LECTURE 9: Open channel flow: Uniform flow, best hydraulic sections, energy principles, Froude number

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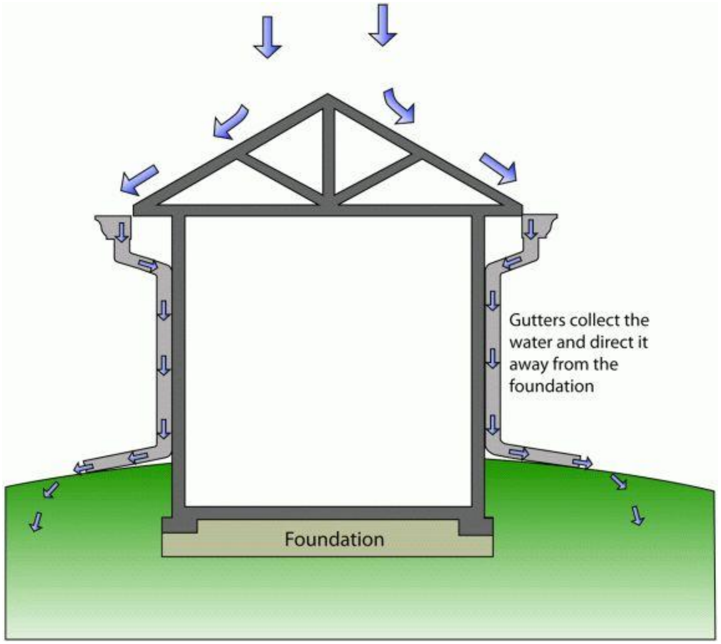
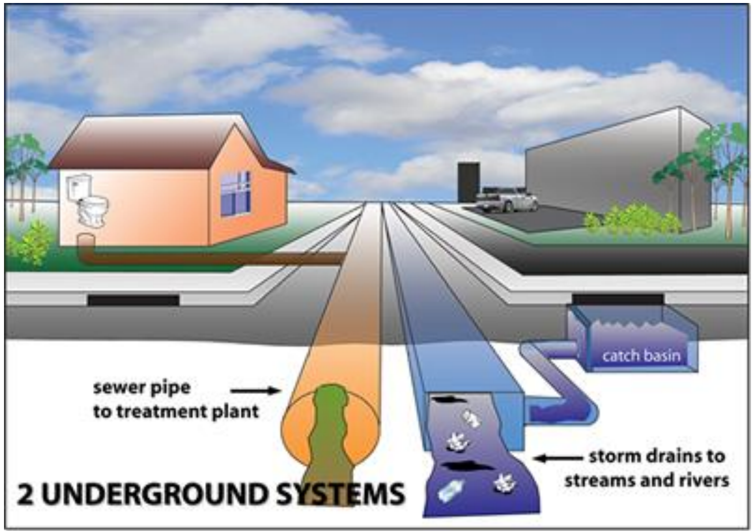
Open channel flow must have a free surface.

Normally free water surface is subjected to atmospheric pressure, which remains relatively constant throughout the entire length of the channel



