DEVELOPMENT OF PRESERVICE TEACHERS IN LINKING MULTIPLE REPRESENTATIONS VIA TECHNOLOGY

M. Fatih Özmantar, Gaziantep University, Turkey
Hatice Akkoç, Marmara University, Turkey
Erhan Bingölbali, Servet Demir, Gaziantep University, Turkey

In this paper, we examine the development of pre-service mathematics teachers' use of multiple representations during teaching in technology-rich environments. The pre-service teachers took part in a preparation program aimed at integration of technology into teaching mathematics. The pre-service teachers' development was scrutinized in terms of their knowledge of and the connections established among the representations. We discuss the educational implications of the study for the design and conduct of successful technology integration programs.

INTRODUCTION

The issue of multiple representations (MRs) are long on the agenda of mathematics education. The MRs became the centre of attention especially after the NCTM standards in 1989. The standards strongly suggested teaching mathematics with different representations so that students would be given different lenses to approach mathematical problems flexibly. Since then, many research attempts were undertaken to investigate the benefit and importance of MRs in teaching and learning mathematics. The researchers repeatedly pointed out that MRs lead to better learning outcomes as they cater for a wider range of students with different ability levels and learning styles (Ainsworth, 1999). Kaput (1989) argues that use of MRs enables one to 'see' complex ideas in a new way and apply them more effectively. Also as different representations stress different aspects of the same concepts, teaching with MRs is found to be useful in making connections (Berthold et al., 2009).

With digital technologies beginning to spread, MRs received a renewed interest. In 2001, for example, the Council's yearbook focused on the roles of representation in school mathematics (Cuoco, 2001). The yearbook attaches considerable importance to the use of digital technologies in establishing links among representations. As NCTM suggests, digital technologies provide visual models or representations that many students are unable to generate through their independent efforts.

Research studies, implicitly or explicitly, regard teachers as an important factor in reaching at the expected benefits from the use of MRs in technology-rich environments (e.g., Alagic & Palenz, 2006). Therefore what teachers know about and how they use MRs as well as how they draw on technology to interrelate MRs become important issues while determining the effect of teaching mathematics with MRs through technology. Despite its importance, there appear a few studies on how (pre-service) teachers employ MRs in technology-rich environments (Juersvich et al., 2009; Alagic & Palenz, 2006). These studies, however, do not give much details as to

2011. In B.Ubuz (Ed.). *Proceedings of the 35th Conference of the International Group for the Psychology of Mathematics Education*, Vol. 3, pp. 313-320. Ankara, Turkey: PME.

how pre-service mathematics teachers (PSMTs) use MRs and how their competence to effectively make use of MRs in technology-rich environments can be developed. In this connection, we will share, here in this paper, some of the results of our research project which focused on technology integration process designed for PSMTs. We will report our findings regarding PSMTs' development with regard to use of MRs in technologically rich environments and interconnecting them for a deeper understanding. Before doing this, however, we first describe the project and background of the study in the next section.

BACKGROUND OF THE STUDY AND METHODOLOGY

This paper stems from a research project for which a course was designed for PSMTs to develop technological pedagogical content knowledge (TPCK) (Mishra & Koehler, 2006). TPCK framework has been recently used to investigate the characteristics of knowledge required by teachers for successful technology integration. It was originated from the notion of "Pedagogical Content Knowledge (PCK)" offered by Shulman (1986). As the importance and potential of technology in teaching and learning is realized, Pierson (2001) has included the technology component into the idea of PCK and considered TPCK as a blend of three categories of knowledge: content, pedagogy and technology. We use TPCK framework with its various components to develop contents for two courses (Methods for Teaching Mathematics II and Technology-Aided Mathematics Teaching) as part of a project for PSMTs in Turkey. The aims of these courses were, broadly speaking, to get PSMTs equipped with the skills of teaching mathematics with the aid of technology at secondary level.

