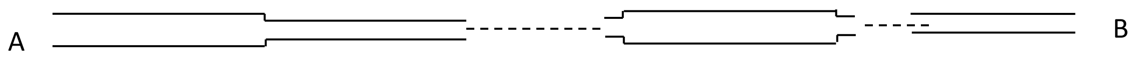
LECTURE 3: PIPELINES and PIPE NETWORKS-II

(Series, Parallel and Branch Piping)

3.1 Series Piping



For many flow situation (e.g. when A and B are point in two reservoir), it is common practice to neglect the kinetic energy terms at A and B.

$$h_L = \left(\frac{P}{\rho . g} + h\right)_A - \left(\frac{P}{\rho . g} + h\right)_B$$

$$= \sum_{i=1}^{N} \overbrace{\left[R_i + \frac{\sum k_{ij}}{2g.A_i{}^2}\right]}^{\overline{Ri}} . Q^2$$

R_i: Resistant coefficient for friction loss

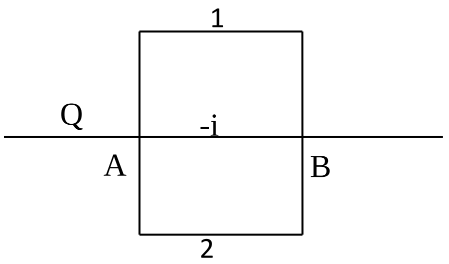
$\frac{\sum k_{ij}}{2g.A_i{}^2}$: Resistant coefficient for minor loss

where $Ri = \frac{fiLi}{2gDiAi^2}$ and Q=ViAi

Type 2: problem (pipe properties and h_L is known, Q = ?) Iterative calculations are needed.

1. Assume complete turbulence
2. L_i, D_i, A_i, e_i => compute $\overline{R_i}$ and $\sum \overline{R_i}$
3. Compute $Q = \sqrt{\frac{h_L}{\sum \overline{R_i}}}$ (Darcy Weisbach Equation)
4. Compute new f_i with new $V_i = \frac{Q}{A_i}$
5. If $|f_{new} - f_{old}|$ small => stop, otherwise go to step 2.

3.2. Parellel Piping



$$Q = \sum_{i=1}^{N} Q_i$$

$$h_L = \left(\frac{P}{\rho.g} + h\right)_A - \left(\frac{P}{\rho.g} + h\right)_B$$

$$h_L = \sum_{i=1}^{N} \left[R_i + \frac{\sum K}{2g.A_i^2}\right].{Q_i}^2$$

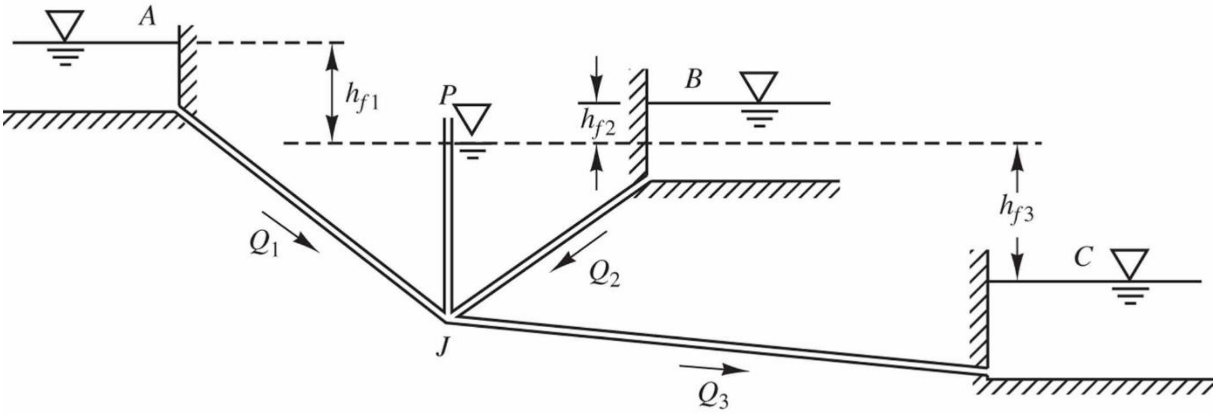
Flowrate for each pipe

h_L in pipe 1 = h_L in pipe 2 = … h_L in pipe N
Unknowns : Qi (i=1, …..N) and hL
Known : Q, Li, ei, Di, Kij

$$Q_i = \sqrt{\frac{hL}{\overline{Ri}}} \rightarrow Q = \sum Q_i = \sqrt{hL} \sum_{i=1}^{N} \frac{1}{\sqrt{\overline{Ri}}} \rightarrow h_L = \left(Q/(\sum \frac{1}{\sqrt{R_i}})\right)^2$$