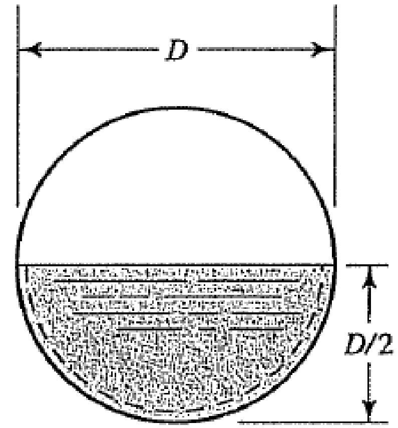
rivers

Rain gutters



Storm and sanitary sewers

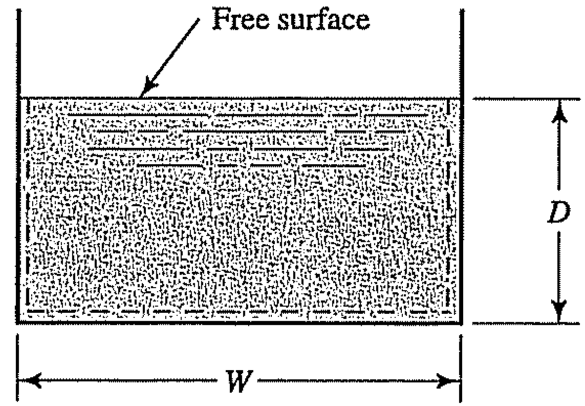
CROSS SECTIONS OF OPEN CHANNEL FLOW



$A = \pi D^2/8$

$WP = \pi D/2$

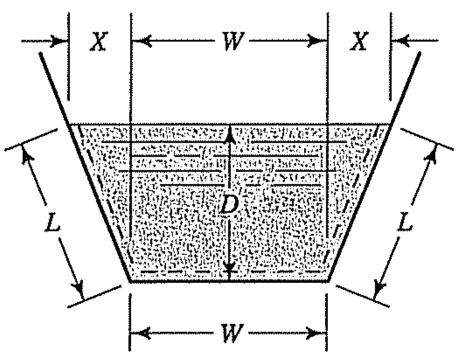
(a) Circular pipe
running half full



$A = WD$

$WP = W + 2D$

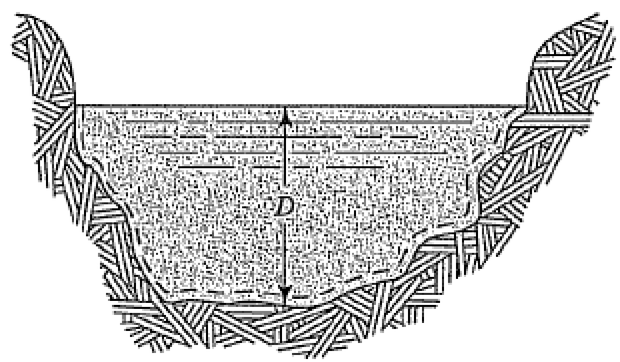
(b) Rectangular
channel



$A = WD + XD$

$WP = W + 2L$

(c) Trapezoidal
channel



A and WP irregular

(d) Natural channel

CLASSIFICATION OF OPEN CHANNEL FLOW

Time as criterion

Steady flow:
Depth of flow does not change with time

Unsteady flow:
Depth of flow does change with time

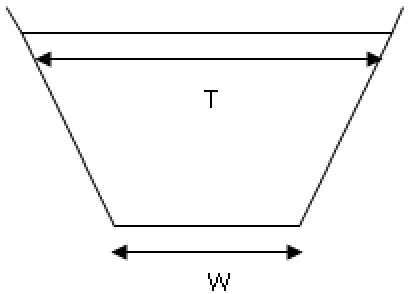
Space as criterion

Uniform flow:
Depth of flow is same at every section

Varied flow:
Depth of flow changes along the length

- Rapidly varied flow
- Gradually varied flow

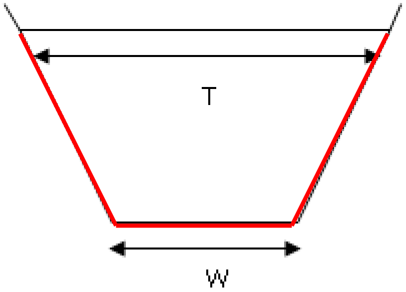
IMPORTANT TERMS IN OPEN CHANNEL FLOW



Top width of the channel = T

Width of the channel = W

Wetted Perimeter (P)



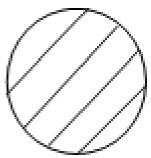
Hydraulic Radius (R)= $\frac{\text{Flow area}}{\text{Wetted perimeter}}$

Hydraulic Depth (D) = $\frac{\text{Flow Area}}{\text{Top Width}}$

Bottom Slope , (S_o)

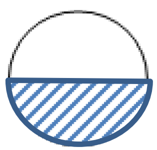
Side Slope, z

HYDRAULIC RADIUS (R)



