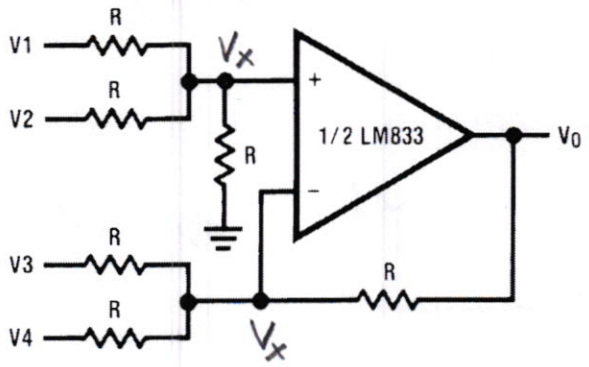
Student Name:_____

EE201 Midterm Exam #1 Dec. 5, 2017

(25 pts.) **1)** Find the output voltage Vo in terms of the input voltages V_1, V_2, V_3, and V_4.



Answer 1:

$$\frac{V_x - V_1}{R} + \frac{V_x - V_2}{R} + \frac{V_x}{R} = 0$$

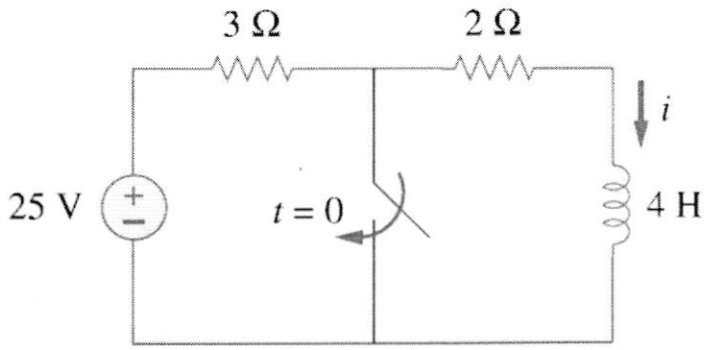
$$\Rightarrow V_x = \frac{V_1 + V_2}{3}$$

$$\frac{V_x - V_3}{R} + \frac{V_x - V_4}{R} + \frac{V_x - V_o}{R} = 0$$

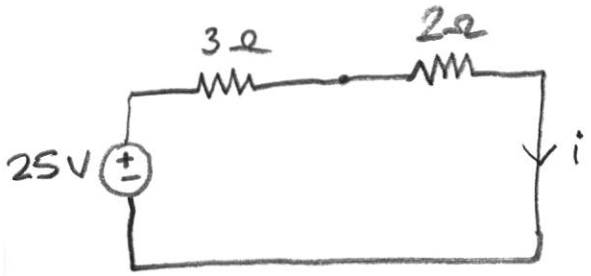
$$\Rightarrow \underline{\underline{V_o = V_1 + V_2 - V_3 - V_4}}$$

Note: Total time allowed is 90 min. Please show all your work and write legibly.

(40 pts.) **2)** Consider the circuit given.
a) Find the initial current $i(0)$.
b) Find the differential equation of $i(t)$ for $t \geq 0$.
c) Solve the differential equation and find $i(t)$.
d) Sketch $i(t)$ for $t \geq 0$.
(Please show all important aspects and units)

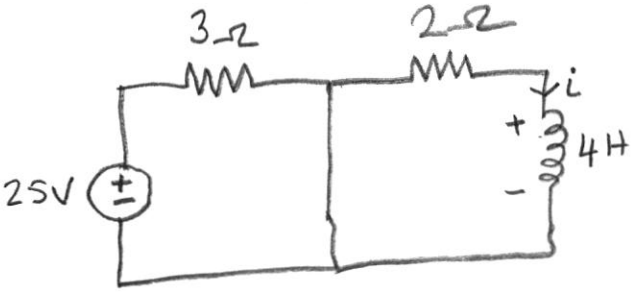


a)



$$\Rightarrow i(0) = \frac{25}{3+2} = 5\ A$$

b)



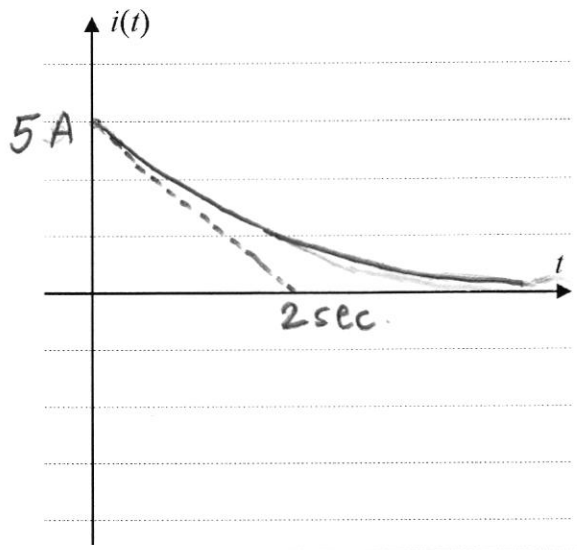
$$4\frac{di}{dt} + 2i = 0$$

$$\Rightarrow \frac{di}{dt} + 0.5i = 0$$

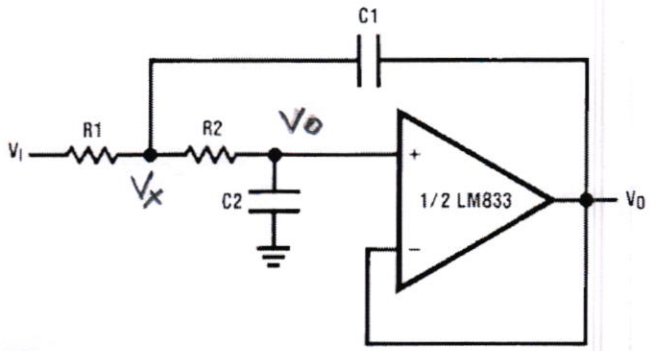
c)

$$i(t) = 5 \cdot e^{-t/2}$$

d)



(35 pts.) **3)** Second order Low Pass Filter (Butterworth) circuit is shown below. Write Node Voltage Analysis equations as necessary. Find a differential equation involving only V_I and V_O voltages.



Answer 3:

$$\frac{V_x - V_I}{R_1} + \frac{V_x - V_o}{R_2} + C_1 \frac{d(V_x - V_o)}{dt} = 0$$

$$\frac{V_o - V_x}{R_2} + C_2 \frac{dV_o}{dt} = 0$$

$$\Rightarrow \quad V_x = R_2 C_2 \frac{dV_o}{dt} + V_o$$

$$\Rightarrow \quad \frac{d(V_x - V_o)}{dt} = R_2 C_2 \frac{d^2 V_o}{dt^2}$$

$$\Rightarrow \left(\frac{1}{R_1} + \frac{1}{R_2}\right)\left(R_2 C_2 \frac{dV_o}{dt} + V_o\right) - \frac{V_I}{R_1} - \frac{V_o}{R_2} + C_1 R_2 C_2 \frac{d^2 V_o}{dt} = 0$$

$$\Rightarrow \quad C_1 C_2 R_1 R_2 \frac{d^2 V_o}{dt^2} + (R_1 + R_2) C_2 \frac{dV_o}{dt} + V_o = V_I$$