This study focuses on a particular component of TPCK, namely knowledge of using MRs of a particular topic with technology. We brought the content aspect into play and aimed to exemplify how this component can be applied with a particular mathematical concept for which we used the concept of derivative. Two workshops were designed: the first aims to develop pedagogical content knowledge (PCK), and the second to develop technological content knowledge (TCK), and (TPCK).

During the PCK workshop, PSMTs were asked to share their existing knowledge of MRs and provided with the knowledge of algebraic, numerical and graphical representations of derivative, the relationships among them, and how to take them into account in teaching. Limitations and affordances of each representation were also discussed. TPCK workshop firstly focused on TCK. Technological content introduced to pre-service teachers is a Turkish version of Graphic Calculus software. The software and an activity book in Turkish (Akkoç, 2006) were given to each PSMTs. Graphic Calculus software provides graphical and numerical representations of derivative at a point which are dynamically linked. During the TPCK workshops, MRs of derivative at a point was introduced. A discussion among PSMT's about the software focused on the following questions:

- What kinds of representations of derivative at a point are available and what kinds of opportunities are there to make links between representations?

- What kinds of opportunities are there to relate different aspects of derivative (derivative as rate of change, derivative as slope and derivative as a limit) using the software?

Forty PSMTs in Istanbul, Turkey, took part in the courses. Completion of the course along with some others would award them a certificate for teaching mathematics in high schools for students aged between 15 and 19. All forty PSMTs enrolled this course which was designed on the basis of TPCK framework as explained hitherto.

During the research, several data collection tools were employed, including: lesson plans, detailed teaching notes and questionnaires. Participants were given open-ended questionnaires before and after the courses to find out their initial understandings of several issues including MRs. The participants were also asked to prepare three lesson plans: one before the PCK workshops, one after the PCK workshops and the last was after the TCPK workshops. All three lesson plans were on the same topic: introduction of derivative. To prepare the plans, the PSMTs were allowed to make use of any textbook they wish and also required to examine the curriculum scripts.

DATA ANALYSIS AND FINDINGS

The concept of derivative at a point can be represented in algebraic, graphical and tabular (numerical) forms. Derivative at a point has three different aspects: (i) the slope of the tangent line to a curve at a particular point, (ii) the limit of the difference quotient and (iii) the instantaneous rate of change (Bingolbali, 2008). All these three are closely interrelated and an understanding of any one of these supports the comprehension of the others. As Ainsworth's (1999) study suggests, constructing a deeper understanding involves interconnecting the MRs of derivative with reference to different aspect of the concept. Hence in our analyses of PSMTs lesson plans, we focused on the MRs of derivative in tandem with its different aspects.

Before the TPCK program started, we wished to find out participant PSMTs' initial ideas on MRs in general. To this end, we collected data via questionnaire where we asked the participants to explain their understanding of MRs. The questionnaire was applied twice: before PCK and after TPCK workshops. Participants' responses involved graphical, tabular and algebraic examples in explaining the meaning of MRs. Before the workshops started there were only 4 (10%) responses within these three categories. The number of PSMTs with regard to these three representations increased dramatically to 36 (90%) after the completion of workshops. Interesting was to see that eight PSMTs considered different symbols used for mathematical expressions as representations of the concepts. After the workshops, however, this misconception disappeared. Before the program, 15 (37.5%) PSMTs left this question unanswered but this figure dropped to none after the workshops. Generally speaking, PSMTs' grasp of the MRs has improved over the course with clear terms.

We now turn our attention to the PSMTs' lesson plans along which participants prepared rather detailed teaching notes. Three sets of lesson plans with teaching notes were analyzed: (1) before the workshops started, (2) after PCK workshops and (3) following TPCK workshops. In analyzing the plans, we aimed to determine whether PSMTs made connections among three MRs (graphical, numerical and algebraic) of derivative and whether they employed technology for that purpose (see Table-1).

