\ 2 & 1 & 4 \end{bmatrix}$ $\quad 2D^T - E = \begin{bmatrix} 2 & -2 & 6 \\ 10 & 0 & 4 \\ 4 & 2 & 8 \end{bmatrix} - \begin{bmatrix} 6 & 1 & 3 \\ -1 & 1 & 2 \\ 4 & 1 & 3 \end{bmatrix} = \begin{bmatrix} -4 & -3 & 3 \\ 11 & -1 & 2 \\ 0 & 1 & 5 \end{bmatrix}$

$$(2D^T - E).A = \begin{bmatrix} -4 & -3 & 3 \\ 11 & -1 & 2 \\ 0 & 1 & 5 \end{bmatrix} \cdot \begin{bmatrix} 3 & 0 \\ -1 & 2 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} -6 & -3 \\ 36 & 0 \\ 4 & 7 \end{bmatrix}$$

b) $[(4.B).C]$ is 2×3 matrix and can not be added B (2×2 matrix)

c) $$-A.C = -\begin{bmatrix} 3 & 0 \\ -1 & 2 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 4 & 2 \\ 3 & 1 & 5 \end{bmatrix} = \begin{bmatrix} -3 & -12 & -6 \\ -5 & 2 & -8 \\ -4 & -5 & -7 \end{bmatrix}$$

$$(-A.C)^T + 5D^T = \begin{bmatrix} -3 & -5 & -4 \\ -12 & 2 & -5 \\ -6 & -8 & -7 \end{bmatrix} + 5\begin{bmatrix} 1 & -1 & 3 \\ 5 & 0 & 2 \\ 2 & 1 & 4 \end{bmatrix} = \begin{bmatrix} 2 & -10 & 11 \\ 13 & 2 & 5 \\ 4 & -3 & 13 \end{bmatrix}$$

d) $$B.A^T = \begin{bmatrix} 4 & -1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 3 & -1 & 1 \\ 0 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 12 & -6 & 3 \\ 0 & 4 & 2 \end{bmatrix}$$

$$B.A^T - 2C = \begin{bmatrix} 12 & -6 & 3 \\ 0 & 4 & 2 \end{bmatrix} - 2\begin{bmatrix} 1 & 4 & 2 \\ 3 & 1 & 5 \end{bmatrix} = \begin{bmatrix} 10 & -14 & -1 \\ -6 & 2 & -8 \end{bmatrix}$$

$$(B.A^T - 2C)^T = \begin{bmatrix} 10 & -6 \\ -14 & 2 \\ -6 & -8 \end{bmatrix}$$

e) $C.C^T = \begin{bmatrix} 1 & 4 & 2 \\ 3 & 1 & 5 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ 4 & 1 \\ 2 & 5 \end{bmatrix} = \begin{bmatrix} 21 & 17 \\ 17 & 35 \end{bmatrix}$

$A^T.A = \begin{bmatrix} 3 & -1 & 1 \\ 0 & 2 & 1 \end{bmatrix} \begin{bmatrix} 3 & 0 \\ -1 & 2 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 11 & -1 \\ -1 & 5 \end{bmatrix}$

$\underbrace{C.C^T - A^T.A}_{X} = \begin{bmatrix} 10 & 18 \\ 18 & 30 \end{bmatrix}$

$B^T.X = \begin{bmatrix} 4 & 0 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 10 & 18 \\ 18 & 30 \end{bmatrix} = \begin{bmatrix} 40 & 72 \\ 26 & 42 \end{bmatrix}$

f) $D^T.E^T = (ED)^T$ so result is 0 (3x3 matrix with all zero)

1.3.24

a) $a_{ij} = i+j$ 4x4 $\begin{bmatrix} 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 6 \\ 4 & 5 & 6 & 7 \\ 5 & 6 & 7 & 8 \end{bmatrix}$

b) $a_{ij} = i^{j-1}$ 4x4 $\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 4 & 8 \\ 1 & 3 & 9 & 27 \\ 1 & 4 & 16 & 64 \end{bmatrix}$

c) $a_{ij} = \begin{cases} 1 \text{ if } |i-j| > 1 \\ -1 \text{ if } |i-j| \leq 1 \end{cases}$

$\begin{bmatrix} -1 & -1 & 1 & 1 \\ -1 & -1 & -1 & 1 \\ 1 & -1 & -1 & -1 \\ 1 & 1 & -1 & -1 \end{bmatrix}$

