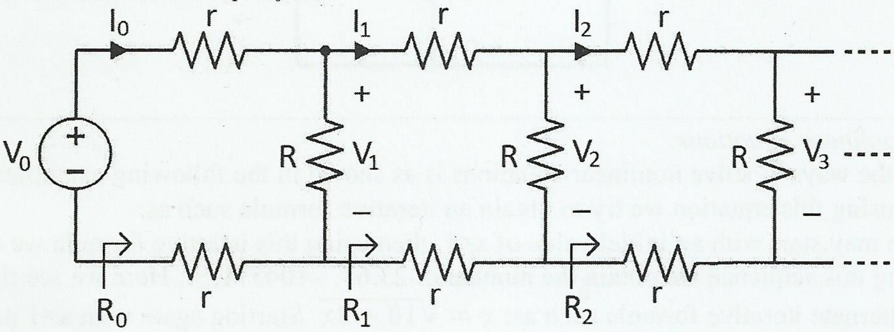
Student Name: ______________________________

EE201 Midterm Exam Nov. 13, 2013

(30 pts.) **1)** Consider the following circuit which models a long cable channel. Resistors repeat indefinitely. Note that the internal resistances are defined as $R_i = V_i/I_i$ for $i \geq 0$, and, $R_0 = R_1 = R_2 = \ldots$

a) Find R_0 in terms of the resistance values r and R.

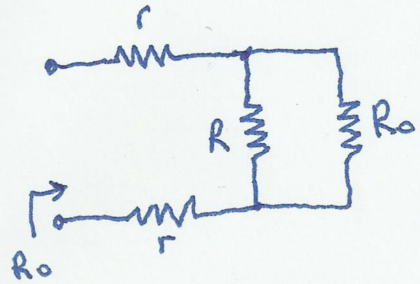
b) Find the attenuation rate V_1/V_0 .



Hint: How to solve problems containing indefinitely repeating terms:
Instead of a circuit example, let us consider a mathematical expression consisting of indefinitely repeating terms: $a = 1 + \frac{1}{1+\frac{1}{1+\cdots}}$. Here, to solve for the value *a,* we need to notice that the same pattern repeats itself. We can write: $a = 1 + \frac{1}{a}$. From this last expression we can solve for *a* as: $a^2 = a + 1 \Rightarrow a = (1 \mp \sqrt{5})/2$. Since $a>1$, $a = 1.618$, which is known to be the golden ratio. You may employ a similar strategy to solve the above problem.

Answer 1:

a)



$$R_0 = 2r + \underbrace{\frac{R \cdot R_0}{R + R_0}}_{R // R_0}$$

(10)

(5) $\Rightarrow R_0^2 - 2rR_0 - 2rR = 0$

(5) $\Rightarrow R_0 = r + \sqrt{r^2 + 2rR}$

b) $I_0 = \frac{V_0}{R_0}$

(5) $V_1 = V_0 - 2r \cdot I_0$ or $V_1 = I_0 \cdot (R // R_0)$

(5) $\Rightarrow \frac{V_1}{V_0} = \frac{R}{R + R_0} = 1 - \frac{2r}{R_0}$

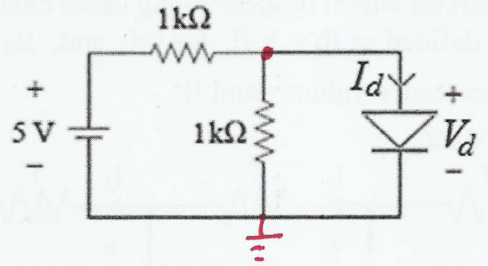
$$\frac{V_1}{V_0} = \underline{\underline{\frac{R}{R + r + \sqrt{r^2 + 2rR}}}} = \underline{\underline{\frac{\sqrt{r^2 + 2rR} - r}{\sqrt{r^2 + 2rR} + r}}}$$

Note: Total time allowed is 110 min. Please show all your work and write legibly.

(20 pts.) **2)** The following circuit contains a nonlinear element such that $I_d = 10^{-3}(e^{5V_d} - 1)$.

a) Obtain an equation containing only the variable V_d.

b) Using the equation obtained in (a), find V_d within 5% error.



Hint: How to solve nonlinear equations:
Among many, one of the ways to solve nonlinear equations is as shown in the following example: $3x + x^4 = 10$. Here, using this equation we try to obtain an iterative formula such as: $x = (10 - x^4)/3$. We may start with an initial value of x=1, then using this iterative formula we obtain the next value as: x=3. Then repeating this sequence we obtain the numbers: -23.67, -104571, … Here we see that it does not work! Now we may try an alternate iterative formula such as: $x = \sqrt[4]{10 - 3x}$. Starting again with x=1 produces: 1.6266, 1.5043, 1.5305, 1.5250, 1.5262, 1.5259, 1.5260, 1.5260 … Therefore we find the solution as x=1.5260.

