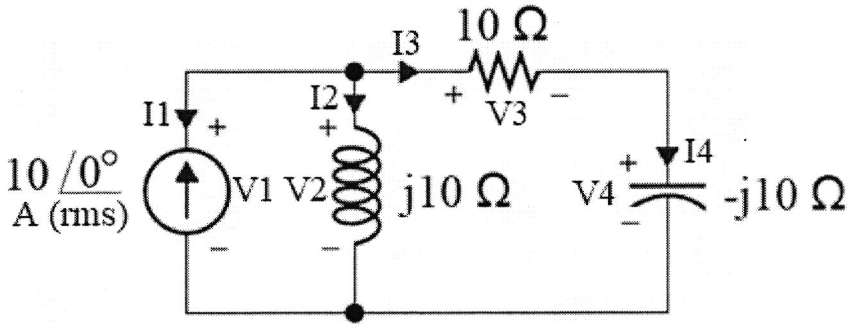
Student Name & Number:________________________________

EE2001 Final Exam Jan. 14, 2020

(24 pts.) **1)** For each element of the circuit below, find the rms phasor voltages (V) and currents (I), together with the average powers *absorbed* (P) and the magnetizing reactive powers *absorbed* (Q). Note that the current and voltage reference directions are indicated for each element. Please fill out the table provided.

Answer 1:



$I_1 = -10$ A

$I_2 = \frac{10 - j10}{j10 + (10 - j10)} \cdot 10 = 10 - j10$ A

$I_3 = I_4 = \frac{j10}{j10 + (10 - j10)} \cdot 10 = j10$ A

$V_1 = V_2 = I_2 \cdot j10 = 100 + j100$ V

$V_3 = I_3 \cdot 10 = j100$ V

$V_4 = I_4 \cdot (-j10) = 100$ V

$S_1 = V_1 \cdot I_1^* = -1000 - j1000$ VA

$S_2 = V_2 \cdot I_2^* = j2000$ VA

$S_3 = V_3 \cdot I_3^* = 1000$ VA

$S_4 = V_4 \cdot I_4^* = -j1000$ VA

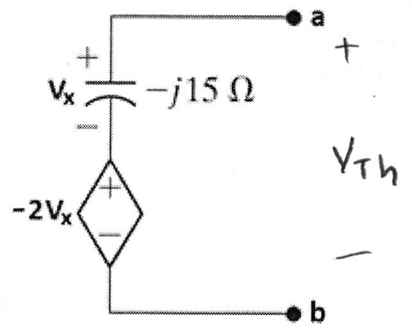
Note that, ΣP and ΣQ are zero for absorbed powers.

	(1) Current Source	(2) Inductor	(3) Resistor	(4) Capacitor
V (Volts rms)	$100 \cdot \sqrt{2} \angle 45°$	$100 \cdot \sqrt{2} \angle 45°$	$100 \angle 90°$	$100 \angle 0°$
I (Ampers rms)	$10 \angle 180°$	$10 \cdot \sqrt{2} \angle -45°$	$10 \angle 90°$	$10 \angle 90°$
P (Watts)	−1000	0	1000	0
Q (VAR)	−1000	2000	0	−1000

Note: Total time allowed is 100 min. Please show all your work and write legibly.

(20 pts.) **2)** Find Thevenin equivalent circuit with respect to the terminals a,b for the circuit shown.

Hint: If V_{TH} value is found to be zero, the circuit is equivalent to a pure impedance. In this case, the equivalent impedance value can be found by applying a voltage source to the terminals, and calculating the current supplied.



