

CSE 123

Introduction to Computing

Lecture 3

Matrix Operations

SPRING 2012

Assist. Prof. A. Evren Tugtas



Textbooks and other references

Following books are used to prepare this lecture

- Larsen, R.W. Engineering with Excel. 3rd ed. New Jersey. Prentice Hall. 2009. *ISBN: 0-13-601775-4.*
(Textbook)
- Billo, E. J. Excel for Scientist and Engineers: Numerical Methods. Wiley. 2007. *ISBN: 978-0471387343*

Worksheet Functions for Working with Matrices

- Matrix is a collection of related values, matrices show up frequently in engineering calculations.
- Matrix manipulations are a natural for Excel Worksheets; the worksheet grid provides columns and rows for the matrix operations
- Matrix calculations can be performed by using array functions rather than menu commands

Worksheet Functions for Working with Matrices

Array: Collection of values organized in rows and columns

- If you introduce data as an array, every element of the array will be subjected to same modifications

Matrix: Collection of related values organized in rows or columns

Vector: Matrix with a single row or column

Worksheet Functions for Working with Matrices

Square matrix: has the same number of rows and columns

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

Main diagonal
OR
Principal diagonal

An Introduction to Matrix Mathematics

Addition/ Substraction

$$A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \quad B = \begin{bmatrix} r & s & t \\ u & v & w \\ x & y & z \end{bmatrix}$$

$$A + B = \begin{bmatrix} a + r & b + s & c + t \\ d + u & e + v & f + w \\ g + x & h + y & i + z \end{bmatrix}$$

$$A + 3 = \begin{bmatrix} a + 3 & b + 3 & c + 3 \\ d + 3 & e + 3 & f + 3 \\ g + 3 & h + 3 & i + 3 \end{bmatrix}$$

An Introduction to Matrix Mathematics

Multiplication/Division

$$A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \quad B = \begin{bmatrix} r & s & t \\ u & v & w \\ x & y & z \end{bmatrix}$$

$$A \cdot B = \begin{bmatrix} ar + bu + cx & as + bv + cy & at + bw + cz \\ dr + eu + fx & ds + ev + fy & dt + ew + fz \\ gr + hu + ix & gs + hv + iy & gt + hw + iz \end{bmatrix}$$

An Introduction to Matrix Mathematics

Matrix Multiplication

$$A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$

$$B = \begin{bmatrix} r & s & t \\ u & v & w \\ x & y & z \end{bmatrix}$$

$$A \times B = \begin{bmatrix} a \times r & b \times s & c \times t \\ d \times u & e \times v & f \times w \\ g \times x & h \times y & i \times z \end{bmatrix}$$

$$A \times 3 = \begin{bmatrix} a \times 3 & b \times 3 & c \times 3 \\ d \times 3 & e \times 3 & f \times 3 \\ g \times 3 & h \times 3 & i \times 3 \end{bmatrix}$$

An Introduction to Matrix Mathematics

Transposition

Formed by exchanging rows and columns of a matrix

$$A = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}$$

$$A^T = \begin{bmatrix} a & d \\ b & e \\ c & f \end{bmatrix}$$

An Introduction to Matrix Mathematics

Inversion

$$AA^{-1} = I \quad \text{I is the unit matrix}$$

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}^{-1} \quad \left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 3 & 4 & 0 & 1 \end{array} \right]$$