1. Assume complete turbulence
2. Calculate $\overline{R_i}$
3. Calculate $h_L = \left(Q/(\sum \frac{1}{\sqrt{R_i}})\right)^2$ i = 1, 2, … , N
4. Calculate $Q_i = \sqrt{\frac{h_L}{\overline{R_i}}}$ i=1,….N
5. Calculate new f_i with $V_i = \frac{Q_i}{A_i}$, $R_{ei} = \frac{V_i.D_i}{\upsilon}$
6. If $|f_{new} - f_{old}|$ small => stop, otherwise go to step 2 (Alternatively, iterate until $|Q_{new} - Q_{old}|$ are small enough)

3.3. Branch Piping



Different flowrates and h_L in each pipe

Contunity: $\sum_{i=1}^{N} Q_i = 0$

If flow is into the junction $Q_i > 0$
If flow is from junction to reservoir $Q_i < 0$

Energy Balance

$$h_{Li} = \bar{R}_i . Q_i^2 = \pm \left(\underbrace{[P/\gamma + h]_i}_{Hi} - \underbrace{[P/\gamma + h]_j}_{Hj} \right)$$

$$\bar{R}_i . Q_i^2 = \mathrm{Sign}(H_i - H_j)(H_i - H_j) \qquad \text{or} \qquad \bar{R}_i . Q_i^2 = |H_i - H_j| \quad i = 1, \ldots N$$

Where $\bar{R}_i = R_i + \frac{(\sum K)_i}{2gA_i^2}$ $\qquad R_i = \frac{f_i . L_i}{2g . D_i . A_i^2}$

$$Q_i = \mathrm{Sign}(H_i - H_j) . \sqrt{\frac{|H_i - H_j|}{\bar{R}_i}}$$

Solution Procedure:

1. **1.** Given L_i, D_i, H_i, $(\sum K)_i$ knowns Q_i, H_j is unknown
2. **2.** Find H_j such that the following equation holds:

$$\sum_{i=1}^{N}\left\{\mathrm{Sign}(H_i - H_j).\sqrt{\frac{|H_i - H_j|}{\frac{1}{2gA_i^2}\cdot\left[f_i\cdot\left(\frac{L_i}{D_i}\right)+(\sum K)_i\right]}}\right\} = 0 \qquad (*)$$

$$A_i = \frac{\pi D_i^2}{4}$$

$$f_i = f_i.(e_i/D_i,\ R_{ei})$$

$$R_{ei} = \frac{V_i.D_i}{\upsilon} = \frac{|Q_i|.D_i}{\upsilon.A_i} = \left(\frac{|H_i - H_j|}{\bar{R}_i}\right)^{1/2}.\frac{D_i}{\upsilon.A_i}$$

Hj is the only independent variable, i.e. (*) can be solved using Bisection, Newton Raphson, etc.

Determination of Q_i and f_i for each value H_j

1. Initialize f_i by assuming complete turbulence

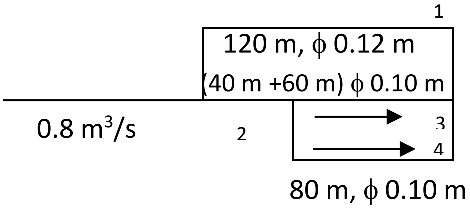
2. Calculate $\bar{R}_i = R_i + \frac{(\sum K)_i}{2gA_i^2}$ $\qquad R_i = \frac{f_i.L_i}{2g.D_i.A_i^2}$

3. $Q_i = \mathrm{Sign}(H_i - H_j).\sqrt{\frac{|H_i - H_j|}{\bar{R}_i}}$ $\qquad R_{ei} = \frac{|Q_i|.D_i}{\upsilon.A_i}$

4. Find new f_i and $\bar{R}_i$

5. If $|f_{new} - f_{old}|$ is small, stop. Otherwise go to step 2.

Example 1: (p. 4.4.5, Hwang 3rd. Edition). Parallel cast iron pipes 1,2, and 3 in Figure below carry a total discharge of 0.8 m^3/s. Determine the flowrate in each pipe. Neglect minor losses.



