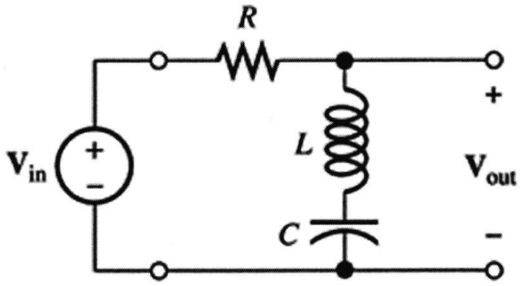
Student Name:____

EE372 Midterm Apr. 07, 2014

(25 pts) 1) Find the transfer function for the circuit shown below, and, obtain the state space representation in the *controllable canonical form.*



$$G(s) = \frac{V_{out}(s)}{V_{in}(s)} = \frac{Ls + \frac{1}{sC}}{R + Ls + \frac{1}{sC}} = \underline{\underline{\frac{LCs^2+1}{LCs^2+RCs+1}}} = 1 - \frac{\frac{R}{L}s}{s^2+\frac{R}{L}s+\frac{1}{LC}}$$

$$\Rightarrow \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{1}{LC} & -\frac{R}{L} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} V_{in}$$

$$V_{out} = \begin{bmatrix} 0 & -\frac{R}{L} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + [1]\, V_{in}$$

(25 pts) 2) Consider the system given by the transfer function

$$G(s) = \frac{25}{s^2 + as + 25}$$

a) Find the value of a to obtain 10% overshoot in the step response. (±1% accuracy is needed.)

<u>Hint:</u> Second order transfer function: $\frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$, and $\%OS = e^{-\zeta\pi/\sqrt{1-\zeta^2}} \times 100$

$$\omega_n^2 = 25 \Rightarrow \omega_n = 5$$

$$\%OS = e^{-\zeta\pi\sqrt{1-\zeta^2}} \times 100 = 10$$

$$\frac{\zeta}{\sqrt{1-\zeta^2}} = 0.733 \Rightarrow \zeta = 0.591$$

$$\Rightarrow a = 2\zeta\omega_n = 5.91$$

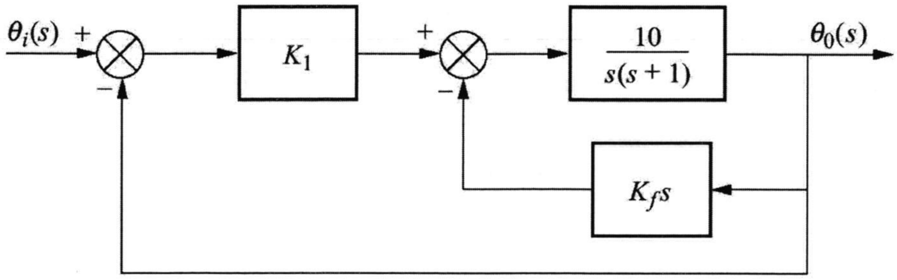
b) If a is chosen as 5, what will be the <u>%OS</u> and <u>settling time</u> for this system? <u>Hint:</u> $T_s = \frac{4}{\zeta\omega_n}$

$$a = 5 \Rightarrow \zeta = \frac{a}{2\omega_n} = 0.5$$

$$\Rightarrow \%OS = 16.3$$

$$\Rightarrow T_s = 1.6 \text{ s}$$

(25 pts) 3) The system shown below is to have the following specifications: K_v=10; ζ=0.5. Find the values of K_1 and K_f required for the specifications of the system to be met.



Hint: The velocity constant: $K_v = \lim_{s\to 0} sG(s)$. Second order transfer function: $\frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$.

$$G(s) = K_1 \cdot \frac{\frac{10}{s(s+1)}}{1+\frac{10K_f \cdot s}{s(s+1)}} = \frac{10K_1}{s^2+s(10K_f+1)}$$

$$K_v = 10 = \lim_{s\to 0} s\frac{10K_1}{s^2+s(10K_f+1)} = \frac{10K_1}{10K_f+1} \Rightarrow \underline{\underline{K_1 = 10K_f+1}}$$

$$\Rightarrow T(s) = \frac{G(s)}{1+G(s)} = \frac{10K_1}{s^2+s(10K_f+1)+10K_1}$$

$$\Rightarrow \omega_n = \sqrt{10K_1}$$

$$\underbrace{2\zeta\omega_n}_{\sqrt{10K_1}} = \underbrace{10K_f+1}_{K_1} \Rightarrow K_1^2 = 10K_1 \Rightarrow \underline{\underline{K_1 = 10}}$$

$$\Rightarrow K_f = \frac{10-1}{10} = \underline{\underline{0.9}}$$

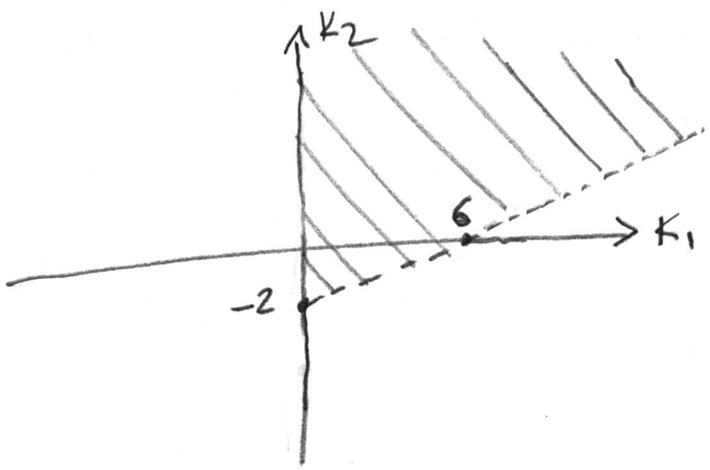
(25 pts) 4) Consider the system transfer function:

$$T(s) = \frac{C(s)}{R(s)} = \frac{K_1}{s^3 + 3s^2 + (2+K_2)s + K_1}$$

a) Find the necessary conditions so that the system is stable. Show the stability region on the (K_1, K_2) plane.

$$\begin{array}{c|cc} s^3 & 1 & 2+K_2 \\ s^2 & 3 & K_1 \\ s^1 & \dfrac{6+3K_2-K_1}{3} & 0 \\ s^0 & K_1 & \end{array}$$

For stability : $6+3K_2-K_1>0$ and $K_1>0$



b) If K_1 is 9, what is the range of K_2 so that the system is stable.

$6+3K_2-9>0 \Rightarrow \underline{\underline{K_2>1}}$