# DE-2013

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**Ch 2.1: Linear Equations**

$$a_0(t)y^{(n)} + a_1(t)y^{(n-1)} + \cdots + a_n(t)y = g(t)$$

- Here if g(t) = 0 homogeneous, non-homogeneous otherwise (driving by a force). You know the equations below already.
- A linear first order ODE has the general form, where p(t), g(t), can be constants and/or variables.

$$\frac{dy}{dt} + p(t)y = g(t)$$

- Constant Coefficient Case: straightforward solution is

$$y' = -ay + b,\ \ \ln\left|y - b/a\right| = -a\,t + C,\ \ y = b/a + ke^{at},\ \ k = \pm e^{C}$$

$$\frac{dy/dt}{y - b/a} = -a \qquad \int \frac{dy}{y - b/a} = -\int a\,dt$$

- Variable Coefficient Case: Method of Integrating Factors.
- **Using the product rule, d(uv)=vdu + udv.** Multiplying the equation by a function $\mu(t)$, so that the entire equation must be easily integrated.

Variable Coefficient Case: Method of Integrating Factors. From the product rule, multiplying the 1st order linear DE by a function $\mu(t)$, so that the resulting equation must be easily integrated. This is the General Case. Proof is an exam question.

$$y' + p(t)y = g(t)$$

$$\mu(t)y' + \mu(t)p(t)y = \mu(t)g(t)$$

$$\frac{d}{dt}[\mu(t)y] =$$

$$\mu(t)\frac{dy}{dt} + \frac{d\mu(t)}{dt}y = \mu(t)g(t)$$

$$\int \frac{d}{dt}[\mu(t)y] = \int [\mu(t)g(t)] + C$$

$$y = \frac{1}{\mu(t)}\left(\int [\mu(t)g(t)] + C\right)$$

$$\frac{d\mu(t)}{dt} = \mu(t)p(t)$$

$$\int \frac{d\mu(t)}{\mu(t)} = \int p(t)dt$$

$$\ln \mu(t) = \int p(t)dt + k$$

$$\mu(t) = e^{\int p(t)dt}$$



## Method of Integrating Factors: Variable Right Side, g(t)

$$y' + ay = g(t)$$

$$\frac{d\mu(t)}{dt} = a\mu(t) ==> \mu(t) = e^{at}$$

$$\mu(t)\frac{dy}{dt} + a\mu(t)y = \mu(t)g(t)$$

$$e^{at}\frac{dy}{dt} + ae^{at}y = e^{at}g(t) ==> \frac{d}{dt}\left[e^{at}y\right] = e^{at}g(t)$$

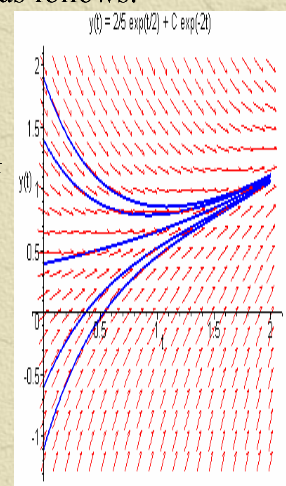
$$y = e^{-at}\int e^{at}g(t)dt + Ce^{-at}$$

## Example 1: $y' + 2y = e^{t/2}$

- Observe that equilibrium solution (of slopes) is shifting due to the t dependence..

$y' + 2y = e^{t/2} \rightarrow y' = 0 \rightarrow y = e^{t/2}/2$

- With $\mu(t) = e^{2t}$, we solve the original equation as follows:




$$y' + 2y = e^{t/2}$$

$$\mu(t)\frac{dy}{dt} + 2\mu(t)y = \mu(t)e^{t/2} \;=>=>\; \mu(t) = e^{2t}$$

$$e^{2t}\frac{dy}{dt} + 2e^{2t}y = e^{5t/2} \;=>=>\; \frac{d}{dt}\left[e^{2t}y\right] = e^{5t/2}$$

$$e^{2t}y = \frac{2}{5}e^{5t/2} + C \;=>=>\; y = \frac{2}{5}e^{t/2} + Ce^{-2t}$$

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## Example 2: General Solution of $y' + \frac{1}{5}y = 5 - t$

$$y = e^{-at}\int e^{at}g(t)dt + Ce^{-at} = e^{-t/5}\int e^{t/5}(5-t)dt + Ce^{-t/5}$$

