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Algebraic generalization strategies of number patterns used by pre-service elementary mathematics teachers

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Abstract

This study investigates algebraic generalization strategies of 147 pre-service elementary mathematics teachers. Pre-service teachers were given five open ended linear and non-linear (quadratic) pattern problems which require them to find the general term of number patterns. Pre-service teachers' strategies to generalize and their use of mathematical models were analyzed. The analysis of data indicated that pre-service teachers described number pattern rules in relation to differences between terms and used visual models without a purpose.

Keywords: Number patterns; mathematics teacher education, generalisation strategies.

1. Introduction

Research on generalization of patterns drew interest over the past decade. This interest emerges from the idea that the structure of mathematics can be observed by searching for patterns and relationships (Hargreaves, Threlfall, Frobisher & Shorrocks-Taylor, 1999). Learning about patterns is crucial for the transition from arithmetic to algebra since it requires making verbal and symbolic generalizations (English & Warren, 1998). The algebraic generalization of a pattern is also important because, according to Radford (2006), "it serves as a warrant to build expressions of elements of the sequence that remains beyond the perceptual field (p. 5)." Numerous studies have been conducted which examined students' strategies of finding rules of number patterns (e.g. Hargreaves et. al; Stacey, 1989). Pre-service teachers' understanding of number patterns has received less attention from the research community. In an attempt to contribute to this growing literature, this study investigates algebraic generalisation strategies of pre-service elementary mathematics teachers.

Lannin, Barker & Townsend's (2006) generalization strategy framework was used in our investigation. Lannin, Barker & Townsend (2006) assembled previous research on generalization strategies and developed 'generalization strategies framework'. Four strategies are defined in this framework, namely; explicit, whole-object, chunking and recursive which were explained below:

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- Explicit: A rule is constructed that allows for immediate calculation of any output value given a particular input value
- Whole-Object: Portion is used as a unit to construct a larger unit using multiples of the unit
- Chunking: Recursive pattern is built on by building a unit onto known values of the desired attribute
- Recursive: Relationship is described that occurs in the situation between consecutive values of the independent variable.

Exposing which strategies are used by pre-service teachers is important because these strategies may be the indicators of their lesson design about patterns. Thus, the present study aims to investigate pre-service mathematics teachers' strategies used in generalising problems and their use of mathematical models that they choose.

2. Method

147 pre-service mathematics teachers were given an essay including five open-ended linear and non-linear (quadratic) pattern problems which require them to find the general term of number patterns. The first three problems are linear pattern problems. The first one (ladder problem) was developed by Stacey (1989) and the other two problems were (triangle dot problem and shaded squares problem) developed by NCTM which were cited in Steele & Johanning (2004). The remaining two problems paid attention to pictorial representation of patterns and to connecting the strategy with the pictorial context.

Data were analyzed to explore pre-service mathematics teachers' responses to these problems, their use of mathematical models, strategies used in implementing them and the explanations they give. It was also analyzed how pre-service teachers' selection of strategies differed according to the difficulty level of the problems. 'Generalization strategies framework' (Lannin, Barker & Townsend, 2006) guided our analysis of strategies used by pre-service teachers. The responses were not analyzed regarding their accuracy, but rather the choices of strategies. The analysis of pre-service teachers' responses will be presented in frequency tables. The following research questions are of our interest:

- What generalization strategies do pre-service teachers use?
- How consistent are strategies used by pre-service teachers to generalize a rule?
- How do pre-service teachers utilize pictorial context to make generalizations?

3. Findings

In the following sections we present the generalization strategies that pre-service teachers used in solving linear and non-linear (quadratic) pattern problems. Five categorizations were used in our analysis. Four of them are the strategies in 'generalization strategies framework' which was described above. An additional category called "null" is used for non-responded problems and/or wrong answers without using any strategy.

3.1. Ladder problem

Ladder problem focuses on a linear relationship and is presented in Figure 1.

With 8 matches, I can make a ladder with 2 rungs like this

With 11 matches, I can make this ladder with 3 rungs.
How many matches are needed for a ladder with 1000 rungs?

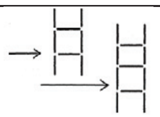


Figure 1. Ladder problem (Stacey, 1989)

Table 1 gives examples of answers from pre-service teachers for each strategy used, frequencies and percentages of the number of pre-service teachers using the strategies.