Answer 2:

Open Circuit:

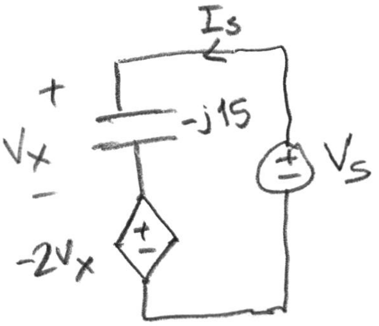
$V_{Th} = V_x - 2V_x = -V_x$

Since the current is zero,

$V_x = (-j15)\cdot 0 = 0$

$\Rightarrow V_{Th} = 0$ (internal voltage is zero)

Applying a voltage source:

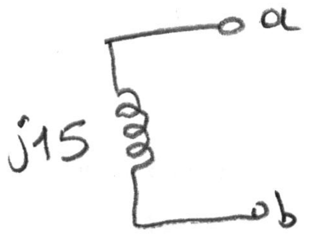


$V_s = V_x - 2V_x = -V_x$

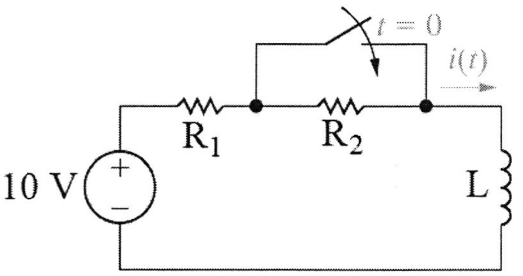
$V_x = (-j15)\cdot I_s$

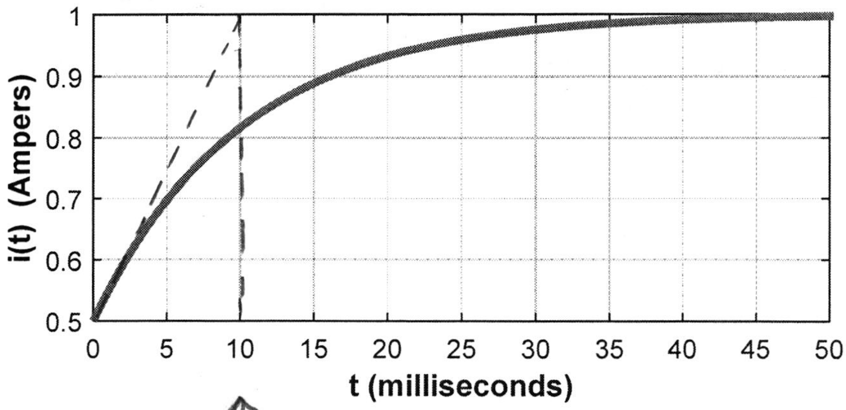
$\Rightarrow V_s = \underbrace{(j15)}_{Z_{Th}} \cdot I_s$

Therefore Thevenin equivalent is:



(21 pts.) **3)** The switch in the circuit shown has been open for a long time. At time t=0 the switch is closed. The resulting current $i(t)$ is depicted below. Estimate the resistor and inductor values (within ±5% error).





Answer 3:

Time constant $\tau = 10\,ms$

$t<0 \Rightarrow i(0) = \dfrac{10}{R_1+R_2} = 0.5\ A$

$t \to \infty \Rightarrow i(\infty) = \dfrac{10}{R_1} = 1\ A$

$t>0 \Rightarrow \tau = \dfrac{L}{R_1} = 10\ ms$

$\Rightarrow R_1 = 10\,\Omega$

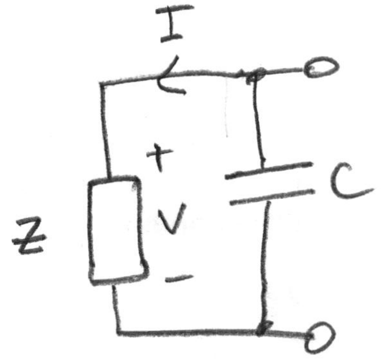
$R_2 = 10\,\Omega$

$L = 100\ mH$

(20 pts.) 4) Consider an inductive load having the voltage and the current as described below:

$$v(t) = V_m \cos(\omega t), \quad \text{and,} \quad i(t) = I_m \cos(\omega t - \theta).$$

Find the capacitor value C (in terms of V_m, I_m, ω, θ), such that, when connected in parallel with the load, will make the load look purely resistive.



