CSE 123
Introduction to Computing

Lecture 3
Matrix Operations

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Textbooks and other references

Following books are used to prepare this lecture

- Larsen, R.W.Engineering with Excel. 3rd ed. New Jersey. Prentice Hall. 2009. *ISBN: 0-13-601775-4*. (Textbook)
- Billo, E. J. Excel for Scientist and Engineers: Numerical Methods. Wiley. 2007. *ISBN: 978-0471387343*

Worksheet Functions for Working with Matrices

- Matrix is a collection of related values, matrices show up frequently in engineering calculations.
- Matrix manipulations are a natural for Excel Worksheets; the worksheet grid provides columns and rows for the matrix operations
- Matrix calculations can be performed by using array functions rather than menu commands

Worksheet Functions for Working with Matrices

Array: Collection of values organized in rows and columns

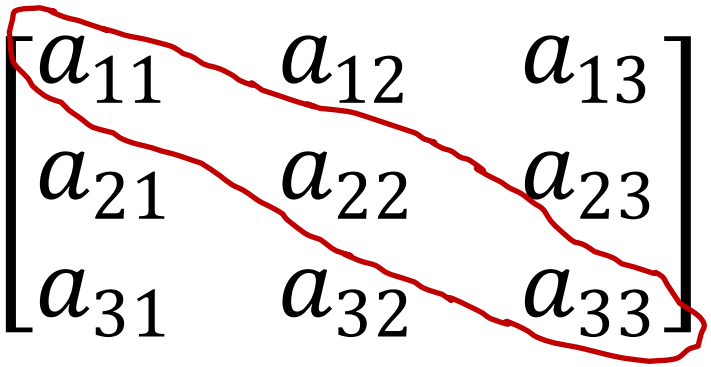
- If you introduce data as an array, every element of the array will be subjected to same modifications

Matrix: Collection of related values organized in rows or columns

Vector: Matrix with a single row or column

Worksheet Functions for Working with Matrices

Square matrix: has the same number of rows and columns



Main diagonal
OR
Principal diagonal

An Introduction to Matrix Mathematics

Addition/Substraction

$$A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \qquad B = \begin{bmatrix} r & s & t \\ u & v & w \\ x & y & z \end{bmatrix}$$

$$A + B = \begin{bmatrix} a + r & b + s & c + t \\ d + u & e + v & f + w \\ g + x & h + y & i + z \end{bmatrix}$$

$$A + 3 = \begin{bmatrix} a + 3 & b + 3 & c + 3 \\ d + 3 & e + 3 & f + 3 \\ g + 3 & h + 3 & i + 3 \end{bmatrix}$$

An Introduction to Matrix Mathematics

Multiplication/Division

$$A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \qquad B = \begin{bmatrix} r & s & t \\ u & v & w \\ x & y & z \end{bmatrix}$$

$$A \cdot B = \begin{bmatrix} ar + bu + cx & as + bv + cy & at + bw + cz \\ dr + eu + fx & ds + ev + fy & dt + ew + fz \\ gr + hu + ix & gs + hv + iy & gt + hw + iz \end{bmatrix}$$

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An Introduction to Matrix Mathematics

Matrix Multiplication

$$A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \qquad B = \begin{bmatrix} r & s & t \\ u & v & w \\ x & y & z \end{bmatrix}$$

$$A \times B = \begin{bmatrix} a \times r & b \times s & c \times t \\ d \times u & e \times v & f \times w \\ g \times x & h \times y & i \times z \end{bmatrix}$$

$$A \times 3 = \begin{bmatrix} a \times 3 & b \times 3 & c \times 3 \\ d \times 3 & e \times 3 & f \times 3 \\ g \times 3 & h \times 3 & i \times 3 \end{bmatrix}$$

An Introduction to Matrix Mathematics

Transposition

Formed by exchanging rows and columns of a matrix

$$A = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}$$

$$A^T = \begin{bmatrix} a & d \\ b & e \\ c & f \end{bmatrix}$$

An Introduction to Matrix Mathematics

Inversion

$AA^{-1} = I$ I is the unit matrix

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}^{-1} \qquad \left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 3 & 4 & 0 & 1 \end{array}\right]$$

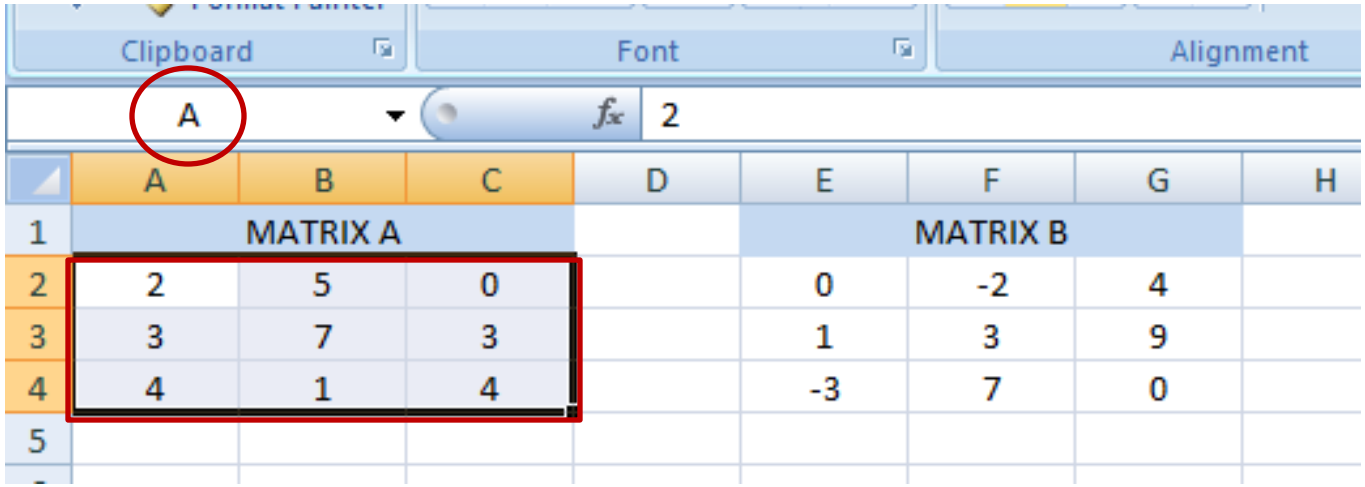
$$\xrightarrow{R2-3R1} \left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 0 & -2 & -3 & 1 \end{array}\right]$$

