INTERPRETATIONS OF THE FORMAL DEFINITION OF DERIVATIVE IN MULTIPLE REPRESENTATIONS

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Multiple representations has been a focus of attention in mathematics education research in recent decades especially with the avaibility of computers and graphical calculators (Confrey, 1994; Kaput, 1992; Keller & Hirsch, 1998; Leinhardt et al., 1990; Yerushalmy, 1991). Curriculums in many countries emphasize the importance of using multiple representations of mathematical concepts (NCTM, 1989; NCTM, 2000).

However, some researchers questioned the meanings given to representations seemingly shared by the mathematics education community (Sierpinska, 1992; Thompson, 1994). Both researchers consider the multiple representations in the case of function. Sierpinska (1992) emphasizes that the awareness of the limitations of each representation is essential for understanding the function concept. Thompson (1994) asserts that the core concept of function, the one and the same general concept for all representations, can not be represented by what are commonly called multiple representations. He claims that students do not realise that something remains the same as they move among different representations, therefore they see each representation as a “topic” to be learned in isolation. Akkoç & Tall (2002) investigated the meanings inherent in multiple representations of functions and found that students’ focus on the defining properties varies from representation to representation.

Multiple representations of the derivative concept has been studied by many researchers (Amoah and Laridon, 2004; Tall, 1997; Asiala, Cotrill & Dubinsky, 1997; Giraldo, Tall & Carvalho, 2003). For instance, Amoah & Laridon (2004) assessed the conceptual understanding of derivative by looking at the succesful use of multiple representations (graphical,numerical and algebraic). They found that there was not any consistency between students’ succes in various representations.

This study investigates the meanings attributed to the formal definition of the derivative concept in various representations such as graphical, algebraic and representation in the real world context. In this paper, we focus on the preliminary results obtained from a questionnaire which were administered to thirty undergraduate students who are in their third years. First, students were asked the formal definition of derivative. Secondly, students were given the following representations and were asked to use the formal definition of derivative to answer the questions:

Algebraic representation: Students were given the function $f(x) = x^2 + 3$ and were asked to prove that the derivative at $x = 2$ is 4.

Graphical representation: The graph of $y = x^2$ and a tangent to the graph at the point $P = (2,4)$ were presented and the students were asked to find the slope of the tangent. They were also presented with a graph and were asked to find the graph of the derivative of the given function.

Real world context: Two problems were asked to students which require to apply the formal definition in the real world context (Stein and Barcellos, 1992). One of them was concerned with the velocity of a falling object at a particular time when the formula for the distance travelled was given. The other was concerned with finding the density of a nonhomogeneous string at a point when the formula for the mass of the string was given.

16.7% of the students gave the formal definition. The formal definition evoked the slope of a tangent to a curve for 30% of the students. In terms of using the formal definition for the representations described above, they were more successful with algebraic representation. 43% of them used the formal definition to find the derivative at a point. 47% of them calculated the derivative using the rules of differentiation instead of proving. Students were also successful in finding the slope of the tangent to the parabola. 60% used the formal definition. However, only 7% of them successfully drew the graph of the derivative function when the graph of the function was presented. For the representation in the real world context, most of the students had difficulties in using the formal definition. Differences in the interpretations of the formal definition of derivative for multiple representations will be discussed in detail during the presentation.

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