$(T = 20°C,\ \ e = 0.026\ mm,\ \ \upsilon = 1.003 \times 10^{-6})\ \rho = 998\ kg/m^3$

Pipe	L	D	e(mm)	e/D	Area, m^2
1	120	0.12	0.026	2.16×10^{-3}	0.0113
2	40	0.10	0.026	2.6×10^{-3}	0.00785
3	60	0.10	0.026	2.6×10^{-3}	0.00785
4	80	0.10	0.026	2.6×10^{-3}	0.00785

1. $Q\ =\ 0.8\ m^3/s = Q_1 + Q_2$
2. $Q_2 = Q_3 + Q_4$
3. H_L in pipe 3 = H_L in pipe 4 (pipe 3//pipe 4)
4. H_L in pipe 1 = H_L in pipe 2 + H_L in pipe 3 or H_L in pipe 4 (4 unknowns, 4 equations)

Assuming complete turbulence conditions

$$H_f = R.Q^2$$

$$f = \frac{1.325}{\left[\ln\left(\left(\frac{e}{D}\right)/3.7\right)\right]^2}$$

$f_1 = 2.39 \times 10^{-2}$ $\quad$ $f_2 = 2.51 \times 10^{-2}$ $\quad$ $f_3 = 2.51 \times 10^{-2}$ $\quad$ $f_4 = 2.51 \times 10^{-2}$

1) Resistance coefficient (only friction loss)

2. $\overline{R} = \dfrac{f.L}{2g.D.A^2}$ $\qquad$ $(\sum K_i) = 0$ $\qquad$ $\overline{R}_i = R_i + \dfrac{(\sum K)_i}{2gA_i^{\,2}}$

$\overline{R_1} = 9534$ $\quad H_{L1} = 9534.Q_1{}^2$

$\overline{R_2} = 8310$ $\quad H_{L2} = 8310.Q_2{}^2$

$\overline{R_3} = 12465$ $\quad H_{L3} = 12465.Q_3{}^2$

$\overline{R_4} = 16620$ $\quad H_{L4} = 16620.Q_4{}^2$

H_L in pipe 3 = H_L in pipe 4

$$12465Q_3{}^2 = 1662Q_4{}^2$$

$$Q_3 = 1.15Q_4 \quad Q_4 = 1/(1.15Q_3)$$

From equation 2

$$Q_2 = Q_3 + Q_4$$

$$Q_2 = 1.15Q_4 + Q_4$$

$$Q_2 = 2.15Q_4 \qquad Q_2 = 2.15(1/(1.15Q_3))$$

$$Q_3 = 0.535Q_2$$

From equation 4

$$9534.Q_1{}^2 = 8310.Q_2{}^2 + 12465.Q_3{}^2$$

$$9534.Q_1{}^2 = 8310.Q_2{}^2 + 12465.(0.535Q_2)^2$$

$$9534.Q_1{}^2 = (8310 + 3568).Q_2{}^2 => \quad Q_1 = 1.116Q_2$$

From equation 1

$$0.8 = Q_1 + Q_2$$

$$0.8 = 1.116Q_2 + Q_2$$

$$Q_2 = 0.378 \text{ m}^3/\text{s}$$

$$Q_1 = 0.8 - 0.378 = 0.422 \text{ m}^3/\text{s}$$

From here

$$Q_3 = 0.505Q_2 = 0.202 \text{ m}^3/\text{s}$$

$$Q_4 = 0.175 \text{ m}^3/\text{s}$$

Check complete turbulence

	Pipe 1	Pipe 2	Pipe 3	Pipe 4
Q	0.422	0.378	0.202	0.175
V	37.31	48.13	25.72	22.28
Re	4.46×10^6	4.798×10^6	2.564×10^6	2.22×10^6
f	0.0239	0.025	0.025	0.025
Diff	0	0	0	0
$\bar{\mathbf{R}}_\mathbf{i}$	9549	8325	12508	16687

$$Q_1 = 0.422 \qquad Q_3 = 0.202$$
$$Q_2 = 0.378 \qquad Q_4 = 0.175$$