$$\xrightarrow{-\frac{1}{2}R2} \left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 0 & 1 & 3/2 & -1/2 \end{array}\right]$$

$$\xrightarrow{R1-2R2} \left[\begin{array}{cc|cc} 1 & 0 & -2 & 1 \\ 0 & 1 & 3/2 & -1/2 \end{array}\right]$$

Worksheet Functions for Working with Matrices

- Performing matrix operations in Excel is very simple.
- Assign name to a range of cells.

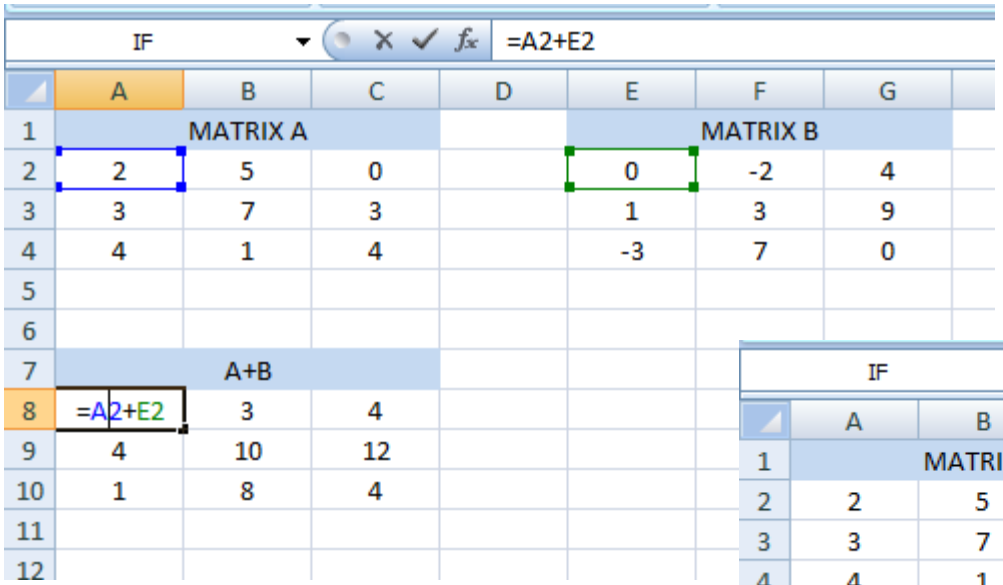