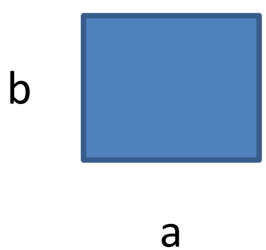
Full flow

Hydraulic radius for full flow ➔ $R = \frac{\pi.D^2/4}{\pi.D} = \frac{D}{4}$

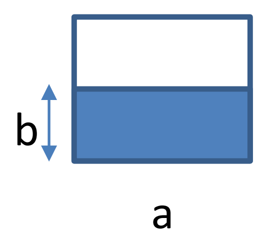


Half - full flow

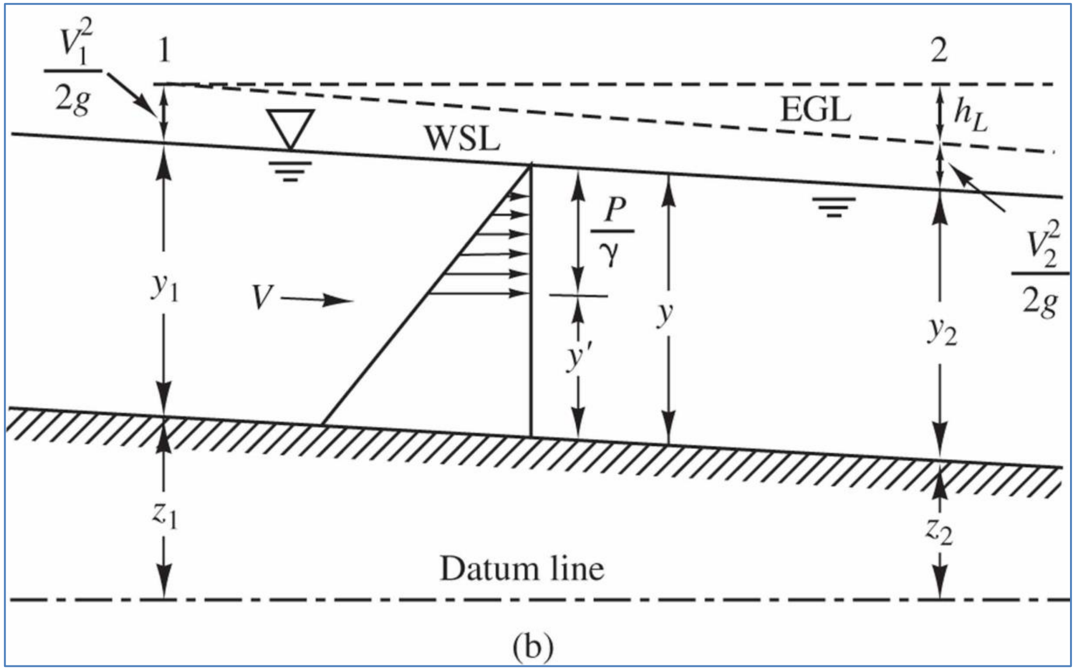
Hydraulic radius for half-full flow ➔ $R = \frac{(\pi.D^2/4)/2}{\pi.D^2/2} = \frac{D}{4}$