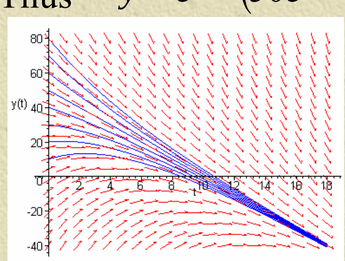
Integrating by parts, udv=d(uv)-vdu

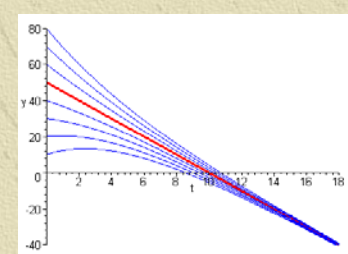
$$\int e^{t/5}(5-t)dt = \int 5e^{t/5}dt - \int te^{t/5}dt$$

$$= 25e^{t/5} - \left[5te^{t/5} - \int 5e^{t/5}dt\right]$$

$$= 50e^{t/5} - 5te^{t/5}$$

- Thus $y = e^{-t/5}\left(50e^{t/5} - 5te^{t/5}\right) + Ce^{-t/5} = 50 - 5t + Ce^{-t/5}$

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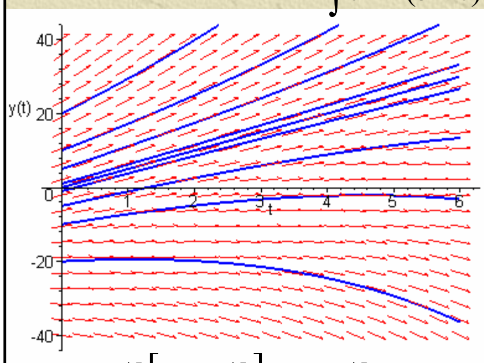
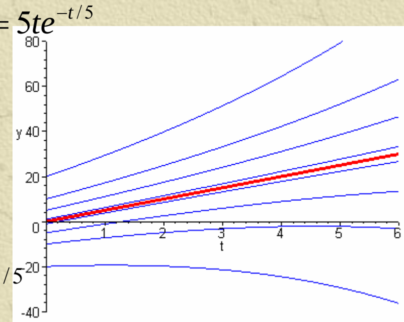
$$y' - \frac{1}{5}y = 5 - t$$

- Equilibrium points y'=0, y= -25 (t=0), and t=5 (y=0)
- Needs integrating by parts,

$$y = e^{-at}\int e^{at} g(t)dt + Ce^{-at} = e^{t/5}\int e^{-t/5}(5-t)dt + Ce^{t/5}$$

$$\int e^{-t/5}(5-t)dt = \int 5e^{-t/5}dt - \int te^{-t/5}dt$$
$$= -25e^{-t/5} - \left[-5te^{-t/5} + \int 5e^{-t/5}dt\right]$$
$$= 5te^{-t/5}$$




$$y = e^{t/5}\left[5te^{-t/5}\right] + Ce^{t/5} = 5t + Ce^{t/5}$$



Example for general case of 1st order DE, IVP probl. EXAM WARNING, linear!!?

$$ty' - 2y = 5t^2, \quad y(1) = 2,$$

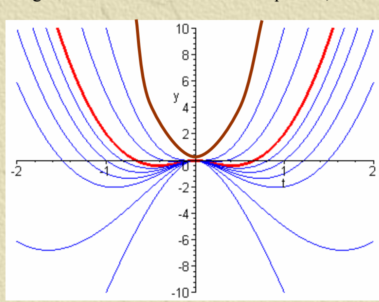
$$y' - \frac{2}{t}y = 5t, \quad \text{for } t \neq 0$$