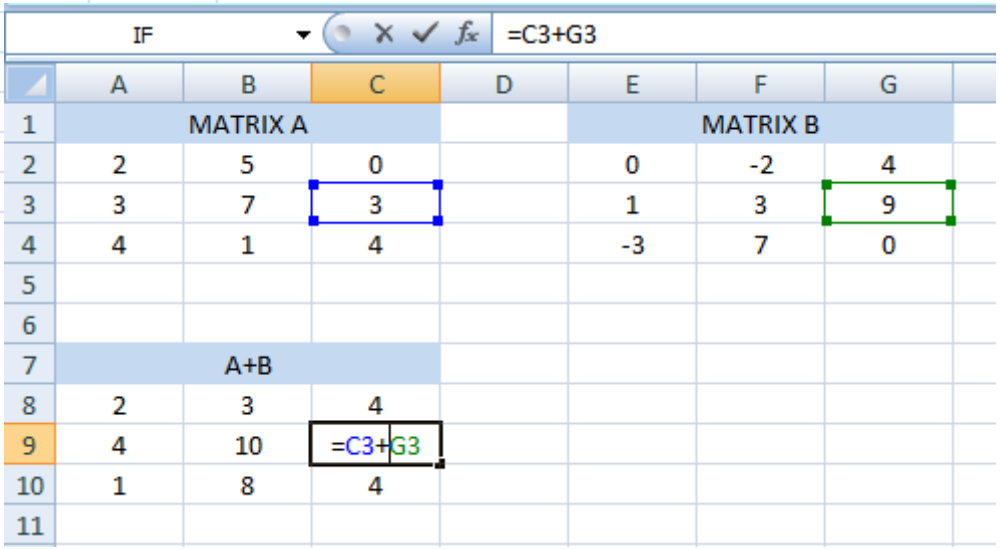
Basic Matrix Operations in Excel

Addition

- Matrices has to be the same size

1) Matrix addition can be conducted element by element





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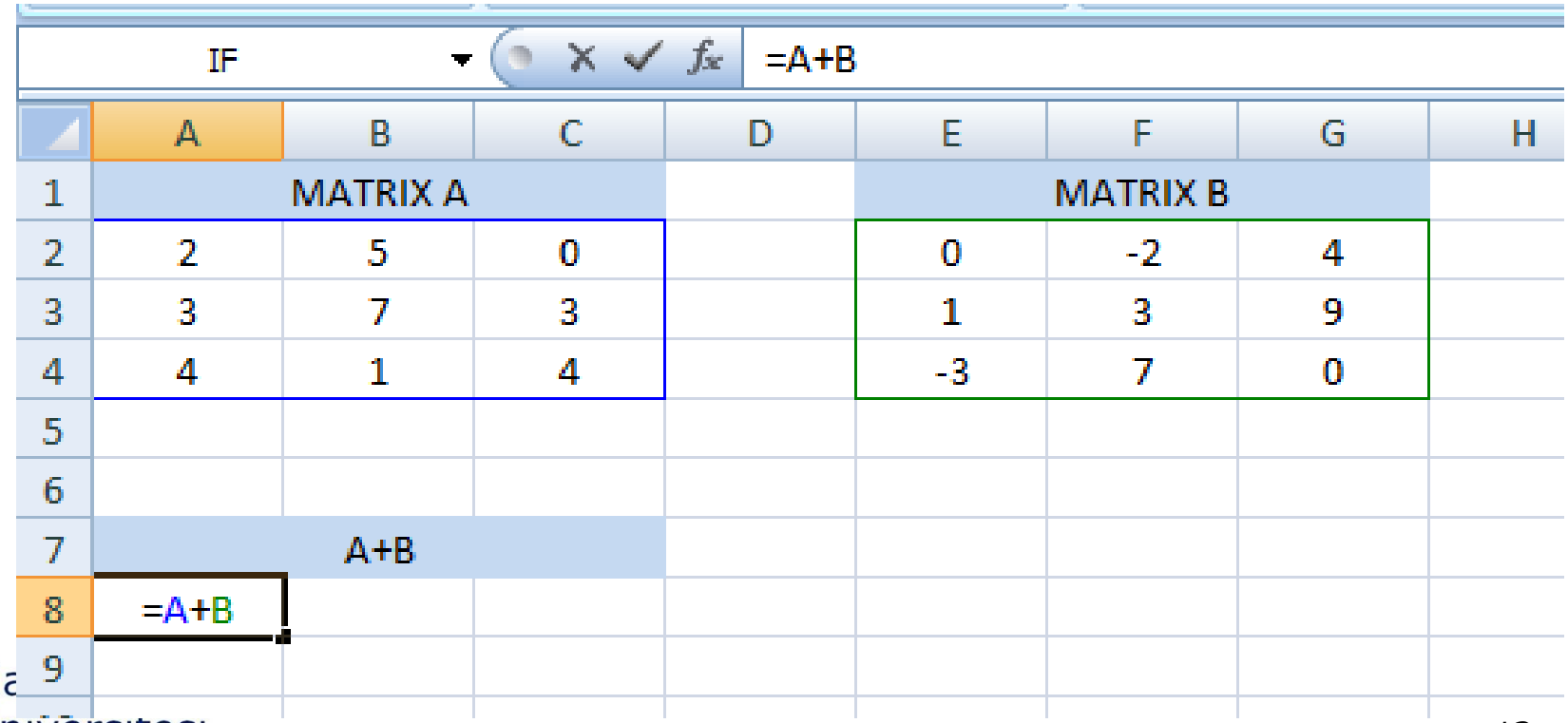
Basic Matrix Operations in Excel
Addition

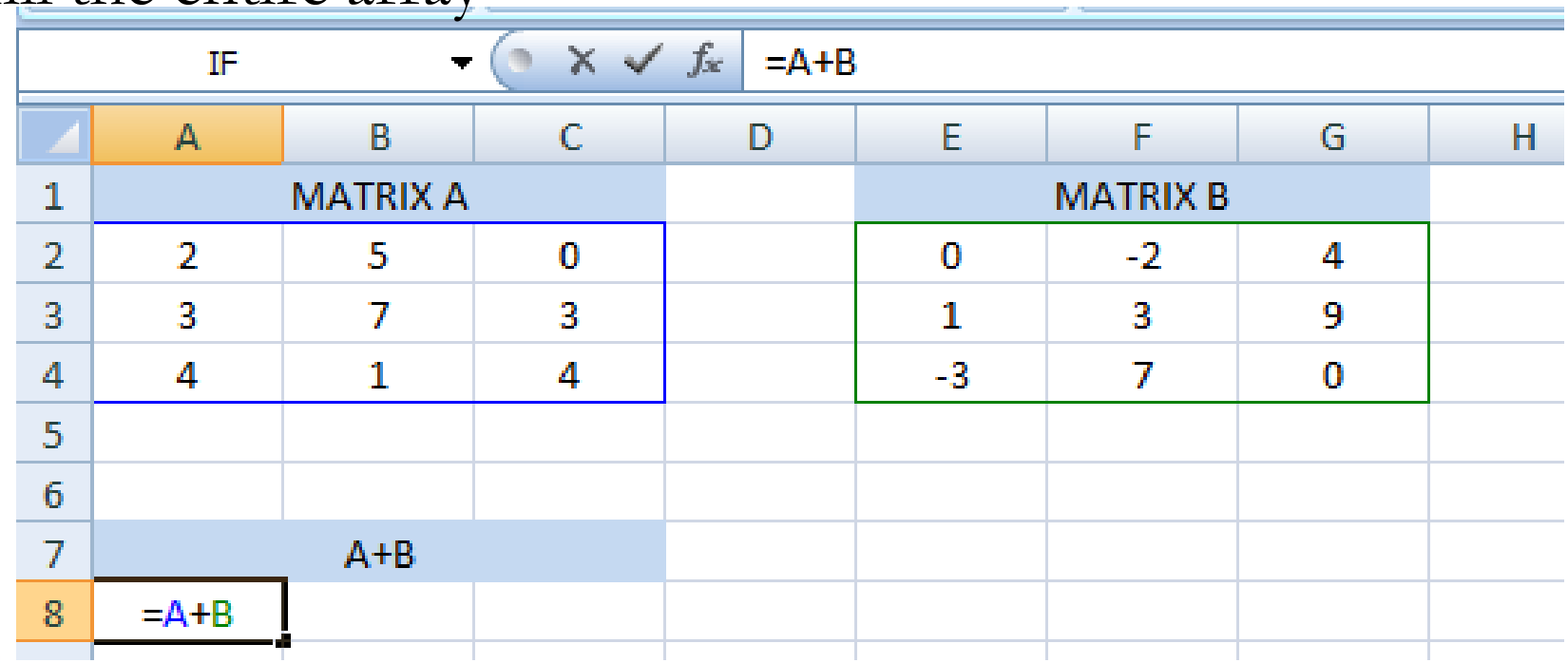
2) Array Math can be used for matrix additon