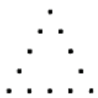
Table 1. Strategies used for ladder problem

Strategy	Illustrative examples	f	%
Explicit	There are three matches for each ladder so I took three times the number of matches, then I added two matches for the beginning of the ladder.	123	84
Whole-Object	There are three matches for one ladder, so 3000 matches are needed for 1000 rungs.	3	2
Chunking	There are three matches for each ladder so I took three times 999, then I add five which is the number of matches in first ladder.	11	7
Null	-	10	7

As can be seen above, “explicit” category is the most frequently used strategy among pre-service teachers. On the other hand, pre-service teachers did not use recursive strategy at all.

3.2. Triangle dot problem

Triangle dot problem which also has a linear pattern is given Figure 2 below.



In the diagram left is a 5-dot triangle. It is a triangle made by using 5 dots on each side. The 5-dot triangle is made using a total of 12 dots. How many dots will be used to make a 13-dot triangle? If n represents the number of dots on each side of an *n*-dot triangle, write an expression to represent how many total dots are in the triangle?

Figure 2. Triangle dot problem (NCTM, 1997 cited in Steele & Johanning, 2004)

Table 2 gives the illustrative examples of pre-service teachers for each strategy used in triangle dot problem and, frequencies and percentages for strategies.

Table 2. Strategies used for triangle dot problem

Strategy	Illustrative examples	f	%
Explicit	There are three dots for each triangle so I took three times the number of the dots, then I subtract three dots in the corners because they were counted more than once.	141	96
Null	-	6	4

The only strategy used for triangle dot problem is, explicit. When the percentage for explicit strategy is compared to the percentage of the same category for the ladder problem it can be seen that the number of pre-service teachers who preferred explicit strategy increased. This increase might have emanated from the fact that the triangle dot problem is less complicated compared to the ladder problem in which input and output values were shifted.

3.3. Shaded squares problem

Shaded squares problem contains a linear pattern and is presented in Figure 3.

In the diagram a 3 x 3 grid of squares is colored so that only outside squares are shaded. This leaves one square on the inside that is not shaded and 8 squares that are shaded. If you had a 25 x 25 grid of squares and only the outside edge of squares are shaded, how many squares would be shaded? If n represents the number of squares on a side and you have all the outside squares of an n x n grid shaded, write an expression representing the total number of shaded squares in the figure.

Figure 3. Shaded squares problem (NCTM, 1998 cited in Steele & Johanning, 2004)

Table 3 gives the illustrative examples of answers given by pre-service teachers for each strategy used in shaded squares problem and, frequencies and percentages of strategies.

Table 4. Strategies used for shaded squares problem

Strategy	Illustrative examples	f	%
Explicit	There are four squares increase for each figures thus I took four times the number of the dots. Then I subtract four squares in	134	91

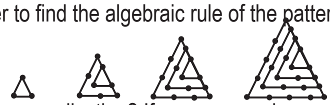
	the corners because they were counted more than one.		
Null	-	13	9

Similar to results for previous problems, explicit is the most preferred strategy among others. Whole-object, chunking and recursive strategies were not used at all in shaded squares problem. So far, it can be concluded that pre-service teachers preferred to use explicit strategy for linear pattern problems.

3.4. Problem 1 in pictorial context

Problem 1 given in pictorial context is a non-linear pattern problem as presented in Figure 4. Pre-service teachers were required to find the rule of the pattern and connect pictorial context with the rule. The rule of pattern can be considered less complicated for a quadratic pattern since the terms are one smaller than square numbers.

A student drew the following model in order to find the algebraic rule of the pattern 3,8,15, 24 …



Can this model help the student to make a generalization? If your answer is yes explain how it can help the student to make a generalization. If your answer is no, create a new pictorial model which can help the student.

Figure 4. Problem 1 in pictorial context

Table 4 gives the sample answers of pre-service teachers for each strategy used in problem 1 in pictorial context and, frequencies and percentages for strategies.