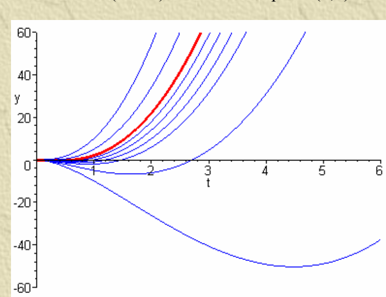
- First put into standard form:
- Integrating Factor $\mu(t) = e^{\int p(t)dt} = e^{-\int \frac{2}{t}dt} = e^{-2\ln|t|} = e^{\ln\left(\frac{1}{t^2}\right)} = \frac{1}{t^2}$

and hence the general and particular solution for $y(1) = 2$, respectively.

$$y = \frac{\int \mu(t)g(t)dt + C}{\mu(t)} = t^2\left[\int \frac{5}{t}dt + C\right] = 5t^2 \ln|t| + Ct^2 \qquad y = 5t^2\left(\ln|t| + 2/5\right)$$

- Integral curves for the differential equation, and a particular solution (in red) for the initial point (1,2).



## Separable DEs: $g(y)dy = f(x)dx$ or $dy/dt = y' = f(x)/g(x)$.

Two Examples and implicit solutions and isoclines. Linearity?

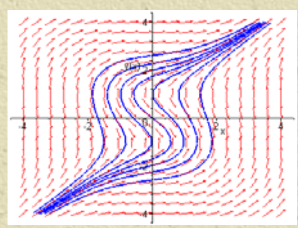
$$\frac{dy}{dx} = \frac{x^2+1}{y^2-1}$$

$$(y^2-1)dy = (x^2+1)dx$$

$$\int (y^2-1)dy = \int (x^2+1)dx$$

$$\frac{1}{3}y^3 - y = \frac{1}{3}x^3 + x + C$$

$$y^3 - 3y = x^3 + 3x + C$$

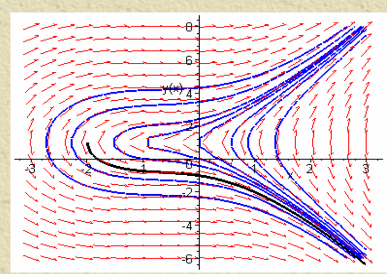


$$\frac{dy}{dx} = \frac{3x^2+4x+2}{2(y-1)}$$

$$2(y-1)dy = (3x^2+4x+2)dx$$

$$2\int (y-1)dy = \int (3x^2+4x+2)dx$$

$$y^2 - 2y = x^3 + 2x^2 + 2x + C$$



$$y^2 - 2y = x^3 + 2x^2 + 2x + C \quad \text{(implicit)}$$

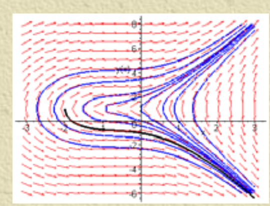
$$y = 1 \pm \sqrt{x^3 + 2x^2 + 2x + C} \quad \text{(explicit)}$$



## In 2nd Example, domain of the solution

- Thus the solutions to the initial value problem

$$\frac{dy}{dx} = \frac{3x^2+4x+2}{2(y-1)}, \quad y(0) = -1$$



are given by

$$y^2 - 2y = x^3 + 2x^2 + 2x + 3 \quad \text{(implicit)}$$

$$y = 1 - \sqrt{x^3 + 2x^2 + 2x + 4} \quad \text{(explicit)}$$

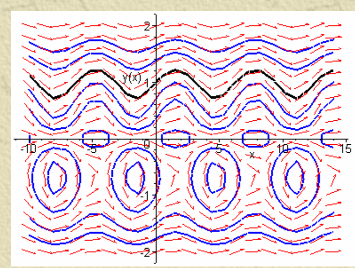
- From explicit representation of $y$, it follows that

$$y = 1 - \sqrt{x^2(x+2) + 2(x+2)} = 1 - \sqrt{(x+2)(x^2+2)}$$

and hence domain of $y$ is $x=(-2, \infty)$. Smaller than -2 negates inside sqrt, and $x = -2$ yields $y = 1$, which makes denominator of $dy/dx$ zero (vertical tangent).