- Any time array math is used, the size of the resulting array must be indicated before entering the array formula



Basic Matrix Operations in Excel
Addition

- Excel requires special character sequence when entering array formulas
- [CTRL-Shift-Enter] after entering the formula, it tells to fill the entire array



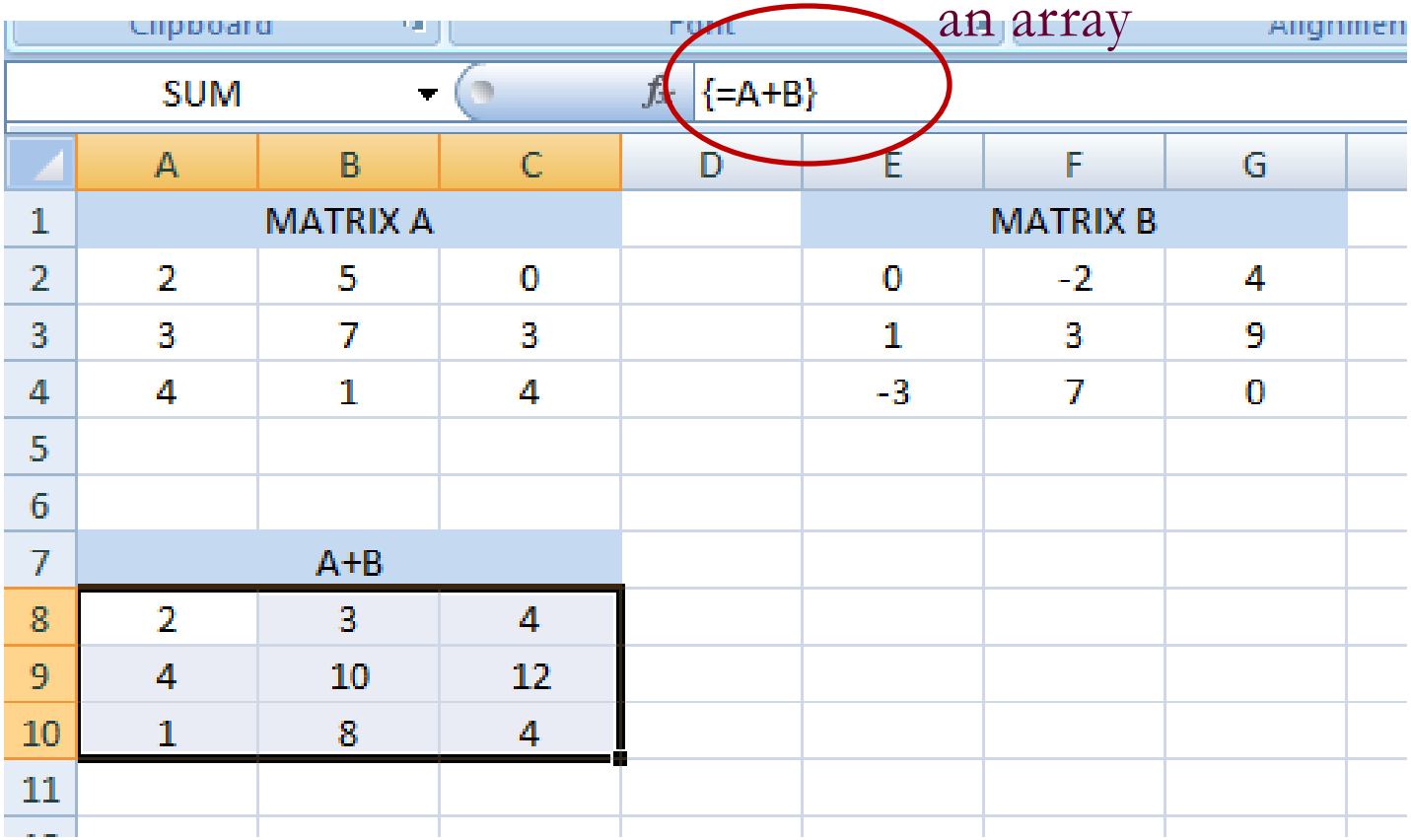
	A	B	C	D	E	F	G	H
1	MATRIX A				MATRIX B			
2	2	5	0		0	-2	4	
3	3	7	3		1	3	9	
4	4	1	4		-3	7	0	
5								
6								
7	A+B							
8	=A+B							

Basic Matrix Operations in Excel

Addition

- After [CTRL-Shift-Enter]

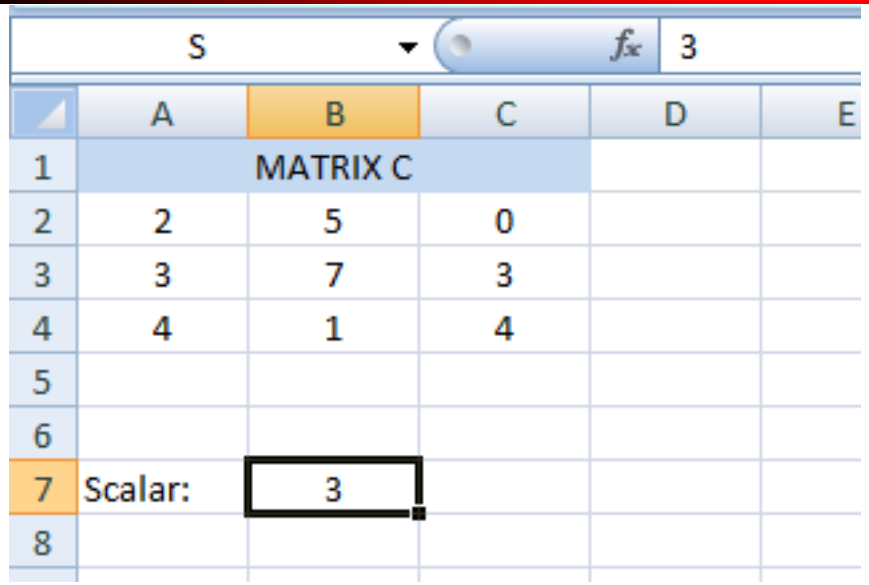
The braces indicate that array math was used and the result is an array