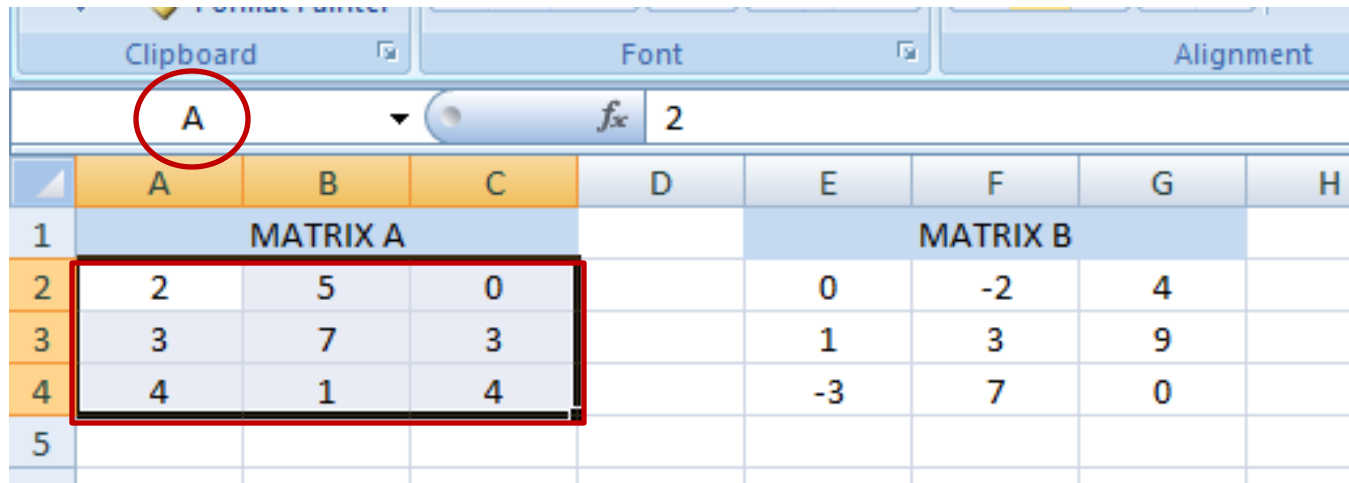
$$\xrightarrow{R2-3R1} \left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 0 & -2 & -3 & 1 \end{array} \right]$$

$$\xrightarrow{-\frac{1}{2}R2} \left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 0 & 1 & 3/2 & -1/2 \end{array} \right]$$

$$\xrightarrow{R1-2R2} \left[\begin{array}{cc|cc} 1 & 0 & -2 & 1 \\ 0 & 1 & 3/2 & -1/2 \end{array} \right]$$

Worksheet Functions for Working with Matrices

- Performing matrix operations in Excel is very simple.
- Assign name to a range of cells.



The screenshot shows the Excel ribbon with the 'Clipboard', 'Font', and 'Alignment' tabs. The 'Clipboard' tab is active, and the 'A' button is circled in red. The worksheet below shows two matrices, MATRIX A and MATRIX B, defined in the range A2:C4. The cell 'A' in the ribbon is circled in red, and the range of cells A2:C4 is highlighted with a red border.

| | A | B | C | D | E | F | G | H |
|---|----------|---|---|---|----------|----|---|---|
| 1 | MATRIX A | | | | MATRIX B | | | |
| 2 | 2 | 5 | 0 | | 0 | -2 | 4 | |
| 3 | 3 | 7 | 3 | | 1 | 3 | 9 | |
| 4 | 4 | 1 | 4 | | -3 | 7 | 0 | |
| 5 | | | | | | | | |

Basic Matrix Operations in Excel

Addition

- Matrices has to be the same size
- 1) Matrix addition can be conducted element by element

| | A | B | C | D | E | F | G |
|----|----------|----|----|---|----------|----|---|
| 1 | MATRIX A | | | | MATRIX B | | |
| 2 | 2 | 5 | 0 | | 0 | -2 | 4 |
| 3 | 3 | 7 | 3 | | 1 | 3 | 9 |
| 4 | 4 | 1 | 4 | | -3 | 7 | 0 |
| 5 | | | | | | | |
| 6 | | | | | | | |
| 7 | A+B | | | | | | |
| 8 | =A2+E2 | 3 | 4 | | | | |
| 9 | 4 | 10 | 12 | | | | |
| 10 | 1 | 8 | 4 | | | | |
| 11 | | | | | | | |
| 12 | | | | | | | |

