Oxygen Profile in Streams

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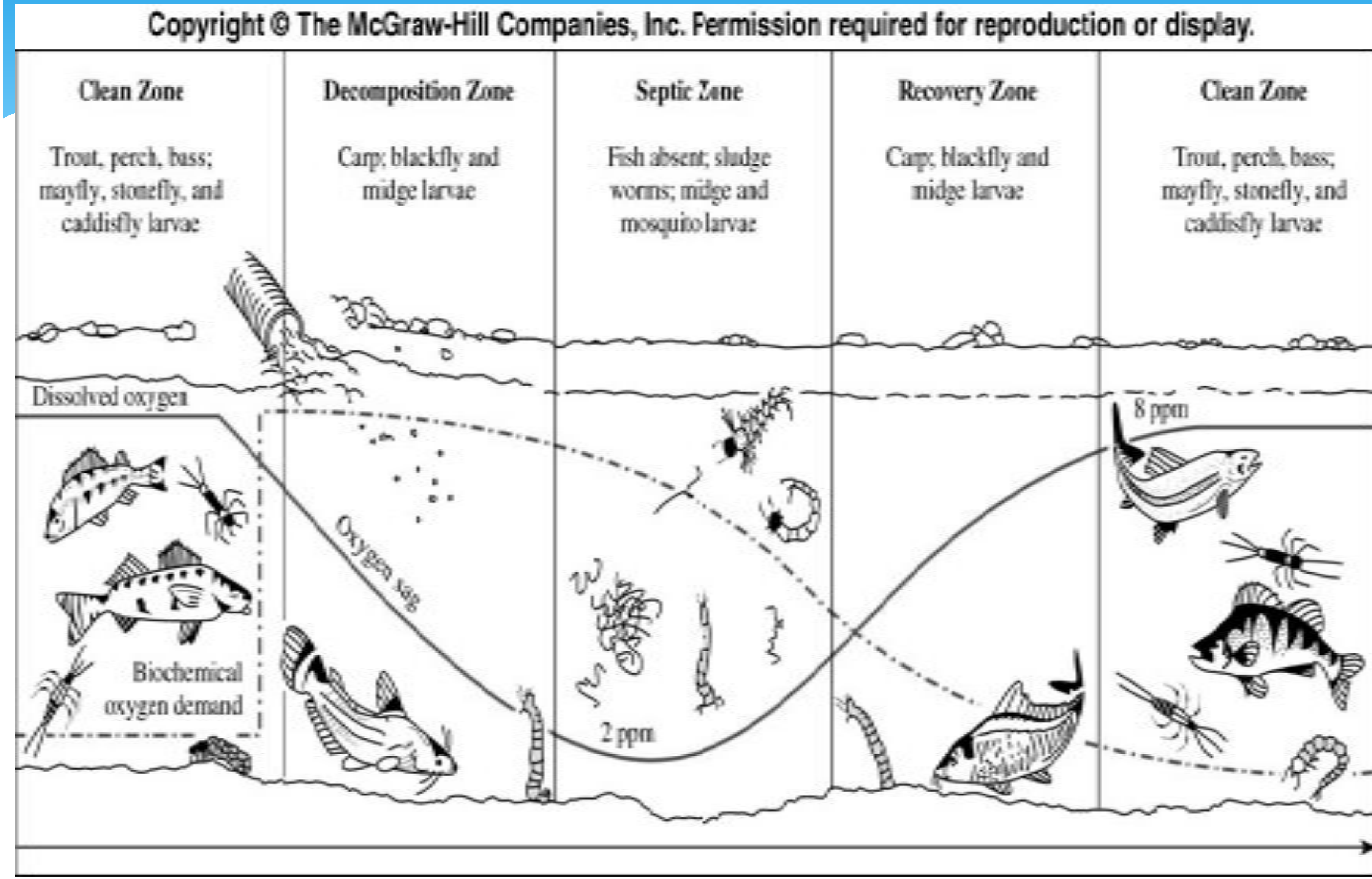
Oxygen depletion in streams