1.4.30

$[A.B.C^T D B A^T \underbrace{C].C^{-1}}_{I} = [A.B^T].C^{-1}$

$AB.C^T.D.B.\underbrace{A^T.(A^T)^{-1}}_{I} = A.B^T.C^{-1}.(A^T)^{-1}$

$A.B.C^T.D.\underbrace{B.B^{-1}}_{I} = A.B^T.C^{-1}.(A^T)^{-1}.B^{-1}$

$(A.B.C^T)^{-1}.(A.B.C^T).D = (A.B.C^T)^{-1}.A.B^T.C^{-1}.(A^T)^{-1}.B^{-1}$

$D = (A.B.C^T)^{-1}.A.B^T.C^{-1}(A^T)^{-1}.B^{-1}$

$D = (C^T)^{-1}.B^{-1}.\underbrace{A^{-1}.A}_{I}.B^T.C^{-1}.(A^T)^{-1}.B^{-1} = (C^T)^{-1}.B^{-1}.B^T.C^{-1}.(A^T)^{-1}.B^{-1}$

1.4.53

a) $A(A^{-1}+B^{-1})B(A+B)^{-1} = (A\cdot A^{-1} + AB^{-1})B(A+B)^{-1}$

$= (I + AB^{-1})B(A+B)^{-1}$

$= B + (A\cdot B^{-1}\cdot B)(A+B)^{-1}$

$= (B+A)(A+B)^{-1}$

$= I$

b) $A^{-1}+B^{-1} \neq (A+B)^{-1}$

1.5.8

a) $E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ b) $E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1/3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

c) $E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$ d) $E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}$

1.5.22

It has no inverse (singular). Because third row contains only zeros.

1.5.36

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix} \quad R_2 = R_2 - R_1 \qquad E_1 = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \quad R_2 \leftrightarrow R_3 \qquad E_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad R_2 = R_2 - R_3 \qquad E_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad R_1 = R_1 - R_2 \qquad E_4 = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

So $E_4 . E_3 E_2 E_1 A = I$ that means $A = (E_4 . E_3 . E_2 . E_1)^{-1} . I$

$$A^{-1} = E_4 . E_3 . E_2 . E_1$$

1.6.16

$$\left[\begin{array}{ccc|c} 1 & -2 & 1 & b_1 \\ -4 & 5 & 2 & b_2 \\ -4 & 7 & 4 & b_3 \end{array}\right] \xrightarrow[R_3 = R_3 - 4R_1]{R_2 = R_2 - 4R_1} \left[\begin{array}{ccc|c} 1 & -2 & -1 & b_1 \\ 0 & -3 & -2 & 4b_1 + b_2 \\ 0 & -1 & 0 & 4b_1 + b_3 \end{array}\right] \xrightarrow[R_2 \leftrightarrow R_3]{R_3 = -R_3}$$

$$\left[\begin{array}{ccc|c} 1 & -2 & -1 & b_1 \\ 0 & 1 & 0 & -4b_1 - b_3 \\ 0 & -3 & -2 & 4b_1 + b_2 \end{array}\right] \xrightarrow[R_3 = -\frac{1}{2} R_3]{R_3 = R_3 + 3R_2} \left[\begin{array}{ccc|c} 1 & -2 & -1 & b_1 \\ 0 & 1 & 0 & -4b_1 - b_3 \\ 0 & 0 & +1 & 4b_1 - \frac{1}{2} b_2 + \frac{3}{2} b_3 \end{array}\right]$$

for all values of b_1, b_2, and b_3, the system is consistent

P-4

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1.6.18 $Ax = x \to Ax = Ix \to Ax - Ix = 0 \quad (A-I)x = 0$

This is homogeneous system.

a)

$$A - I = \left[\begin{array}{ccc|c} 1 & 1 & 2 & 0 \\ 2 & 1 & -2 & 0 \\ 3 & 1 & 0 & 0 \end{array}\right] \xrightarrow[R_3 = R_3 - 3R_1]{R_2 = R_2 - 2R_1} \left[\begin{array}{ccc|c} 1 & 1 & 2 & 0 \\ 0 & -1 & -6 & 0 \\ 0 & -2 & -6 & 0 \end{array}\right] \xrightarrow{R_2 = -R_2} \left[\begin{array}{ccc|c} 1 & 1 & 2 & 0 \\ 0 & 1 & 6 & 0 \\ 0 & 2 & -6 & 0 \end{array}\right] \xrightarrow{R_3 = R_3 - 2R_2}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 0 \\ 0 & 1 & 6 & 0 \\ 0 & 0 & 6 & 0 \end{array}\right] \xrightarrow{R_3 = 1/6 R_3} \left[\begin{array}{ccc|c} 1 & 1 & 2 & 0 \\ 0 & 1 & 6 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