| | A | B | C | D | E | F | G |
|----|----------|----|--------|---|----------|----|---|
| 1 | MATRIX A | | | | MATRIX B | | |
| 2 | 2 | 5 | 0 | | 0 | -2 | 4 |
| 3 | 3 | 7 | 3 | | 1 | 3 | 9 |
| 4 | 4 | 1 | 4 | | -3 | 7 | 0 |
| 5 | | | | | | | |
| 6 | | | | | | | |
| 7 | A+B | | | | | | |
| 8 | 2 | 3 | 4 | | | | |
| 9 | 4 | 10 | =C3+G3 | | | | |
| 10 | 1 | 8 | 4 | | | | |
| 11 | | | | | | | |

Basic Matrix Operations in Excel

Addition

- 2) Array Math can be used for matrix addition
- Any time array math is used, the size of the resulting array must be indicated before entering the array formula

| | A | B | C | D | E | F | G | H |
|---|----------|---|---|---|----------|----|---|---|
| 1 | MATRIX A | | | | MATRIX B | | | |
| 2 | 2 | 5 | 0 | | 0 | -2 | 4 | |
| 3 | 3 | 7 | 3 | | 1 | 3 | 9 | |
| 4 | 4 | 1 | 4 | | -3 | 7 | 0 | |
| 5 | | | | | | | | |
| 6 | | | | | | | | |
| 7 | A+B | | | | | | | |
| 8 | =A+B | | | | | | | |
| 9 | | | | | | | | |

Basic Matrix Operations in Excel

Addition

- Excel requires special character sequence when entering array formulas
- [CTRL-Shift-Enter] after entering the formula, it tells to fill the entire array

| | A | B | C | D | E | F | G | H |
|---|----------|---|---|---|----------|----|---|---|
| 1 | MATRIX A | | | | MATRIX B | | | |
| 2 | 2 | 5 | 0 | | 0 | -2 | 4 | |
| 3 | 3 | 7 | 3 | | 1 | 3 | 9 | |
| 4 | 4 | 1 | 4 | | -3 | 7 | 0 | |
| 5 | | | | | | | | |
| 6 | | | | | | | | |
| 7 | A+B | | | | | | | |
| 8 | =A+B | | | | | | | |

Basic Matrix Operations in Excel

Addition

- After [CTRL-Shift-Enter]

The braces indicate that array math was used and the result is an array

| | A | B | C | D | E | F | G |
|----|----------|----|----|---|----------|----|---|
| 1 | MATRIX A | | | | MATRIX B | | |
| 2 | 2 | 5 | 0 | | 0 | -2 | 4 |
| 3 | 3 | 7 | 3 | | 1 | 3 | 9 |
| 4 | 4 | 1 | 4 | | -3 | 7 | 0 |
| 5 | | | | | | | |
| 6 | | | | | | | |
| 7 | A+B | | | | | | |
| 8 | 2 | 3 | 4 | | | | |
| 9 | 4 | 10 | 12 | | | | |
| 10 | 1 | 8 | 4 | | | | |
| 11 | | | | | | | |

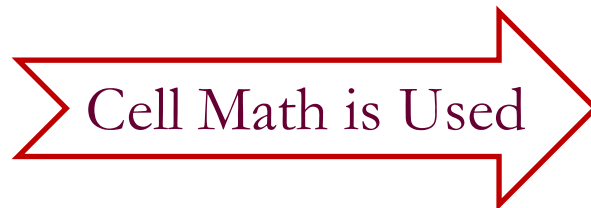
Basic Matrix Operations in Excel

Multiplying by a scalar

| | A | B | C | D | E |
|---|----------|---|---|---|---|
| 1 | MATRIX C | | | | |
| 2 | 2 | 5 | 0 | | |
| 3 | 3 | 7 | 3 | | |
| 4 | 4 | 1 | 4 | | |
| 5 | | | | | |
| 6 | | | | | |
| 7 | Scalar: | 3 | | | |
| 8 | | | | | |