	First plans	Second plans	Third plans
Categories	N (%)	N (%)	N (%)
MRs of derivative (Graphical (G), Numerical (N) and Algebraic (A)) are not linked.	34 (85.0)	9 (22.5)	1 (2.5)
Only one pair of representations are linked (any one of G-N, G-A, or N-A)	3 (7.5)	5 (12.5)	5 (12.5)
Only two pairs of representations are linked (any two of G-N, G-A, and N-A)	1 (2.5)	8 (20.0)	1 (2.5)
Three pairs of representations are linked (pairs of G-N, G-A, and N-A are all present)	0	2 (5.0)	2 (5.0)
All three representations are interconnected to one another (G-N, G-A, & N-A and G-N-A are present)	0	12 (30.0)	27 (67.5)
No response	2 (5.0)	4 (10.0)	4 (10.0)
Total (N=40)	40	40	40

Table-1. Frequency analysis of the links among different representations

It is worth noting that 85% of initial lesson plans did not link MRs of derivative. However, this figure drops to 22.5% after PCK workshops and to a remarkable 2.5% following the TPCK workshops. There is also an outstanding increase in the number of plans which links all three MRs of derivative. There was not a single plan that linked all three MRs of derivative at the beginning; yet this figure increases to 30% and to 67.5% after, respectively, PCK and TPCK workshops. Please note that all the links among the MRs of derivative in the third lesson plans (after TPCK workshops) were established through technology (Graphical Calculus software).

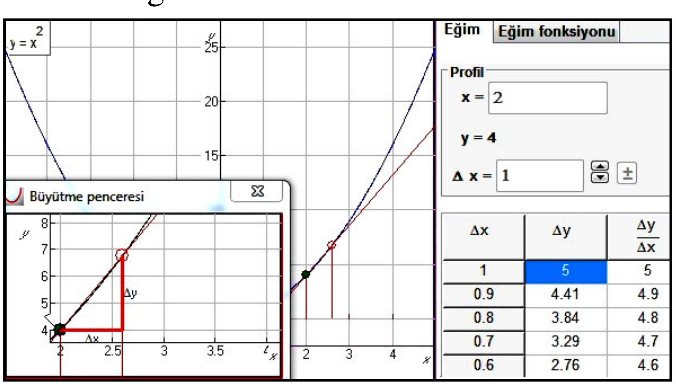

Figure 1. Screen-shot of Graphic Calculus

Before presenting our analysis of PSMTs in relating three aspects of derivative using MRs, let us provide an example from one particular PSMT's third lesson plan in detail. Figure 1 presents a screen-shot using Graphic Calculus software taken from the PSMT's lesson plan. As can be seen from the figure, the software provides numerical and graphical representations of derivative at a point and dynamic links between them. As the software calculates the average rates of change, it draws the secant lines spontaneously. The PSMT used this activity in her third lesson plan and explained what she aimed with it in the open-ended questionnaire she filled. She mentioned that the numerical values of rates of change could be represented graphically by the secant lines through pairs of points. She also mentioned that using the zoom-in tool a right triangle could be constructed on which the rate of change represents the slope of the secant line. She later explained that values for rates of change in the table approach to a number and this promotes an intuitive understanding of limit She added that secant lines approach to the tangent line and rates of change approach to instantaneous rate of change. This way she related three different aspects of derivative using MRs.

We analyzed forty PSMTs' lesson plans with detailed teaching notes in a similar way (just as described above) to see if they related different aspects through MRs of derivative (se Table-2).

	First plans	**Second plans**	**Third plans**
Categories	N (%)	N (%)	N (%)
None of the aspects of the derivative is explicitly addressed.	28 (70.0)	6 (15.0)	0
Only one aspect addressed: any one of Derivative-Rate of Change (D-RoC), Derivative-Limit (D-L) or Derivative-Slope (D-S) relationship focused.	8 (20.0)	6 (15.0)	2 (5.0)
Two aspects are addressed: any two of D-RoC, D-L, D-S emphasized.	1 (2.5)	6 (15.0)	2 (5.0)
All three aspects are addressed but not interconnected	0	2 (5.0)	2 (5.0)
All three aspects are both addressed and interconnected	0	19 (47.5)	30 (75.0)
Unanswered	3 (7.5)	1 (2.5)	4 (10.0)
Total (N=40)	40	40	40

