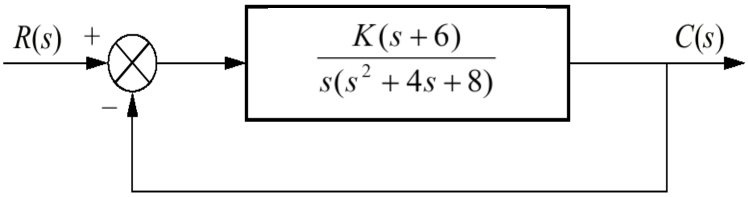
Student Name:___________________________________

EE372 Final Solutions May 27, 2014

(30 pts) 1) .Sketch the root locus for the unity feedback system shown below. Find the asymptotes. For which values of K the total system will be stable?



Answer:

We have a zero at –6, and poles at 0 and -2±j2. The root locus is shown in the figure.

The asymptote intercept point is

$$\sigma_a = \frac{(-2-2+0)-(-6)}{3-1} = 1$$

The asymptote angles are 90° and 270°.

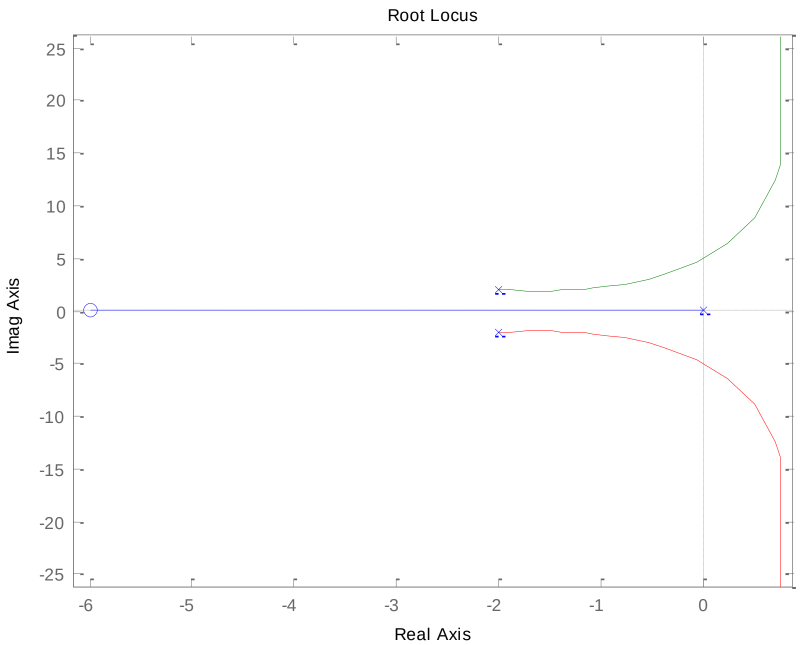
The characteristic polynomial of the closed-loop system is:

$s^3+4s^2+(8+K)s+6K.$

The Routh table is as shown below:

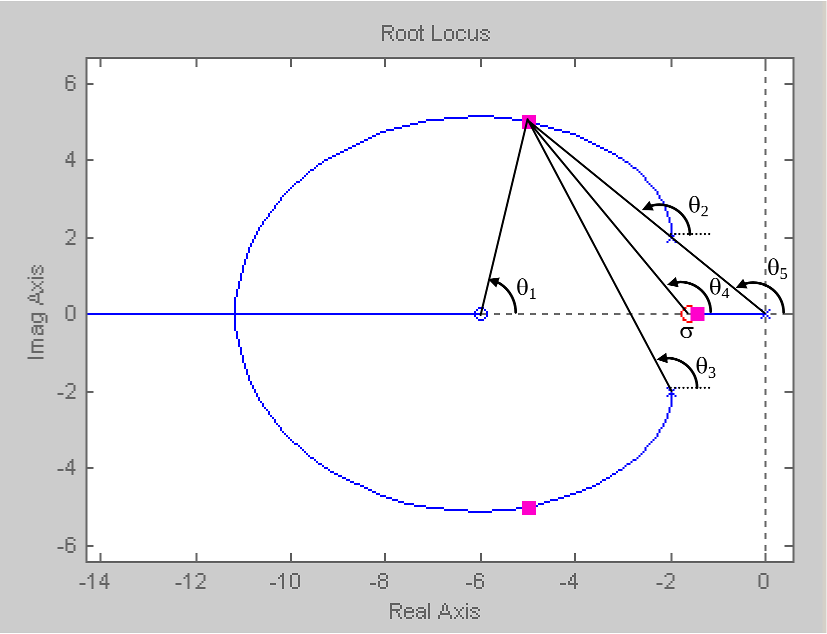
s^3	1	8+K
s^2	4	6K
s^1	32-2K	0
s^0	6K	

Therefore, to have a stable system, K must be between 0 and 16.