	A	B	C	D	E	F	G
1	MATRIX A				MATRIX B		
2	2	5	0		0	-2	4
3	3	7	3		1	3	9
4	4	1	4		-3	7	0
5							
6							
7	A+B						
8	2	3	4				
9	4	10	12				
10	1	8	4				
11							

Basic Matrix Operations in Excel

Multiplying by a scalar

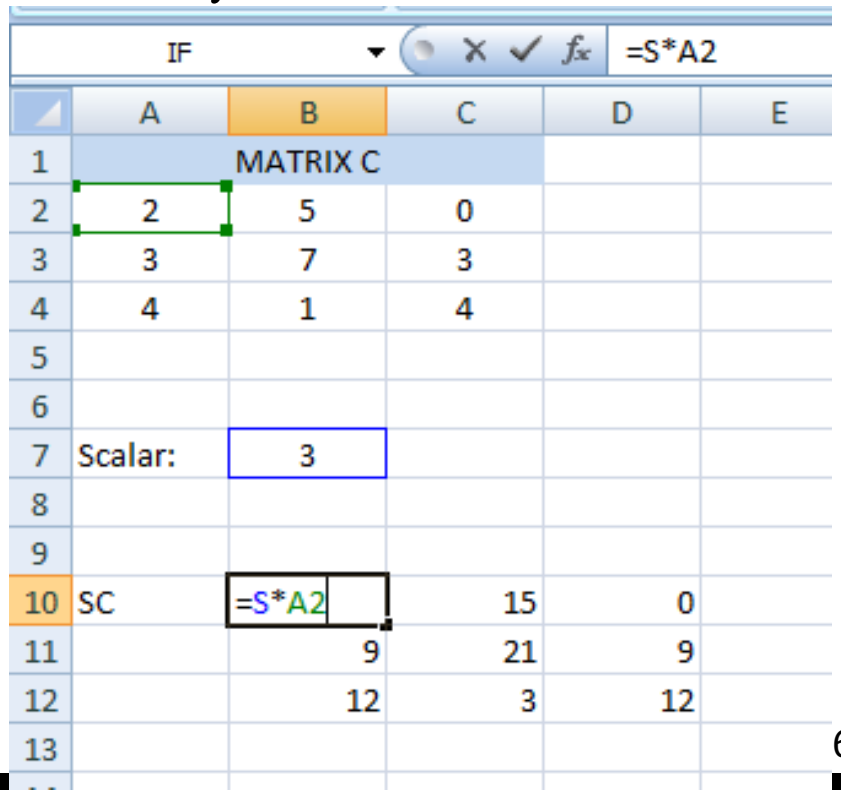


S				
	A	B	C	D	E
1		MATRIX C			
2	2	5	0		
3	3	7	3		
4	4	1	4		
5					
6					
7	Scalar:	3			
8					

S for scalar

- Multiplication can be done using;
  - Cell math
  - Array math

Cell Math is Used



IF		=S*A2		
	A	B	C	D	E
1		MATRIX C			
2	2	5	0		
3	3	7	3		
4	4	1	4		
5					
6					
7	Scalar:	3			
8					
9					
10	SC	=S*A2	15	0	
11		9	21	9	
12		12	3	12	
13					

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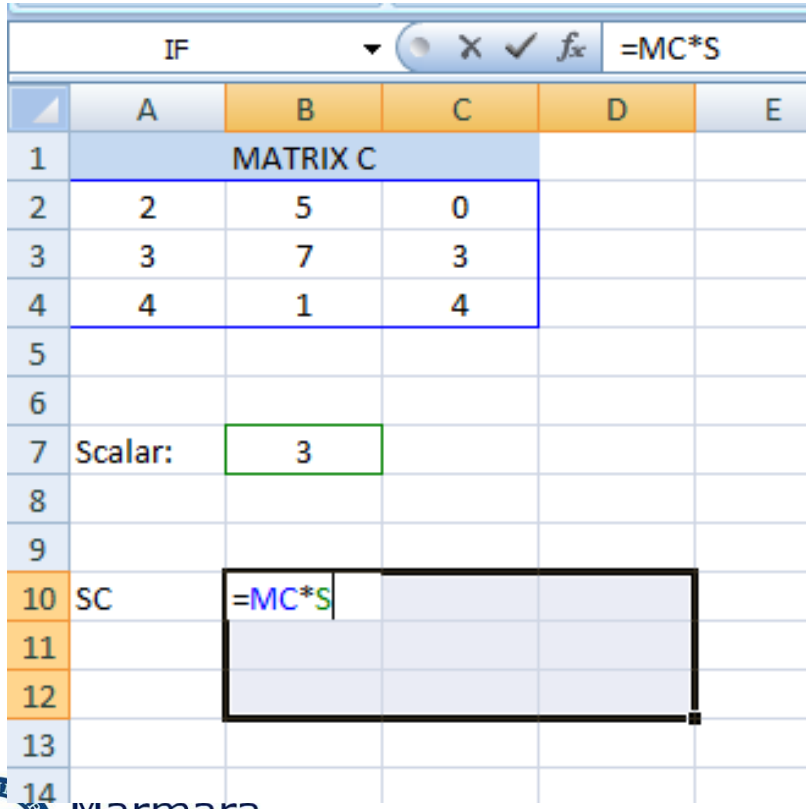
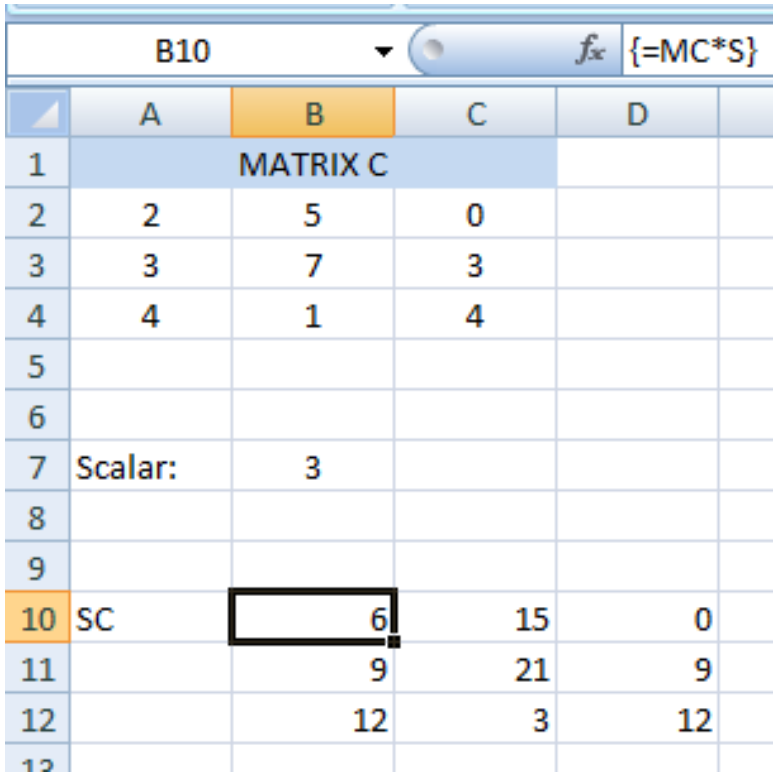
Basic Matrix Operations in Excel