- Conversely, domain of $y$ can be estimated by locating vertical tangents on graph (useful for implicitly defined solutions).

$$y' = \frac{y\cos x}{1+3y^3}, \quad y(0)=1 \qquad\qquad \ln|y| + y^3 = \sin x + C$$





# Ch 2.4: Differences Between Linear and Nonlinear Equations

- Recall that a first order ODE has the form $y' = f(t,y)$, and is linear if $f$ is linear in $y$, and nonlinear if $f$ is nonlinear in $y$ (regardless of $t$).
- Examples: $y' = ty - e^t$, $y' = ty^2$.
- First order linear and nonlinear equations differ in a number of ways:
  - The theory describing existence and uniqueness of solutions, and corresponding domains, are different.
  - **Solutions to linear equations can be expressed in terms of a general solution**, which is not usually the case for nonlinear equations.
  - **Linear equations have explicitly defined solutions while nonlinear equations typically do not**, and nonlinear equations **may or may not have implicitly defined solutions**.
- For both types of equations, numerical and graphical construction of solutions are important.

## Linearity = multiplicity (scalability) and additivity (superposition).

- Linearity Definiton: (with respect to dependent variable, therefore the degree of the independent variables as coefficients of the derivations is nor a concern.)
- Scalability af(x)=f(ax);
- Superposition, y=u+v, f(u)+f(v)?, f(u+v)=f(u)+f(v)
- f(au + bv) = f(au) + f(bv) = af(u) + bf(v)
- Example: $L(z) = z''' - z + k^3 z$
  - $((a + b)z)''' - (a + b)z + k^3(a + b)z = (az)''' + (bz)''' - az - bz + k^3 az + k^3 bz = af(z)+bf(z)$
  - So its's linear.
- We can find it to look degree of functions f z too. Degree of z is 1 and not any trig combinations is involved.
- $L(y) = y' - y + y^2$
- $(a + b)y'- (a + b)y + ((a + b)y)^2 \neq ay' + by' - ay - by + (ay)^2 + (by)^2$
- So it's not linear.and the degree of y is 2, indicating nonlinearity.



## Theorem 2.4.1

- Consider the linear first order initial value problem:

$$\frac{dy}{dt} + p(t)y = g(t), \quad y(0) = y_0$$

If the functions $p$ and $g$ are continuous on an open interval $(\alpha, \beta)$ containing the point $t = t_0$, then there exists a unique solution $y = \phi(t)$ that satisfies the IVP for each $t$ in $(\alpha, \beta)$.

- **Proof:**

$$y = \frac{\int_{t_0}^{t} \mu(t) g(t) dt + y_0}{\mu(t)}, \quad \text{where } \mu(t) = e^{\int_{t_0}^{t} p(s) ds}$$

## Theorem 2.4.2

- Consider the nonlinear first order initial value problem:

$$\frac{dy}{dt} = f(t, y), \quad y(0) = y_0$$

- Suppose $f$ and $\partial f/\partial y$ are continuous on some open rectangle $(t, y) \in (\alpha, \beta) \times (\gamma, \delta)$ containing the point $(t_0, y_0)$. Then in some interval $(t_0 - h, t_0 + h) \subseteq (\alpha, \beta)$ there exists a unique solution $y = \phi(t)$ that satisfies the IVP.
- Since there is no general formula for the solution of arbitrary nonlinear first order IVPs, this proof is difficult, and beyond the scope of this course.
- It turns out that conditions stated in Thm 2.4.2 are sufficient but not necessary to guarantee existence of a solution, **and continuity of *f* ensures existence but not uniqueness of $\phi$.**

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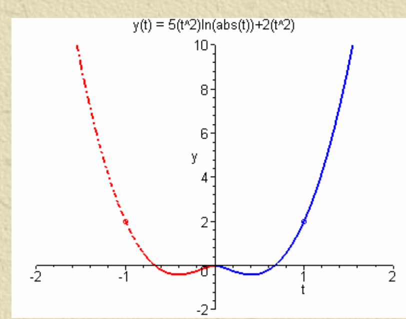
## Example 1: Linear IVP

