MATH2056 Linear Algebra Spring 2014 Quiz 3

1. Are there values of r and s for which

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & r-2 & 2 \\ 0 & s-1 & r+2 \\ 0 & 0 & 3 \end{bmatrix}$$

has rank 1? Has rank 2? If so, find those values.

2. Find a matrix P that diagonalizes A and compute $P^{-1}AP$

$$A = \begin{bmatrix} 2 & 0 & -2 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

ANSWERS

1) Rank = 2 if $r=2$ and $s=1$ (second column is all 0)

Rank = 1 Impossible (Third column never be 0)

2) Since A is triangular, A has eigenvalues $\lambda_1 = 2$ and $\lambda_2 = 3$

$\lambda_1 = 2$ gives $\lambda I - A = \begin{bmatrix} 0 & 0 & 2 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ reduces to $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

general solution is $x_1 = s$ $x_2 = 0$ $x_3 = 0$

$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = s\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ so $p_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ is a basis

$\lambda_2 = 3$ gives $\lambda I - A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

general solution is $x_1 = -2s$ $x_2 = t$ $x_3 = s$

$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = s\begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix} + t\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ so $p_2 = \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix}$ $p_3 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

So $P = \begin{bmatrix} 1 & -2 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

$\uparrow$ P_1 from λ_1, $\uparrow$ P_2 from λ_2, $\uparrow$ P_3 from λ_3

So $P^{-1}.A.P = \underbrace{\begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}}$

Diagonal matrix with the diagonal entries from eigen values of A

Question 1 explanation

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & r-2 & 2 \\ 0 & s-1 & r+2 \\ 0 & 0 & 1 \end{bmatrix}$

first and last row have leading 1

According to values of r and s last row may be all 0 $(r=-1, s=1)$, but it is not possible to eleminate last two rows because with the values $r=-1$ and $s=1$ second row becomes $\begin{bmatrix} 0 & -3 & 2 \end{bmatrix}$

So rank = 1 not possible

for rank = 2 we have to get 2 rows with all 0

$r=2$ and $s=1$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 2 \\ 0 & 0 & 4 \\ 0 & 0 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$