S for scalar

- Multiplication can be done using;
 - Cell math
 - Array math



| | A | B | C | D | E |
|----|----------|-------|----|----|---|
| 1 | MATRIX C | | | | |
| 2 | 2 | 5 | 0 | | |
| 3 | 3 | 7 | 3 | | |
| 4 | 4 | 1 | 4 | | |
| 5 | | | | | |
| 6 | | | | | |
| 7 | Scalar: | 3 | | | |
| 8 | | | | | |
| 9 | | | | | |
| 10 | SC | =S*A2 | 15 | 0 | |
| 11 | | 9 | 21 | 9 | |
| 12 | | 12 | 3 | 12 | |
| 13 | | | | | |

Basic Matrix Operations in Excel

Multiplying by a scalar

- Name the matrix range (MC in this case)
- Determine the size of the matrix
- Write the formula

| | A | B | C | D | E |
|----|----------|-------|---|---|---|
| 1 | MATRIX C | | | | |
| 2 | 2 | 5 | 0 | | |
| 3 | 3 | 7 | 3 | | |
| 4 | 4 | 1 | 4 | | |
| 5 | | | | | |
| 6 | | | | | |
| 7 | Scalar: | 3 | | | |
| 8 | | | | | |
| 9 | | | | | |
| 10 | SC | =MC*S | | | |
| 11 | | | | | |
| 12 | | | | | |
| 13 | | | | | |
| 14 | | | | | |

| | A | B | C | D |
|----|----------|----|----|----|
| 1 | MATRIX C | | | |
| 2 | 2 | 5 | 0 | |
| 3 | 3 | 7 | 3 | |
| 4 | 4 | 1 | 4 | |
| 5 | | | | |
| 6 | | | | |
| 7 | Scalar: | 3 | | |
| 8 | | | | |
| 9 | | | | |
| 10 | SC | 6 | 15 | 0 |
| 11 | | 9 | 21 | 9 |
| 12 | | 12 | 3 | 12 |
| 13 | | | | |

Basic Matrix Operations in Excel

Multiplying two matrices

- Number of columns in the first matrix should be equal to the number of rows in the second matrix
- Matrix multiplication array function;
=MMULT(first matrix, second matrix)
- Press [CTRL-Shift-Enter] not just Enter

| | A | B | C | D | E | F | G |
|----|----------|-----------------------|----|----|----------|----|---|
| 1 | MATRIX D | | | | MATRIX E | | |
| 2 | 2 | 5 | 0 | | 0 | -2 | 4 |
| 3 | 3 | 7 | 3 | | 1 | 3 | 9 |
| 4 | 4 | 1 | 4 | | -3 | 7 | 0 |
| 5 | | | | | | | |
| 6 | | | | | | | |
| 7 | D x E | =MMULT(D,E) | | 53 | | | |
| 8 | | MMULT(array1, array2) | | 75 | | | |
| 9 | | -11 | 23 | 25 | | | |
| 10 | | | | | | | |

Basic Matrix Operations in Excel

Transposing a Matrix

- Two ways to transpose a matrix
 - As values
 - Using array function `TRANSPOSE()`

Basic Matrix Operations in Excel

Transposing a Matrix – As Values

- Copy the Matrix
- Indicate the cell that will contain the top left corner of the results matrix
- Open the paste special dialog
- Click Transpose check box

The screenshot shows an Excel spreadsheet with a 4x3 matrix labeled 'MATRIX A' in cells A2:C5. The values are: Row 2: 2, 5, 0; Row 3: 3, 7, 3; Row 4: 4, 1, 4. The 'Paste Special' dialog box is open, showing the 'Paste' section with 'All' selected. In the 'Operation' section, 'None' is selected. The 'Transpose' checkbox is checked. The 'Skip blanks' checkbox is unchecked. The 'Paste Link' button is disabled, and the 'OK' and 'Cancel' buttons are visible.