(40 pts) 2) Consider the same system as in problem 1. Design a PD compensator so that the root locus passes from -5±j5. Find the necessary zero position of the compensator. Draw the resulting root locus. What must be the corresponding gain for that point.
Answer:

We would like the root locus to pass from -5±j5. To satisfy this, the angle condition must be satisfied:



$\theta_1-\theta_2-\theta_3+\theta_4-\theta_5=180+360N$

where

$\theta_1 = \tan^{-1}(5/1) = 78.69°$,
$\theta_2 = 180-\tan^{-1}(3/3) = 135°$,
$\theta_3 = 180-\tan^{-1}(7/3) = 113.19°$,
$\theta_5 = 180-\tan^{-1}(5/5) = 135°$.

Therefore we have $\theta_4 =180+(-360)-78.69+135+113.19+135=124.5°$.

We can find the zero location using:

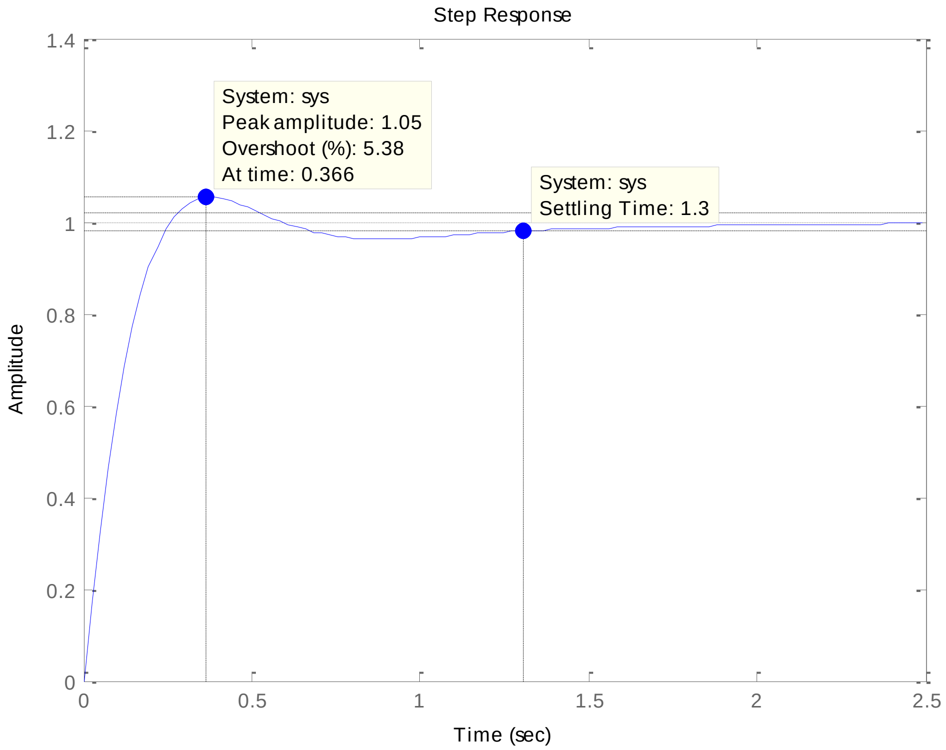
$$\sigma = -5 + \frac{5}{\tan(180-124.5)} = -1.56$$

Therefore the required PD compensator is: $G_C(s) = K(s+1.56)$.

Corresponding gain can be found using:

$$K = \frac{\prod \text{pole lengths}}{\prod \text{zero lengths}} = \frac{\sqrt{(3^2+3^2)(3^2+7^2)(5^2+5^2)}}{\sqrt{(1^2+5^2)(5^2+(5-1.56)^2)}} = 7.38$$

The step response is shown below.



(30 pts) 3) Consider the system in Problem 1. The feedforward part can be described by the state equations below:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -8 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 6 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

a) Is the system controllable? Explain.
b) Is the system observable? Explain.
c) Design a state feedback control to place the closed loop poles at: s = -5 –5 j, s = -5 +5 j, s = -10.

<u>Answer:</u>

a) $C_M = [B\ AB\ A^2B] = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & -4 \\ 1 & -4 & 8 \end{bmatrix}$, and, $\det(C_M) = -1 \neq 0$, therefore controllable.

b) $O_M = \begin{bmatrix} C \\ CA \\ CA^2 \end{bmatrix} = \begin{bmatrix} 6 & 1 & 0 \\ 0 & 6 & 1 \\ 0 & -8 & 2 \end{bmatrix}$, and, $\det(O_M) = 120 \neq 0$, therefore observable.

c) The state feedback control:

$$u = -Kx = -[k_1 \quad k_2 \quad k_3]x$$

$$A - BK = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -k_1 & -8 - k_2 & -4 - k_3 \end{bmatrix}$$

$$\det\big(sI - (A - BK)\big) = s^3 + (4 + k_3)s^2 + (8 + k_2)s + k_1$$

The desired characteristic equation:

$$(s + 5 + 5j)(s + 5 - 5j)(s + 10) = s^3 + 20s^2 + 150s + 500$$

Therefore:

$$K = [500 \quad 142 \quad 16].$$