Table 4. Strategies used for problem 1 pictorial context

Strategy	Illustrative examples	f	%
Explicit	If I add 1 to each term, I will find square numbers. First term begins with 3, thus the general term is $(n+1)^2-1$.	63	43
Recursive	$a_1=3$; $a_2=a_1+(2.2+1)$; $a_3=a_2+(2.3+1)$; $a_4=a_3+(2.3+1)\ldots$ $a_n=a_{n-1}+(2n-1)$	44	30
Null	-	40	27

Pre-service teachers' strategy selections were dramatically changed in this non-linear problem. Recursive strategy was rarely used in linear problems, on the other hand, thirty percent of pre-service teachers used this strategy to solve problem 1 in pictorial context. Whole-object and chunking strategies still were not used.

3.5. Problem 2 in pictorial context

Problem 2 in pictorial context requires to find the rule of the pattern 5, 9, 17, 29 … and to model the pattern. The pattern given in this problem is more complicated than problem 1 in pictorial context. Table 5 gives the illustrative examples of each strategy used by pre-service teachers and, frequencies and percentages of strategies.

Table 5. Strategies used for problem 2 in pictorial context

Strategy	Illustrative examples	f	%
Explicit	$a_1=5$; $a_2=5+4.1$; $a_3=5+4.1+4.2$; $a_4=5+4.1+4.2+4.3\ldots$ $a_n=5+4[(1+2+3+\ldots+(n+1)]$; It equals to $5+2(n^2-n)$.	35	24
Recursive	First term is 5.If I add first term to 4. (2-1), I found the second term. Thus; $a_1=5$; $a_2=a_1+4(2-1)$; $a_3=a_2+4(3-1)$; $a_4=a_3+4(3-1)\ldots$ $a_n=a_{n-1}+4(n-1)$	66	45
Null	-	40	31

From the increase of the null responses it can be claimed that this was a challenging problem for pre-service teachers. In addition to that, when the percentage for explicit strategy is compared to the same strategy used in problem 1 in pictorial content, it can be seen that that percentages of recursive strategy considerably increased.

Pre-service teachers' pictorial representations were also analyzed. The analysis indicated that neither of them was able to construct a model which helps them to find the rule of the given patterns. Some of the given responses were presented in Figure 5 below:

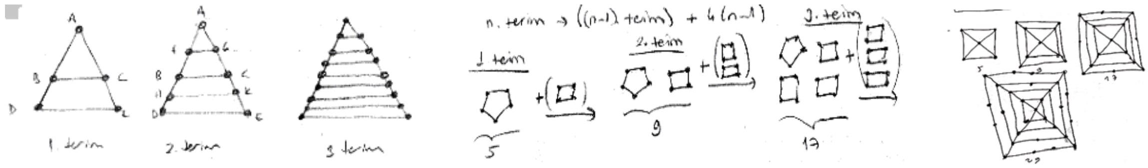


Figure 5. Pre-service teachers' representations of patterns in problem 2 in pictorial context

As can be seen from Figure 5, pre-service teachers modeled the patterns without a purpose. They just drew some shapes which have as same number of corners (or dots, figure etc.) as terms of the pattern. Even the pre-service teachers who could find the rule correctly failed to model the pattern.

4. Discussion and Conclusion

The analysis of data indicated that pre-service mathematics teachers preferred explicit strategy utmost. However they described the rules in relation to differences between terms and used recursive strategy when the relationship between terms of the pattern was non-linear (quadratic). Using recursive strategy allows finding the following term but prevents to see the general structure of the pattern. Although finding the following term of a pattern indicates some degree of generalization, it falls short of an explicit generalization expressed in mathematical forms (Carraher et al, 2008). It was also found that pre-service mathematics teachers used visual models as an accessory not in the way that facilitates finding the general term of the number pattern. Orton, Orton & Roper (1999) reported same results obtained from an exam in University of Leeds. They stated that adults in teacher preparing program were given some pictorial patterns which have quadratic rule and were asked to predict the number of dots for the *n*th picture of sequences. Results showed that recursive strategy is used commonly, and they did not use pictures as guidance in finding *n*th term. Yeşildere&Akkoç (2009) investigated the development of two pre-service mathematic teachers' pedagogical content knowledge with special attention to finding the rule of number patterns and reached similar findings in their research. They found that participants did not use visual models of the patterns in an effective way; to discover the rule of the pattern. Pre-service teachers should be provided with content knowledge of patterns, both on the strategies that can be used and the pictorial representations that helps to generalize.

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