Table-2. Frequency analysis of the aspects of derivative addressed in the plans

As seen in Table-2, 70% of the PSMTs did not address any aspect of derivate in their first lesson plans. However, this figure decreased to 15% and to none following the completion of respectively PCK and TPCK workshops. A notable difference is an increase in the number of those who both addressed and interconnected different aspects via MRs of derivative. This figure, increased from zero to 47.5% after PCK and to an outstanding 75% after TPCK workshops. No less important is the fact that all the PSMTs employed technology for their lesson plans and devised ways to employ it while interrelating different aspects through MRs of derivative.

DISCUSSION

PSMTs participating in our research has shown great improvements not only in their knowledge of MRs but also in establishing connections among MRs and doing this with the help of technology. Participants' responses before the workshops provide evidence that they knew little about the issue of MRs and in fact many could not explain the meaning of MRs or even some considered mathematical symbols as representations. However, following the TPCK workshops, their responses convincingly suggest that they developed insights into the issue of MRs. The way that MRs were employed in the first lesson plans suggested to us that PSMTs could not appreciate the importance of MRs in constructing deeper understanding of derivative as they were not able to use representations in a connected manner, at least, in the context of teaching derivative. Having completed the TPCK workshops, PSMTs were not only able to explain the meaning of MRs but also to started to use MRs by interconnecting them with regard to teaching derivative. They also showed great developments in making use of technology while establishing interconnections among the MRs and relating MRs to different aspects of derivative.

The question of interest here is: why is the professed development on the part of PSMTs regarding the use of MRs in technology rich environments important? Our answer is "for at least two reasons". First, as Moreno & Mayer (1999) point out, establishment of the links among the MRs of mathematical concepts present exceptional potentials for learners to achieve deeper understandings and this is cited as one of the most important reasons to justify the use of MRs in mathematics instruction. However, several studies (such as that of Juersvich et al., 2009) suggest that the links among the MRs are not often established by teachers during the instruction. Hence this heavy burden is left to the shoulders of the learners. In fact, our analysis of the first lesson plans support this argument that 85% of our participants did not make connections among MRs in their lesson plans. Unless these connections are not established, the "expected" deeper understanding is not readily available due to the fact that learners often focus on one type of representations or fail to connect them (Berthold et al., 2009). Hence if the links among the MRs is not explicitly considered during the instruction, then use of MRs does not necessarily result in a better understanding. Considering that 67.5% of the lesson plans prepared after TPCK workshops explicitly attended to the connections among the MRs of derivative, PSMTs in our project seemed to have grasped the significance of making the links among the MRs an explicit focus of their instruction and also were able to make use of the potential that technology offers.

Secondly, the observed development of our participants with regard to the use of MRs through technology underlines certain features for the design of successful technology integration programs: the content, method of delivery of the content and hands-on activities (see also Hew & Brush, 2006). The content of PCK and TPCK workshops included the issue of MRs. During the workshops, particular attention was paid to the examples of MRs in different topics of mathematics (limit and functions etc.), functions of MRs, affordances and limitations of particular representations, significance of the connections among MRs, representational power of the specific technology employed during the workshops (i.e. Graphic Calculus), and the strengths and weaknesses of this particular technology regarding MRs of derivative.

In delivering the content, we employed TPCK framework which shaped our method to deliver the content during the workshops. We initially focused on the definition, the value of employing MRs for teaching and provided particular examples of MRs from different topics (Pedagogical Knowledge). Later, we attended to the ways for the utilization of MRs, their functions, the necessity and benefit of interconnecting representations in the context of derivative (Pedagogical Content Knowledge). Then technology dimension was brought into attention for a discussion of affordances and limitations that the particular software, Graphic Calculus, has for the MRs of derivative (Technological Content Knowledge). Afterwards, we brought into play the particular topic under consideration (derivative) and encouraged our participants come up with ways as to how to combine MRs of derivative with the help of technology, how to explicate the aspects of derivative via MRs, how technology can make explicit the links among the MRs and aspects of derivative (Technological Pedagogical Content Knowledge). Our participants' development suggest that this way of delivering the content was, at least in our case, effective.