$R = \frac{(a \times b)}{2a+2b}$



$R = \frac{(a \times b)}{2b+a}$

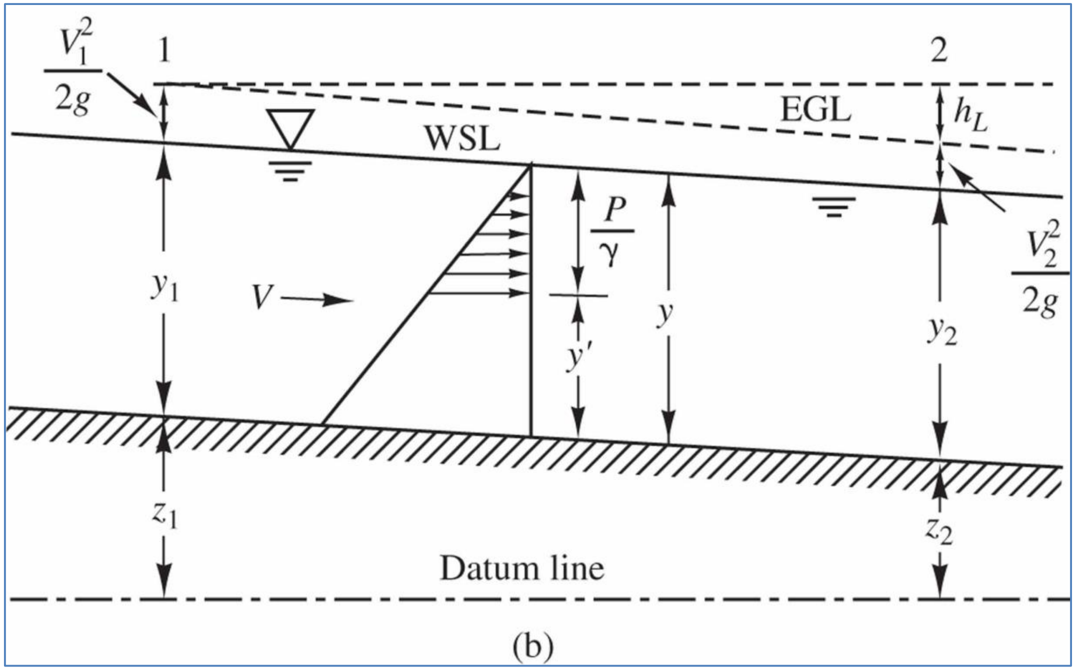


Open channel flow must have a free surface.

Normally free water surface is subjected to atmospheric pressure, which remains relatively constant throughout the entire length of the channel.

$$z_1 + \frac{V_1{}^2}{2g} + \frac{P_1}{\gamma} = z_2 + \frac{V_2{}^2}{2g} + \frac{P_2}{\gamma} + h_L$$

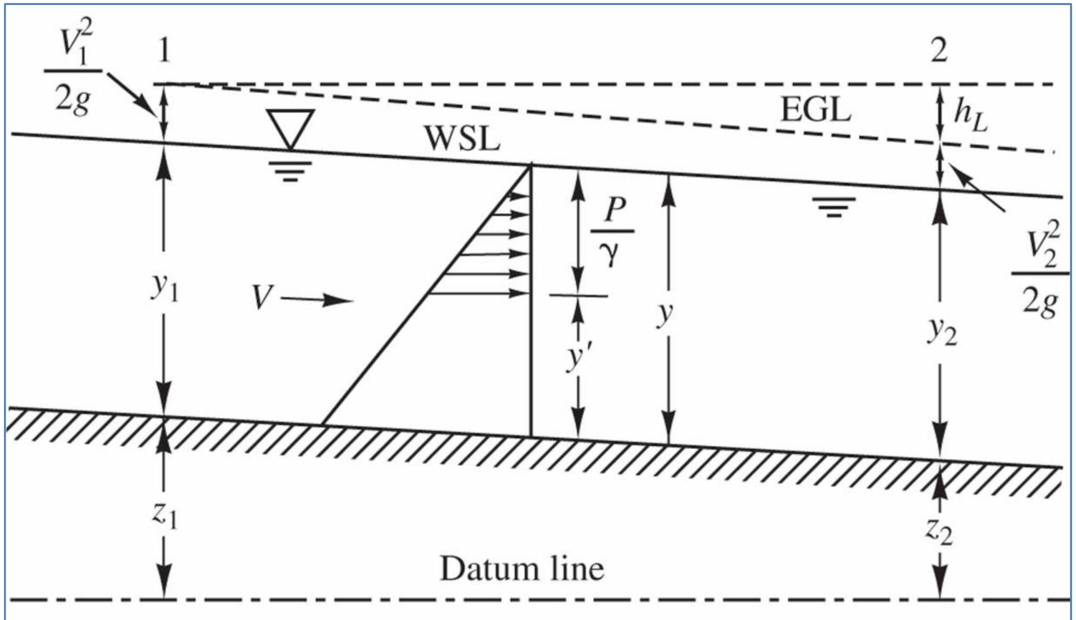
ENERGY EQUATION IN OPEN CHANNEL



The pressure distribution in any section is directly proportional to the depth measured from the free water surface.
In this case, the water surface lines corresponds to the hydraulic gradient line in pipe flow.

$$z_1 + \frac{V_1^2}{2g} + y_1 = z_2 + \frac{V_2^2}{2g} + y_2 + h_L$$

UNIFORM STEADY FLOW IN OPEN CHANNEL



(b)

Fluid surface is parallel to the slope of the channel bottom.

Slope of the fluid surface (Sw) // slope of the channel bottom (S)

Slope of the channel should be constant. If the cross section or slope of the channel is changing then varied flow will occur.