Answer 2:

a) (9) $\dfrac{V_d - 5}{1000} + \dfrac{V_d}{1000} + I_d = 0$

(5) $\Rightarrow \quad 2V_d + e^{5V_d} - 6 = 0$

b) (6) $V_d = 0.3347\ V \qquad (0.31 \ldots 0.36$ ok.)

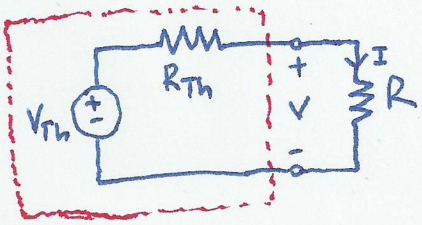
$I_d = 4.3306\ mA$

(using $V_d \leftarrow \frac{1}{5} \ln(6 - 2V_d)$)

(25 pts.) **3)** A power generating linear circuit produces 720 W when 5 Ohms or 20 Ohms connected to it. What is the maximum power that could be obtained from such generator? Which resistor value must be used for that?

Hint:
Use a Thevenin equivalent model for the circuit and find the parameters V_{TH} and R_{TH} using the given data.

Answer 3:



5 Ω Connected:

$$P_R = \frac{V^2}{R} = I^2 R$$

$$\Rightarrow V = \sqrt{720 \cdot 5} = 60\,V$$

$$I = \sqrt{\frac{720}{5}} = 12\,A$$

$$\Rightarrow V_{Th} = R_{Th} \cdot I + V$$

(6) $V_{Th} = R_{Th} \cdot 12 + 60$

20 Ω Connected

$$V = \sqrt{720 \cdot 20} = 120\,V$$

$$I = \sqrt{\frac{720}{20}} = 6\,A$$

(6) $\Rightarrow V_{Th} = R_{Th} \cdot 6 + 120$

or: $P_R = \left(\frac{V_{Th}}{R_{Th} + R}\right)^2 \cdot R$

(6) $\Rightarrow R_{Th} = 10\,\Omega$

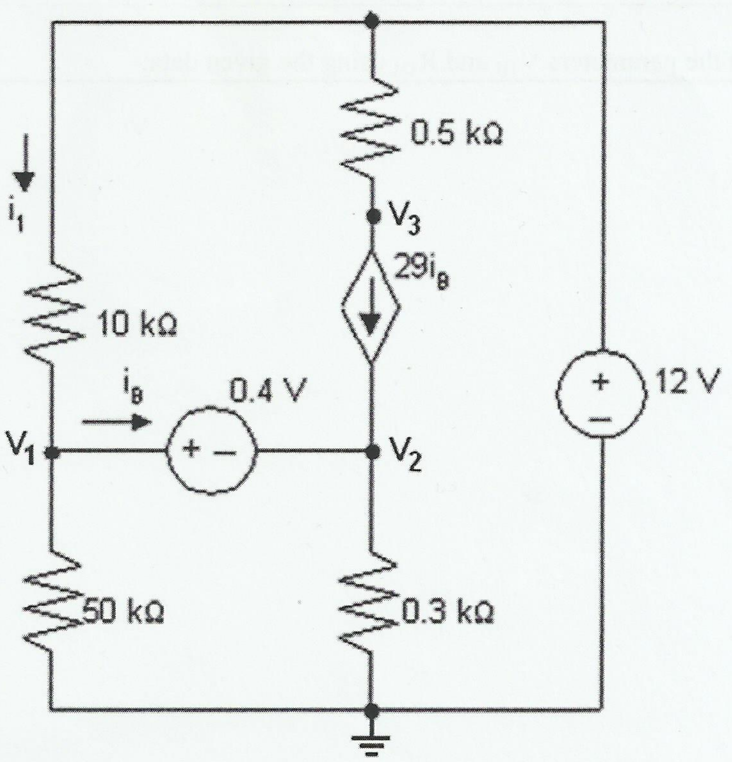
$V_{Th} = 180\,V$

(3) To get the maximum power $R = R_{Th} = 10\,\Omega$

$$\Rightarrow V_{max} = \frac{V_{Th}}{2} = 90\,V$$

(4) $\Rightarrow P_{R\,max} = \frac{90^2}{10} = 810\,W$

(25 pts.) **4)** For the circuit shown, write five equations which are necessary to solve for the unknowns V_1, V_2, V_3, i_B, and i_1. **Do not** solve the equations; just write them correctly and completely.



Equation 1:

(5) $$\frac{V_1}{50K} + \frac{V_1 - 12}{10K} + i_B = 0$$

Equation 2:

(5) $$\frac{V_2}{0.3K} - i_B - 29 i_B = 0$$

Equation 3:

(5) $$\frac{V_3 - 12}{0.5K} + 29 i_B = 0$$

Equation 4:

(5) $$V_1 - V_2 = 0.4$$

Equation 5:

(5) $$\frac{12 - V_1}{10K} = i_1$$