- Recall the initial value problem from Chapter 2.1 slides:

$$ty' - 2y = 5t^2, \quad y(1) = 2 \quad \Rightarrow \quad y = 5t^2 \ln|t| + 2t^2$$

- The solution to this initial value problem is defined for $t > 0$, the interval on which $p(t) = -2/t$ is continuous.
- If the initial condition is $y(-1) = 2$, then the solution is given by same expression as above, but is defined on $t < 0$.
- In either case, Theorem 2.4.1 guarantees that solution is unique on corresponding interval.

Question what is the interval here for thr 2.41.

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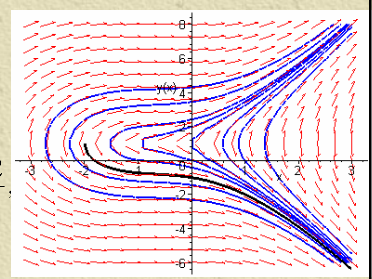
## Example 2: Nonlinear IVP

- Consider nonlinear initial value problem from Ch 2.2:

$$\frac{dy}{dx} = \frac{3x^2 + 4x + 2}{2(y-1)}, \quad y(0) = -1$$



- The functions $f$ and $\partial f/\partial y$ are given by

$$f(x, y) = \frac{3x^2 + 4x + 2}{2(y-1)}, \quad \frac{\partial f}{\partial y}(x, y) = -\frac{3x^2 + 4x + 2}{2(y-1)^2},$$

  and are continuous except on line $y = 1$.
- Thus possible to draw an open rectangle about (0, -1) on which $f$ and $\partial f/\partial y$ are continuous, as long as it doesn't cover $y = 1$.
- How wide is rectangle? Recall solution defined for $x > -2$, with

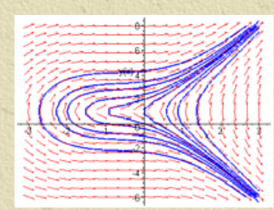
$$y = 1 - \sqrt{x^3 + 2x^2 + 2x + 4}$$



## Example 2: Change Initial Condition SKIP

- Our nonlinear initial value problem is

$$\frac{dy}{dx} = \frac{3x^2 + 4x + 2}{2(y-1)}, \quad y(0) = -1$$



  with

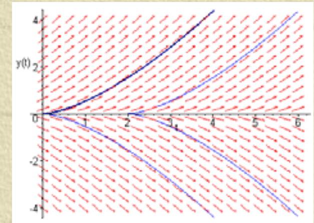
$$f(x, y) = \frac{3x^2 + 4x + 2}{2(y-1)}, \quad \frac{\partial f}{\partial y}(x, y) = -\frac{3x^2 + 4x + 2}{2(y-1)^2},$$

  which are continuous except on line $y = 1$.
- If we change initial condition to $y(0) = 1$, then Theorem 2.4.2 is not satisfied. Solving this new IVP, we obtain

$$y = 1 \pm \sqrt{x^3 + 2x^2 + 2x}, \quad x > 0$$

- **Thus a solution exists but is not unique.**

## Example 3: (!!!linear) IVP (Very simple to draw tangents)



- Consider initial value problem

$$y' = y^{1/3}, \ y(0) = 0 \qquad (t \geq 0)$$

- The functions *f* and ∂*f*/∂*y* are given by

$$f(t, y) = y^{1/3}, \ \frac{\partial f}{\partial y}(t, y) = \frac{1}{3} y^{-2/3}$$