Multiplying by a scalar

- Name the matrix range (MC in this case)
- Determine the size of the matrix
- Write the formula



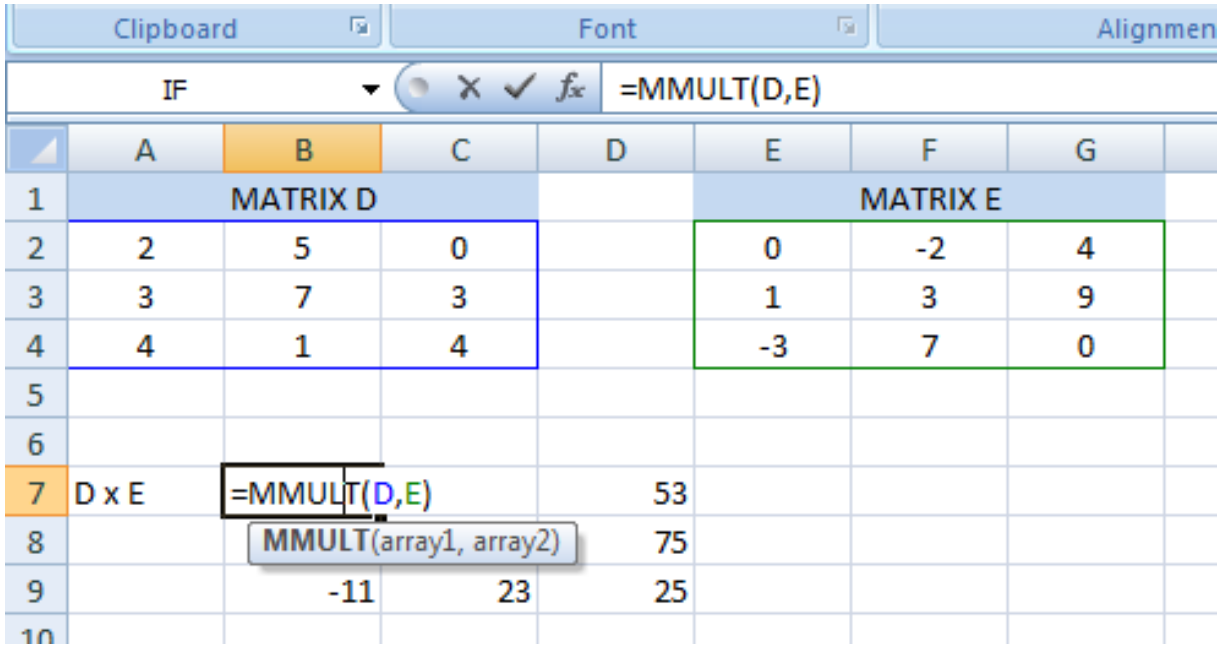


Basic Matrix Operations in Excel

Multiplying two matrices

- Number of columns in the first matrix should be equal to the number of rows in the second matrix
- Matrix multiplication array function;

  =MMULT(first matrix, second matrix)
- **Press [CTRK-Shift-Enter] not just Enter**