Oxygen level in surface waters

* Two mechanisms are known to contribute oxygen to surface waters:
  * dissolution of oxygen from the atmosphere (reaearation)
  * production of oxygen by algal photosynthesis

Oxygen Deficit

* *Oxygen deficit is represented mathematically by*

$$D = c_s - c$$

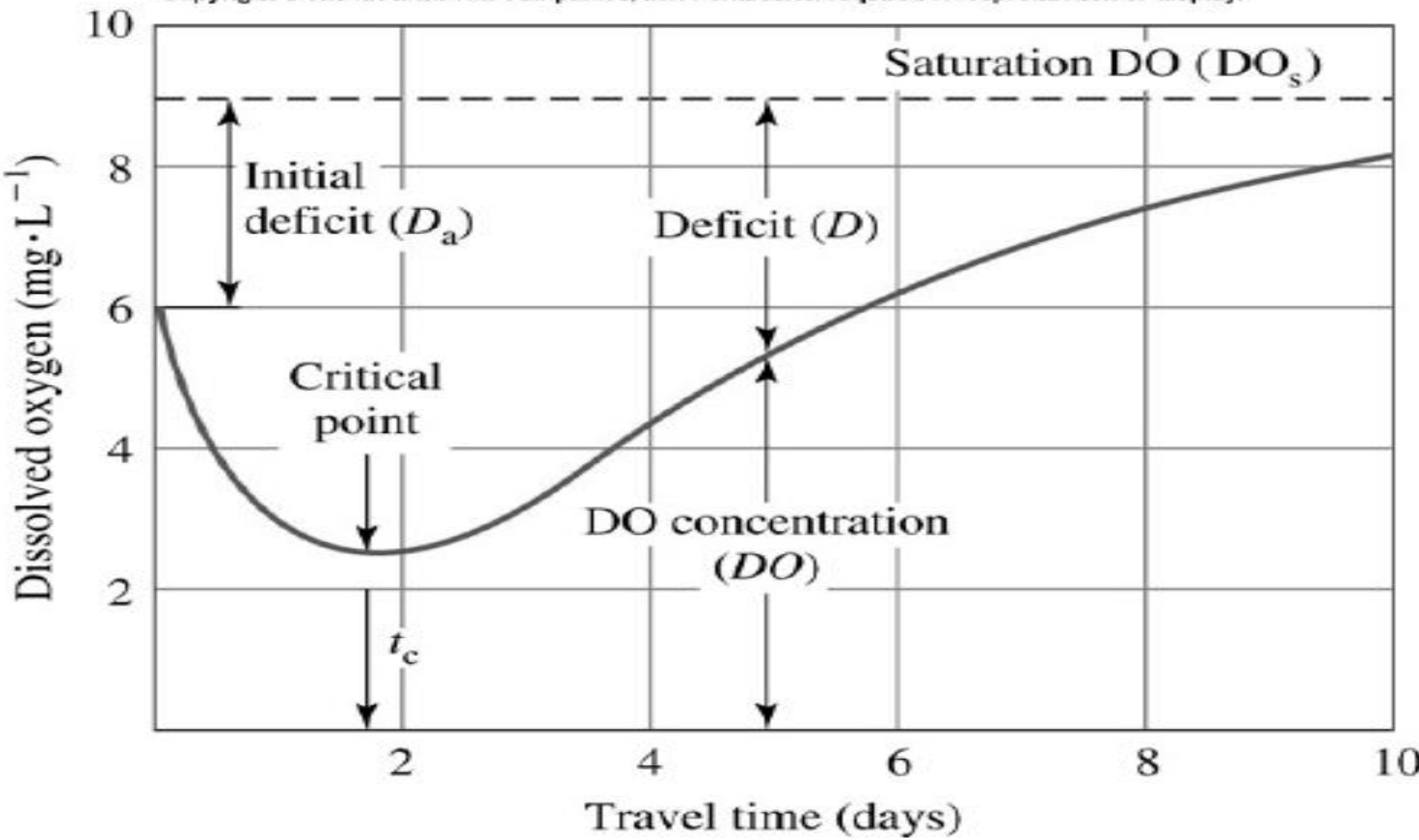
* *For constant equilibrium conditions, i.e. C_s does not change, the rate of change in the deficit:*