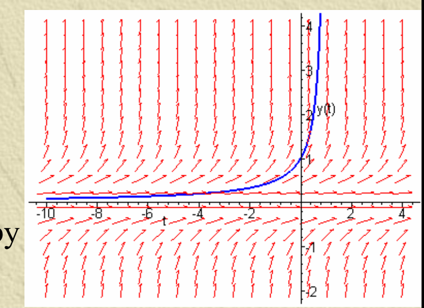
- Thus *f* continuous everywhere, **but ∂*f*/∂*y* doesn't exist at *y* = 0,** and hence Theorem 2.4.2 is not satisfied. Solutions exist but are not unique. Separating variables and solving, we obtain

$$y^{-1/3} dy = dt \ \Rightarrow \ \frac{3}{2} y^{2/3} = t + c \ \Rightarrow \ y = \pm\left(\frac{2}{3} t\right)^{3/2}, \ t \geq 0$$

- Positive since t cannot be negative due to sqrt
- If initial condition is not on *t*-axis where y=0, then Theorem 2.4.2 does guarantee existence and uniqueness.



## SKIP to exactness.
## Example 4: !!!linear IVP



- Consider initial value problem

$$y' = y^2, \ y(0) = 1$$

- The functions *f* and ∂*f*/∂*y* are given by

$$f(t, y) = y^2, \ \frac{\partial f}{\partial y}(t, y) = 2y$$

- Thus *f* and ∂*f*/∂*y* are continuous at *t* = 0, so Thm 2.4.2 guarantees that solutions exist and are unique.
- Separating variables and solving, we obtain

$$y^{-2} dy = dt \ \Rightarrow \ -y^{-1} = t + c \ \Rightarrow \ y = \frac{-1}{t + c} \ \Rightarrow \ y = \frac{1}{1 - t}$$

- The solution *y*(*t*) is defined on (-∞, 1). Note that the singularity at *t* = 1 is not obvious from original IVP statement.

## Interval of Definition: Linear and Nonlinear Cases

- By Theorem 2.4.1, the solution of a linear initial value problem exists throughout any interval about $t = t_0$ on which $p$ and $g$ are continuous.
- Vertical asymptotes or other discontinuities of solution can only occur at points of discontinuity of $p$ or $g$. However, solution may be differentiable at points of discontinuity of $p$ or $g$.

- In the nonlinear case, the interval on which a solution exists may be difficult to determine. The solution $y = \phi(t)$ exists as long as $(t,\phi(t))$ remains within rectangular region indicated in Theorem 2.4.2. This is what determines the value of $h$ in that theorem. Since $\phi(t)$ is usually not known, it may be impossible to determine this region. Furthermore, any singularities in the solution may depend on the initial condition as well as the equation.



## General Solutions

- For a first order linear equation, it is possible to obtain a solution containing one arbitrary constant, from which all solutions follow by specifying values for this constant.
- For nonlinear equations, such general solutions may not exist. That is, even though a solution containing an arbitrary constant may be found, there may be other solutions that cannot be obtained by specifying values for this constant.
- Consider Example 4: The function $y = 0$ is a solution of the differential equation, but it cannot be obtained by specifying a value for $c$ in solution using separation of variables:

$$\frac{dy}{dt} = y^2 \quad \Rightarrow \quad y = \frac{-1}{t+c}$$

## Explicit Solutions: Linear Equations

- By Theorem 2.4.1, a solution of a linear initial value problem

$$y' + p(t)y = g(t),\ \ y(0) = y_0$$

exists throughout any interval about $t = t_0$ on which $p$ and $g$ are continuous, and this solution is unique.

- The solution has an explicit representation,

$$y = \frac{\int_{t_0}^{t} \mu(t)g(t)dt + y_0}{\mu(t)}, \quad \text{where } \mu(t) = e^{\int_{t_0}^{t} p(s)ds},$$

and can be evaluated at any appropriate value of $t$, as long as the necessary integrals can be computed.