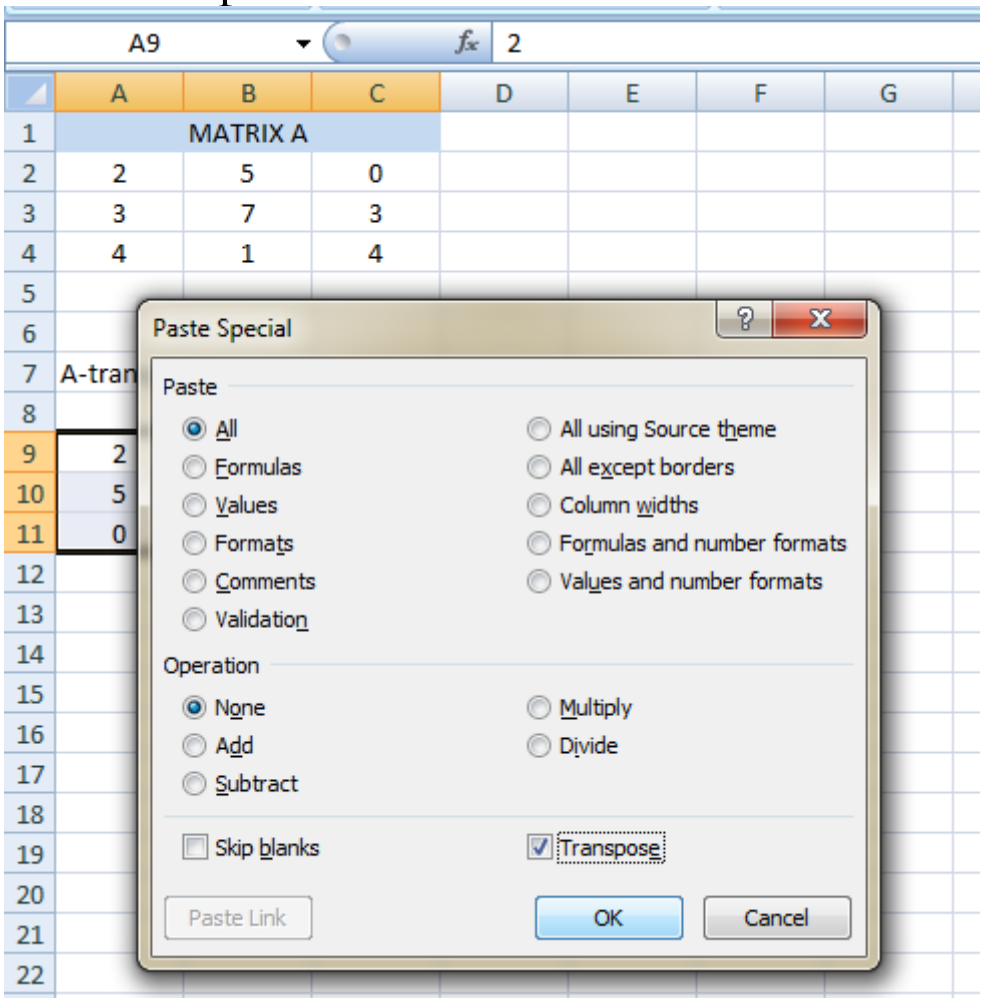
Basic Matrix Operations in Excel
Transposing a Matrix

- Two ways to transpose a matrix
  - As values
  - Using array function TRANSPOSE()

Basic Matrix Operations in Excel

Transposing a Matrix – As Values

- Copy the Matrix
- Indicate the cell that will contain the top left corner of the results matrix
- Open the paste special dialog
- Click Transpose check box



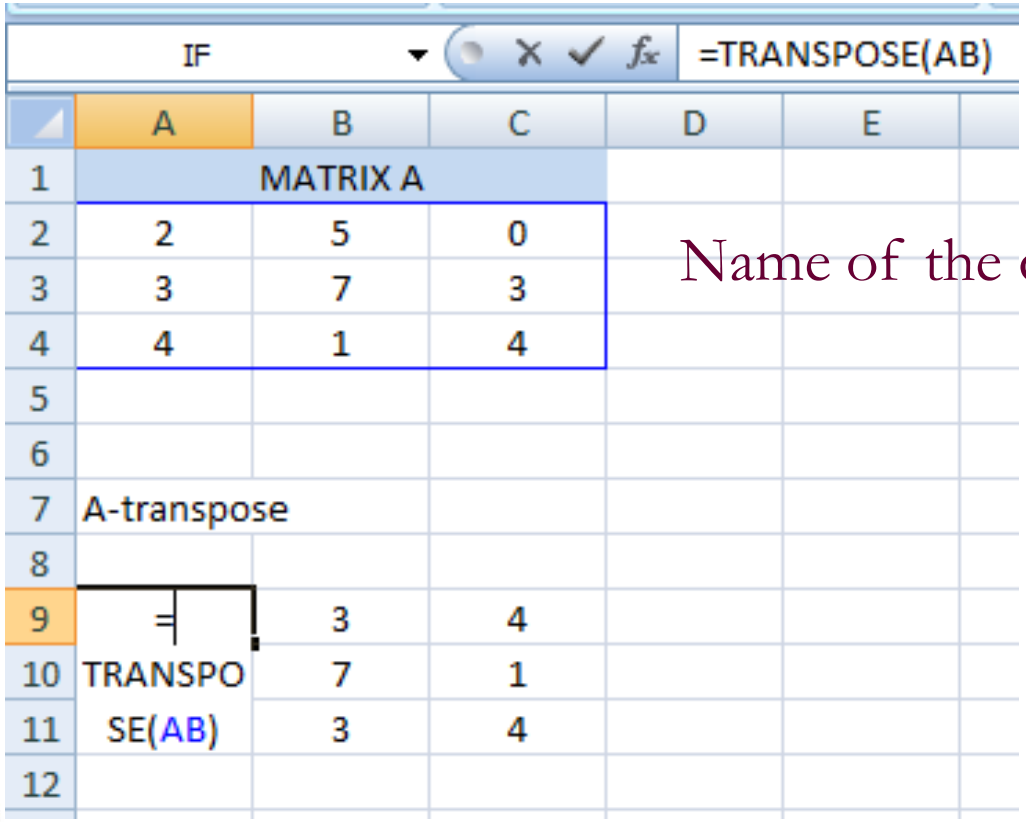
Basic Matrix Operations in Excel

Transposing a Matrix – Using an array function

- Use the following function;

=TRANSPOSE(matrix)

- After entering the function, press [CTRL-Shift-Enter]