| | A | B | C | D | E | F | G |
|----|----------|---|---|---|---|---|---|
| 1 | MATRIX A | | | | | | |
| 2 | 2 | 5 | 0 | | | | |
| 3 | 3 | 7 | 3 | | | | |
| 4 | 4 | 1 | 4 | | | | |
| 5 | | | | | | | |
| 6 | | | | | | | |
| 7 | A-tran | | | | | | |
| 8 | | | | | | | |
| 9 | 2 | | | | | | |
| 10 | 5 | | | | | | |
| 11 | 0 | | | | | | |
| 12 | | | | | | | |
| 13 | | | | | | | |
| 14 | | | | | | | |
| 15 | | | | | | | |
| 16 | | | | | | | |
| 17 | | | | | | | |
| 18 | | | | | | | |
| 19 | | | | | | | |
| 20 | | | | | | | |
| 21 | | | | | | | |
| 22 | | | | | | | |

Basic Matrix Operations in Excel

Transposing a Matrix – Using an array function

- Use the following function;

=TRANSPOSE(matrix)

- After entering the function, press [CTRL-Shift-Enter]

| | A | B | C | D | E |
|----|-------------|---|---|---|---|
| 1 | MATRIX A | | | | |
| 2 | 2 | 5 | 0 | | |
| 3 | 3 | 7 | 3 | | |
| 4 | 4 | 1 | 4 | | |
| 5 | | | | | |
| 6 | | | | | |
| 7 | A-transpose | | | | |
| 8 | | | | | |
| 9 | = | 3 | 4 | | |
| 10 | TRANSP | 7 | 1 | | |
| 11 | SE(AB) | 3 | 4 | | |
| 12 | | | | | |

Name of the cell range is AB

Basic Matrix Operations in Excel

Inverting a Matrix

- Only square matrices can be inverted
- Use MINVERSE(matrix) function
- Press [CTRL-Shift-Enter]

The screenshot shows an Excel spreadsheet with the following data:

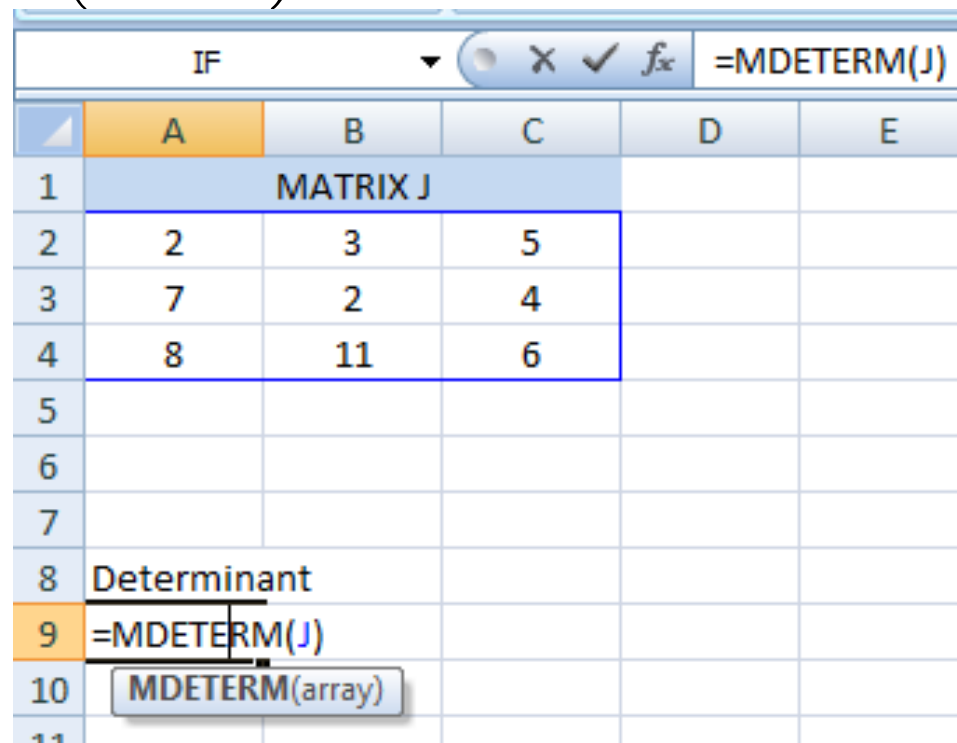
| | A | B | C | D | E |
|----|-----------------|------|-------|-----|-------|
| 1 | MATRIX I | | | | |
| 2 | 2 | 5 | 0 | | |
| 3 | 3 | 7 | 3 | | |
| 4 | 4 | 1 | 4 | | |
| 5 | | | | | |
| 6 | | | | | |
| 7 | | | | | |
| 8 | Inverse | | | | |
| 9 | =MINVERSE(I) | | | 0.3 | |
| 10 | MINVERSE(array) | | | 16 | -0.12 |
| 11 | -0.5 | 0.36 | -0.02 | | |
| 12 | | | | | |
| 13 | | | | | |