$V = V_m \angle 0^\circ$

$I = I_m \angle -\theta$

Impedance: $\Rightarrow Z = \frac{V}{I} = \frac{V_m}{I_m} \angle \theta$

Admittance: $Y = \frac{I_m}{V_m} \angle -\theta = \frac{I_m}{V_m} \cdot \cos(\theta) - j\frac{I_m}{V_m} \sin(\theta)$

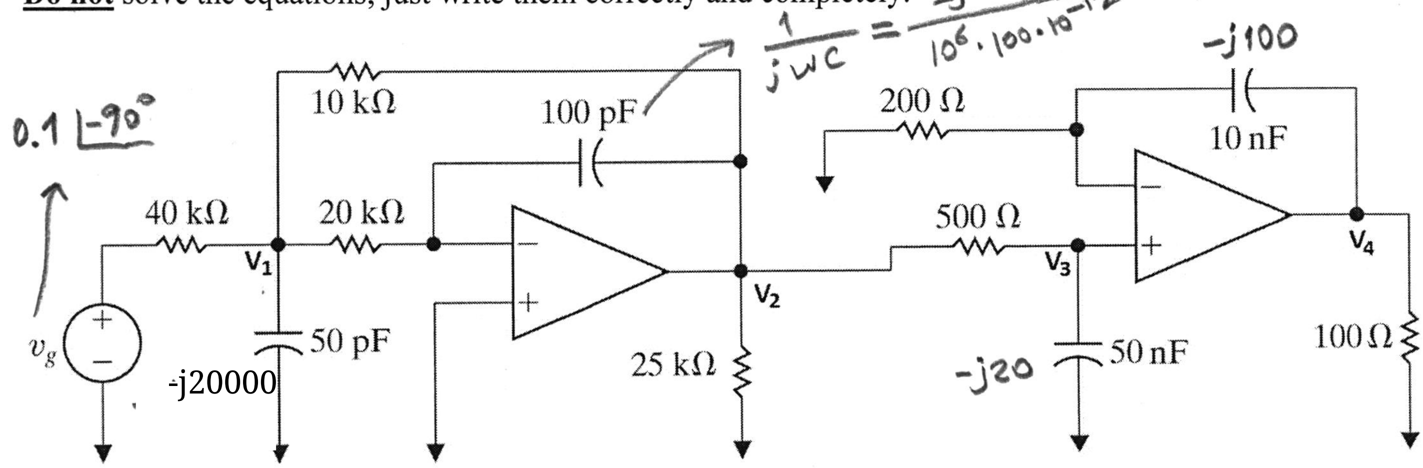
Admittance of C: $Y_c = jwC$

$Y + Y_c$ (in parallel) must be a real number.

$\Rightarrow wC = \frac{I_m}{V_m} \cdot \sin(\theta)$

$\Rightarrow \boxed{C = \frac{I_m}{w V_m} \cdot \sin(\theta)}$

(25 pts.) **5)** For the circuit shown, $v_g(t) = 0.1\sin(10^6 t)$. Write four equations which are necessary to solve for the phasor voltages V_1, V_2, V_3, and V_4. Also, write an expression (in terms of the variables used) for the amount of the complex power generated by the input voltage source v_g.
Do not solve the equations; just write them correctly and completely.



Equation 1:

$$\frac{V_1 + 0.1j}{40000} + \frac{V_1}{-j20000} + \frac{V_1 - V_2}{10000} + \frac{V_1 - 0}{20000} = 0$$

Equation 2:

$$\frac{0 - V_1}{20000} + \frac{0 - V_2}{-j10000} = 0$$

Equation 3:

$$\frac{V_3 - V_2}{500} + \frac{V_3}{-j20} = 0$$

Equation 4:

$$\frac{V_3}{200} + \frac{V_3 - V_4}{-j100} = 0$$

An expression for the complex power generated by the source v_g:

$$S_g = 0.5 V_g \cdot I_g^* = (-0.1j) \cdot \left(\frac{-0.1j - V_1}{40000}\right)^* \text{*0.5}$$