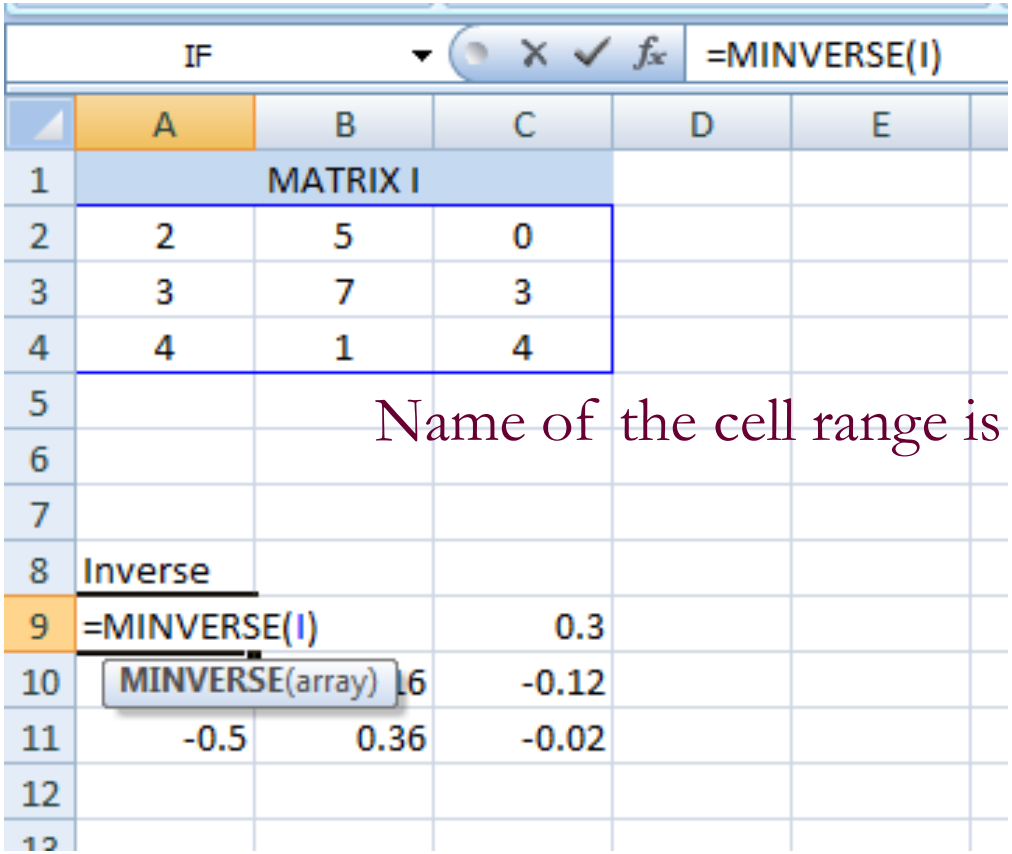


Name of the cell range is AB

Basic Matrix Operations in Excel

Inverting a Matrix

- Only square matrices can be inverted
- Use MINVERSE(matrix) function
- Press [CTRL-Shift-Enter]



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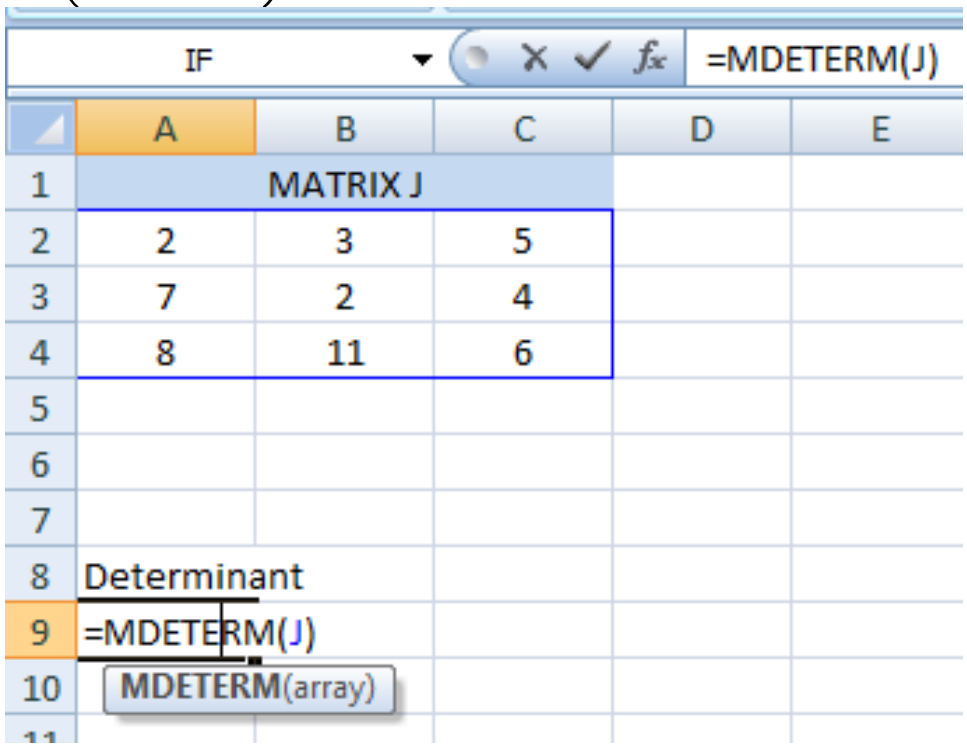
Basic Matrix Operations in Excel
Matrix Determinant

- Determinant is a single value calculated from a matrix and it is often used in solving systems of equations
- Determinant can only be calculated for square matrices
- Use MDETERM(matrix) function



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Solving Systems of Linear Equations

- Matrix operations can be used to solve systems of linear algebraic equations

$$3x_1+2x_2+4x_3=5$$
$$2x_1+5x_2+3x_3=17$$
$$7x_1+2x_2+2x_3=11$$

- Write the equations in matrix form

Solving Systems of Linear Equations

$$3x_1+2x_2+4x_3=5$$

$$2x_1+5x_2+3x_3=17$$

$$7x_1+2x_2+2x_3=11$$

$$[x]=\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix} \qquad [a]=\begin{bmatrix}3&2&4\\2&5&3\\7&2&2\end{bmatrix}$$

$$[r]=\begin{bmatrix}5\\17\\11\end{bmatrix}$$

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Solving Systems of Linear Equations

- [x]=[coefinv][rhs]