## Explicit Solution Approximation

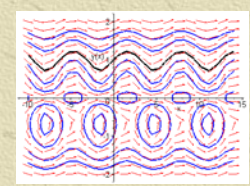
- For linear first order equations, an explicit representation for the solution can be found, as long as necessary integrals can be solved.
- If integrals can't be solved, then numerical methods are often used to approximate the integrals.

$$y = \frac{\int_{t_0}^{t} \mu(t)g(t)dt + C}{\mu(t)}, \quad \text{where } \mu(t) = e^{\int_{t_0}^{t} p(s)ds}$$

$$\int_{t_0}^{t} \mu(t)g(t)dt \approx \sum_{k=1}^{n} \mu(t_k)g(t_k)\Delta t_k$$

## Implicit Solutions: Nonlinear Equations

- For nonlinear equations, explicit representations of solutions may not exist.
- As we have seen, it may be possible to obtain an equation which implicitly defines the solution. If equation is simple enough, an explicit representation can sometimes be found.
- Otherwise, numerical calculations are necessary in order to determine values of *y* for given values of *t*. These values can then be plotted in a sketch of the integral curve.
- Recall the following example from Ch 2.2 slides:



$$y' = \frac{y \cos x}{1 + 3y^3}, \quad y(0) = 1 \;\Rightarrow\; \ln y + y^3 = \sin x + 1$$

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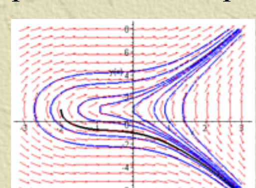
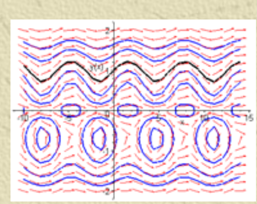
## Direction Fields

- In addition to using numerical methods to sketch the integral curve, the nonlinear equation itself can provide enough information to sketch a direction field.
- The direction field can often show the qualitative form of solutions, and can help identify regions in the *ty*-plane where solutions exhibit interesting features that merit more detailed analytical or numerical investigations.
- Chapter 2.7 and Chapter 8 focus on numerical methods.





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## Ch 2.6: Exact Equations (chain rule!!!).

- Consider a first order ODE of the form

$$M(x,y)+N(x,y)y'=0$$

- Suppose there is a function $\psi$ such that

$$\psi_x(x,y)=M(x,y),\ \ \psi_y(x,y)=N(x,y)$$

and such that $\psi(x,y) = c$ defines $y = \phi(x)$ implicitly. Then

$$\frac{\partial\psi}{\partial x}+\frac{\partial\psi}{\partial y}\frac{dy}{dx}=\frac{d}{dx}\psi\left[x,\phi(x)\right]$$ and hence the original ODE becomes

$$\frac{d}{dx}\psi\left[x,\phi(x)\right]=0$$

- Thus $\psi(x,y) = c$ defines a solution implicitly.
- In this case, the ODE is said to be **exact**.



## Theorem 2.6.1- Continuity and Existence of $\psi$ and the condition of Exactness.

- Suppose an ODE can be written in the form

$$M(x,y)+N(x,y)y'=0 \qquad (1)$$

where the functions $M$, $N$, $M_y$ and $N_x$ are all continuous in the rectangular region $R$: $(x, y) \in (\alpha, \beta) \times (\gamma, \delta)$. Then Eq. (1) is an **exact** differential equation iff

$$M_y(x,y)=N_x(x,y),\ \ \forall(x,y)\in R \qquad (2)$$

- That is, there exists a function $\psi$ satisfying the conditions

$$\psi_x(x,y)=M(x,y),\ \ \psi_y(x,y)=N(x,y) \qquad (3)$$

iff $M$ and $N$ satisfy Equation (2). Think here.. How to solve it.

## Example 1: Exact Equation (1 of 4)

- Consider the following differential equation.

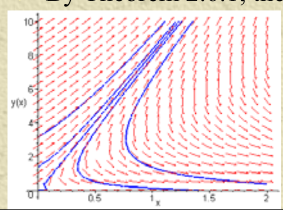
$$\frac{dy}{dx} = -\frac{x+4y}{4x-y} \Leftrightarrow (x+4y)+(4x-y)y' = 0$$