System has unique solution where $x_1 = x_2 = x_3 = 0$

b) $Ax = 4x \to (A - 4I)x = 0$

$$A - 4I = \left[\begin{array}{ccc|c} -2 & 1 & 2 & 0 \\ 2 & -2 & -2 & 0 \\ 3 & 1 & -3 & 0 \end{array}\right] \xrightarrow{R_1 \leftrightarrow R_2} \left[\begin{array}{ccc|c} 2 & -2 & -2 & 0 \\ -2 & 1 & 2 & 0 \\ 3 & 1 & -3 & 0 \end{array}\right] \xrightarrow{R_1 = 1/2 R_1} \left[\begin{array}{ccc|c} 1 & -1 & -1 & 0 \\ -2 & 1 & 2 & 0 \\ 3 & 1 & -3 & 0 \end{array}\right] \xrightarrow[R_3 = R_3 - 3R_1]{R_2 = R_2 + 2R_1}$$

$$\left[\begin{array}{ccc|c} 1 & -1 & -1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 4 & 0 & 0 \end{array}\right] \xrightarrow[R_3 = R_3 - 4R_2]{R_2 = -R_2} \left[\begin{array}{ccc|c} 1 & -1 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \xrightarrow{R_1 = R_1 + R_2} \left[\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

Let $x_3 = t$, $x_2 = 0$, $x_1 = t$ (General solution)

1.7.24 The matrix is symmetric iff

$$\left.\begin{aligned} a - 2b + 2c &= 3 \\ 2a + b + c &= 0 \\ a + \quad c &= -2 \end{aligned}\right\}$$

You may solve it by Gauss-Jordan elimination
$a = 11$, $b = -9$, $c = -13$

1.7.26 A triangular matrix is invertible iff its diagonal entries are all nonzero

So x may be any real number except 1/2, 1/3, 1/4

1.7.37 a) If A is invertible and skew-symmetric then $(A^{-1})^T = (A^T)^{-1} = (-A)^{-1} = -A^{-1}$ so A^{-1} is also skew symmetric

b) Let A and B be skew-symmetric matrices

$$(A^T)^T = (-A)^T = -A^T$$

$$(A+B)^T = A^T + B^T = -A - B = -(A+B)$$

$$(A-B)^T = A^T - B^T = -A + B = -(A-B)$$

$$(k.A)^T = k.(A^T) = k.(-A) = -k.A$$

c) from the hint, it's sufficient to prove that $\frac{1}{2}(A+A^T)$ is symmetric and $\frac{1}{2}(A+A^T)$ is skew-symmetric

$$\frac{1}{2}(A+A^T)^T = \frac{1}{2}(A^T+(A^T)^T) = \frac{1}{2}(A+A^T)$$

Thus $1/2\,(A+A^T)$ is symmetric.

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MÜHENDİSLİK FAKÜLTESİ

CEVAP KAĞIDI

ANSWER SHEET

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1.7.40

$A^T = -A$

$$\begin{bmatrix} 0 & -2 & -3 \\ 2a-3b+c & 0 & -5 \\ 3a-5b+5c & 5a-8b+6c & d \end{bmatrix} = \begin{bmatrix} 0 & -2a+3b-c & -3a+5b+5c \\ 2 & 0 & -5a+8b-6c \\ 3 & 5 & -d \end{bmatrix}$$

Solve

$$\left.\begin{aligned} -2a+3b-c &= -2 \\ -3a+5b-5c &= -3 \\ 2a-3b-c &= 2 \\ -5a+8b-6c &= -5 \\ 3a-5b+5c &= 3 \\ 5a-8b+6c &= 5 \\ d &= -d \end{aligned}\right\}$$

$d=0$

$$\left.\begin{aligned} 2a-3b-c &= 2 \\ 3a-5b+5c &= 3 \\ 5a-8b+6c &= 5 \end{aligned}\right\}$$ Solve the system

$c=t \quad b=7t \quad a=1+10t \quad d=0$

(parametric solution)

GOOD LUCK!

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