While delivering the content, an extra care was taken to ensure that our participants find opportunities to get involved into hands-on activities. We aimed to provide PSMTs with a "space" where they can explore their own ideas, in their own ways with the technological tools available to them. Therefore, the PSMTs were guided by thought provoking questions but they were given responsibilities to discover in group-work sessions. We believe that PSMTs should be given chances to discover alternatives by themselves. Only then can they gain insights and develop competencies for an effective integration process of technology. Considering all these we argue that technology integration programs could be successful when they have at least these three as features in the design and conduct of the courses.

We end our discussion with a final implication of our findings. Having witnessed the PSMTs development and their detailed lesson plans, we are convinced that an effective teaching depends very much on the use of MRs regardless of the teaching medium. Hence we think that MRs and interconnections among them should be one of the foci of technology integration programs. (Pre-service) teachers should be given a chance to realize the important roles that MRs play for a robust understanding and the potential that technology offers interconnecting different representations.

Acknowledgement

This study is part of a project (project number 107K531) funded by TUBITAK.

REFERENCES

Ainsworth, S.E. (1999). A functional taxonomy of multiple representations, *Computers and Education*, 33(2/3), 131-152.

Akkoç, H. (2006) *Bilgisayar Destekli Matematik Öğretimi: Grafik Analiz Yaklaşımı: İlköğretim İkinci Kademe ve Liseler İçin*. Toroslu Kitaplığı: İstanbul.

Alagic, M. & Palenz, D. (2006). Teachers Explore Linear and Exponential Growth: Spreadsheets as Cognitive Tools, *Journal of Technology and Teacher Education*, 14 (3), 633-649.

Berthold, K., Eysink, T.H.S., & Renkl, A. (2009). Assisting self-explanation prompts are more effective than open prompts when learning with multiple representations. *Instructional Science*, 37: 345-363.

Bingölbali, E. (2008). Türev kavramına ilişkin öğrenme zorlukları ve kavramsal anlama için öneriler. Özmantar, M.F., Bingölbali, E., Akkoç, H. (Eds.) *Matematiksel Kavram Yanılgıları ve Çözüm Önerileri* (pp. 223-255), Pegem Akademi, Ankara.

Cuoco, A. A. (2001) *Yearbook of the National Council of the Teachers of Mathematics: The Roles of Representation in School Mathematics* (pp.173-185). Reston: The Council.

Hew, K.F. & Brush, T. (2007) Integrating technology into K-12 teaching and learning: current knowledge gaps and recommendations for future research. *Educational Technology Research and Development*, 55:223–252.

Juersivich, N., Garofalo, J. & Fraser, V. (2009). Student Teachers' Use of Technology-Generated Representations: Exemplars and Rationales, *Journal of Technology and Teacher Education*, 17, 2, 149-173.

Kaput, J. J. (1989). Linking representations in the symbol systems of algebra. In Wagner, & Kieran (Eds.), *Research Issues in the Learning and Teaching of Algebra* (pp. 167-194).

Mishra, P. & Koehler, M.J. (2006). Technological pedagogical content knowledge: A framework for teacher knowledge. *Teachers College Record*, 108(6), 1017–1054.

Moreno, R., & Mayer, R. E. (1999). Multimedia-supported metaphors for meaning making in mathematics. *Cognition and Instruction*, 17, 215–248.

National Council of Teachers of Mathematics (1989). *Curriculum and Evaluation Standards for School Mathematics.* Reston, VA.: The Council.

Pierson, M. E. (2001). Technology integration practice as a function of pedagogical expertise. *Journal of Research on Computing in Education*, 33(4), 413-429.

Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher*, 15, 4–14.