- Then $M(x,y) = x+4y,\ N(x,y) = 4x-y$
  and hence $M_y(x,y) = 4 = N_x(x,y) \Rightarrow$ ODE is exact
- From Theorem 2.6.1, $\psi_x(x,y) = x+4y,\ \psi_y(x,y) = 4x-y$
- Thus $\psi(x,y) = \int \psi_x(x,y)dx = \int (x+4y)dx = \frac{1}{2}x^2 + 4xy + C(y)$

$$\psi_y(x,y) = 4x-y = 4x+C'(y) \Rightarrow C'(y) = -y \Rightarrow C(y) = -\frac{1}{2}y^2 + k$$

$$\psi(x,y) = \frac{1}{2}x^2 + 4xy - \frac{1}{2}y^2 + k = c$$

- By Theorem 2.6.1, the solution is given implicitly by $x^2 + 8xy - y^2 = c$




## Example 2: $(y\cos x + 2xe^y) + (\sin x + x^2e^y - 1)y' = 0$

$$M(x,y) = y\cos x + 2xe^y,\ N(x,y) = \sin x + x^2e^y - 1$$

$$M_y(x,y) = \cos x + 2xe^y = N_x(x,y) \Rightarrow \text{ODE is exact}$$

- From Theorem 2.6.1,

$$\psi_x(x,y) = M = y\cos x + 2xe^y,\ \psi_y(x,y) = N = \sin x + x^2e^y - 1$$

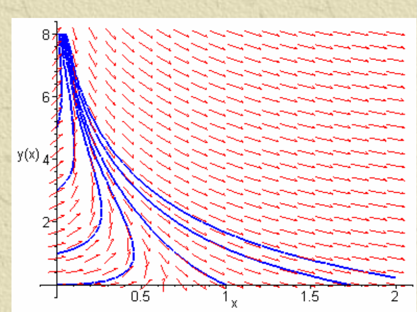
$$\psi(x,y) = \int \psi_x(x,y)dx = \int (y\cos x + 2xe^y)dx = y\sin x + x^2e^y + C(y) = c$$

$$\psi_y(x,y) = \sin x + x^2e^y - 1$$

$$= \sin x + x^2e^y + C'(y)$$

$$C'(y) = -1 \Rightarrow C(y) = -y + k$$

$$\psi(x,y) = y\sin x + x^2e^y - y + k = c$$



## Example 3: Non-Exact Equation Treated by Integrating Factors. Interesting therefore potential Exam Question

- It is **sometimes** possible to convert a inexact DE into an exact equation by treating with a suitable integrating factor $\mu(x,y)$:

$$M(x,y) + N(x,y)y' = 0$$

$$\mu(x,y)M(x,y) + \mu(x,y)N(x,y)y' = 0$$

- For this equation to be exact, we need

$$(\mu M)_y = (\mu N)_x \Leftrightarrow M\mu_y - N\mu_x + (M_y - N_x)\mu = 0$$

- This partial differential equation may be difficult to solve. If $\mu$ is a function of $x$ alone, then $\mu_y = 0$ and hence we solve

$$\frac{d\mu}{dx} = \frac{M_y - N_x}{N}\mu,$$

provided right side is a function of $x$ only. Similarly if $\mu$ is a function of $y$ alone. See text for more details.



## Non-Exact Equation Example treated.

- Consider the following non-exact differential equation.

$$(3xy + y^2) + (x^2 + xy)y' = 0$$

- Seeking an integrating factor, we solve the linear equation

$$\frac{d\mu}{dx} = \frac{M_y - N_x}{N}\mu \Leftrightarrow \frac{d\mu}{dx} = \frac{\mu}{x} \Rightarrow \mu(x) = x$$

- Multiplying our differential equation by $\mu$, we obtain the exact equation

$$(3x^2y + xy^2) + (x^3 + x^2y)y' = 0,$$

which has its solutions given implicitly by

$$x^3y + \frac{1}{2}x^2y^2 = c$$

- Exam question, and HW, 27.b, 29, 29, 30 at page 73 and 74, solve bernoullie problems at least two to prove that eq reduces to a 1rst order linear DE.



```
 y = dsolve('Dy=1+y^2',)

y =

tan(t+C1)


>> y = dsolve('Dy=1+y^2','y(0)=1‘, ‘t’)

y =

tan(t+1/4*pi)


>> diff(y, 't')

ans =

1+tan(t+1/4*pi)^2

```