Name of the cell range is I

Basic Matrix Operations in Excel

Matrix Determinant

- Determinant is a single value calculated from a matrix and it is often used in solving systems of equations
- Determinant can only be calculated for square matrices
- Use `MDETERM(matrix)` function



The screenshot shows an Excel spreadsheet with the following content:

| | A | B | C | D | E |
|----|--------------------------|----|---|---|---|
| 1 | MATRIX J | | | | |
| 2 | 2 | 3 | 5 | | |
| 3 | 7 | 2 | 4 | | |
| 4 | 8 | 11 | 6 | | |
| 5 | | | | | |
| 6 | | | | | |
| 7 | | | | | |
| 8 | Determinant | | | | |
| 9 | <code>=MDETERM(J)</code> | | | | |
| 10 | MDETERM(array) | | | | |
| 11 | | | | | |

Solving Systems of Linear Equations

- Matrix operations can be used to solve systems of linear algebraic equations

$$3x_1 + 2x_2 + 4x_3 = 5$$

$$2x_1 + 5x_2 + 3x_3 = 17$$

$$7x_1 + 2x_2 + 2x_3 = 11$$

- Write the equations in matrix form

Solving Systems of Linear Equations

$$3x_1 + 2x_2 + 4x_3 = 5$$

$$2x_1 + 5x_2 + 3x_3 = 17$$

$$7x_1 + 2x_2 + 2x_3 = 11$$

$$[x] = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$[a] = \begin{bmatrix} 3 & 2 & 4 \\ 2 & 5 & 3 \\ 7 & 2 & 2 \end{bmatrix}$$

$$[r] = \begin{bmatrix} 5 \\ 17 \\ 11 \end{bmatrix}$$

Solving Systems of Linear Equations

- $[x] = [\text{coefinv}] [\text{rhs}]$

IF =MMULT(coefinv, rhs)

| | A | B | C | D | E | F | G | H | I | J |
|----|--------------|-------|-------|-------|---|---|---|-----------------------|----|---|
| 1 | | | | | | | | | | |
| 2 | | | | | | | | | | |
| 3 | Coefficients | 3 | 2 | 4 | | | | rhs | 5 | |
| 4 | | 2 | 5 | 3 | | | | | 17 | |
| 5 | | 7 | 2 | 2 | | | | | 11 | |
| 6 | | | | | | | | | | |
| 7 | | | | | | | | | | |
| 8 | | | | | | | | | | |
| 9 | Coeff invert | -0.05 | -0.05 | 0.18 | | | | =MMULT(coefinv, rhs) | | |
| 10 | | -0.22 | 0.28 | 0.01 | | | | MMULT(array1, array2) | | |
| 11 | | 0.40 | -0.10 | -0.14 | | | | -1.30769 | | |
| 12 | | | | | | | | | | |
| 13 | | | | | | | | | | |