$$\frac{dD}{dt} = -\frac{dc}{dt}$$

The deficit increases at the same rate that oxygen is used

* *The dissolved oxygen deficit is the main driving force for reaeration*

DO sag definitions



Streeter-Phelps Model*

Mass Balance for the Model

Not a Steady-state situation

rate O_2 accum. = rate O_2 in – rate O_2 out + produced – consumed

rate O_2 accum. = rate O_2 in – 0 + 0 – rate O_2 consumed

Kinetics

Both reoxygenation and deoxygenation are 1st order

* Streeter, H.W. and Phelps, *E.B. Bulletin* #146, USPHS (1925)

Sag Curve

* The oxygen deficit in a stream is a function of
  * Oxygen utilization
  * Reaeration

$$\frac{dD}{dt} = r_D + r_R$$

  * r_D=Rate of BOD exertion
  * r_R=Rate of reaeration

Kinetics for Streeter-Phelps Model

- **Deoxygenation**

L = BOD remaining at any time

dL/dt = Rate of deoxygenation equivalent to rate of BOD removal

dL/dt = $-k_1L$ for a first order reaction

k_1 = deoxygenation constant, f'n of waste type and temp.

$$-\frac{d[L]}{dt} = kL$$

$$\int_{C_0}^{C} \frac{dL}{L} = -k\int_0^t dt$$

$$\ln\frac{L}{L_0} = -kt \quad or \quad \frac{L}{L_0} = e^{-kt} \; -> \; L = L_0\, e^{-kt}$$

Developing the Streeter-Phelps

Rate of reoxygenation = k_2D

D = deficit in D.O.

k_2 = reoxygenation constant*

$$k_2 = \frac{3.9 v^{1/2} \left([1.025]^{(T-20)} \right)^{1/2}}{H^{3/2}}$$

Where

- T = temperature of water, ºC
- H = average depth of flow, m
- ν = mean stream velocity, m/s

D.O. deficit

= saturation D.O. – D.O. in the water

There are many correlations for this. The simplest one, used here, is from O'Connor and Dobbins, 1958

* or

Table 3-2 Reaeration constants	
Water body	Ranges of k_R at 20°C, base e
Small ponds and backwaters	0.1-0:23
Sluggish streams and large lakes	0.23-0.35
Large streams of low velocity	0.35-0.46
Large streams of normal velocity	0.46-0.69
Swift streams	0.69-1.15
Rapids and waterfalls	Greater than 1.15
Source: Peavy, Rowe and Tchobanoglous, 1985	

Combining the kinetics

Net rate of change of oxygen deficiency, dD/dt

$$dD/dt = k_1L - k_2D$$

where $L = L_0e^{-k_1t}$

OR

$$dD/dt = k_1L_0e^{-k_1t} - k_2D$$

Integration and substitution

The last differential equation can be integrated to:

$$D = \frac{k_1 L_o}{k_2 - k_1}\left(e^{-k_1 t} - e^{-k_2 t}\right) + D_o e^{-k_2 t}$$

It can be observed that the minimum value, D_c is achieved when dD/dt = 0:

$$\frac{dD}{dt} = k_1 L_o e^{-k_1 t} - k_2 D = 0$$

$$D_c = \frac{k_1}{k_2} L_o e^{-k_1 t}$$, since D is then D_c

Substituting this last equation in the first, when D = D_c and solving for t = t_c:

$$t_c = \frac{1}{k_2 - k_1} \ln\left\{\frac{k_2}{k_1}\left[1 - \frac{D_o(k_2 - k_1)}{k_1 L_o}\right]\right\}$$