ANALYSIS OF OPEN CHANNEL FLOW

- Chezy Equation,
- Manning's Equation (derived from Chezy Equation)
- Uniform flow equations: flow area does not change with the length of channel

CHEZY EQUATION

First Formula for uniform open channel flow.

$$V = C.\sqrt{R.S}$$

Velocity m/s

Chezy's constant

S = Slope of HGL for uniform flow $S = S_o$

$$C = \frac{(23+(0.00155/S))+(1/n)}{(1+n/R).(23+(0.00155/S))}$$

$$C_m = \frac{100\sqrt{R}}{m+\sqrt{R}}$$

m = Depends on pipe material

m = 0.35 for concrete pipe

m = 0.25 for vitrified clay pipe

Emprical relation for Chezy's constant

$$V = (\frac{1}{n}.R^{1/6}).R^{1/2}.S^{1/2}$$

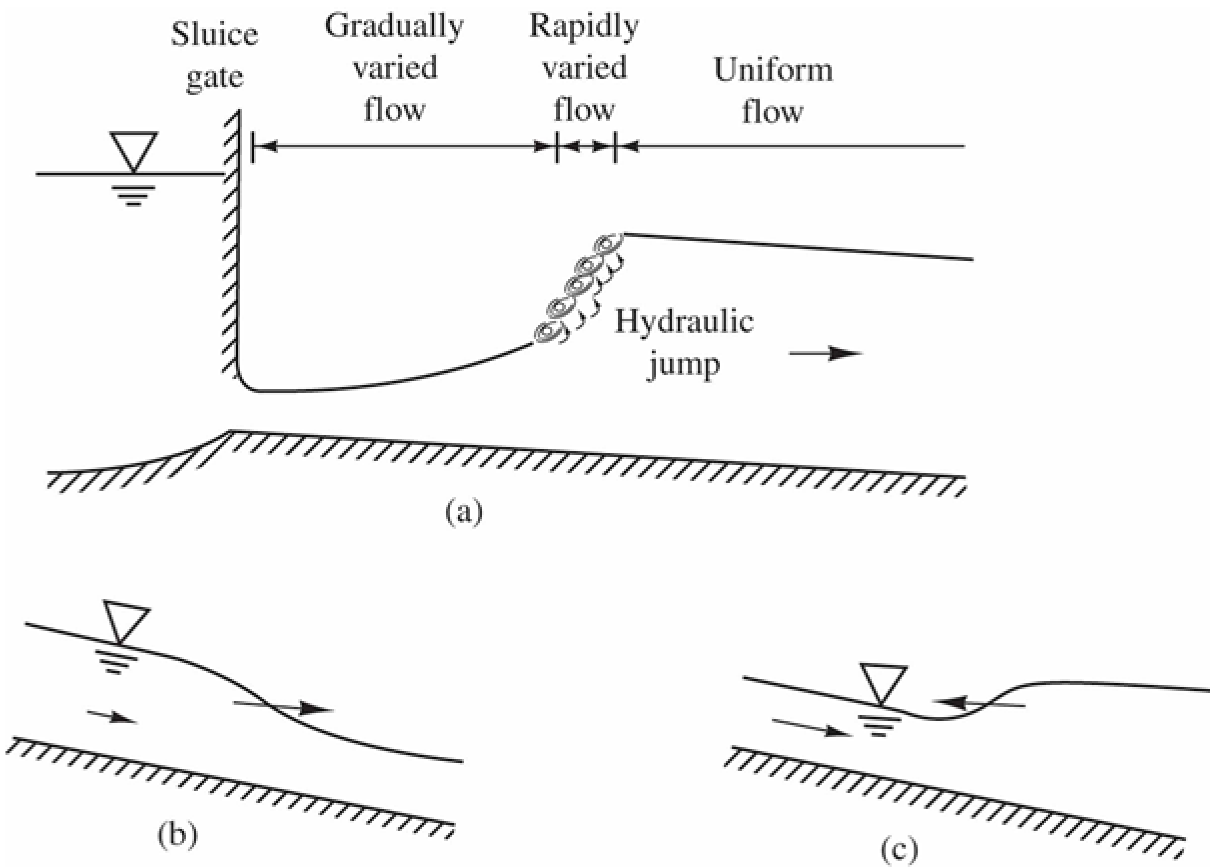
$$V = \frac{1}{n}.R^{2/3}.S^{1/2} \longrightarrow \text{Slope of EGL} = \frac{h_L}{L}$$

Velocity m/s

Manning's Constant

Hydraulic Radius

Figure 6.2 Classifications of open-channel flow: (a) gradually varied flow (GVF), rapidly varied flow (RVF), and uniform flow (UF); (b) unsteady varied flow; (c) unsteady varied flow



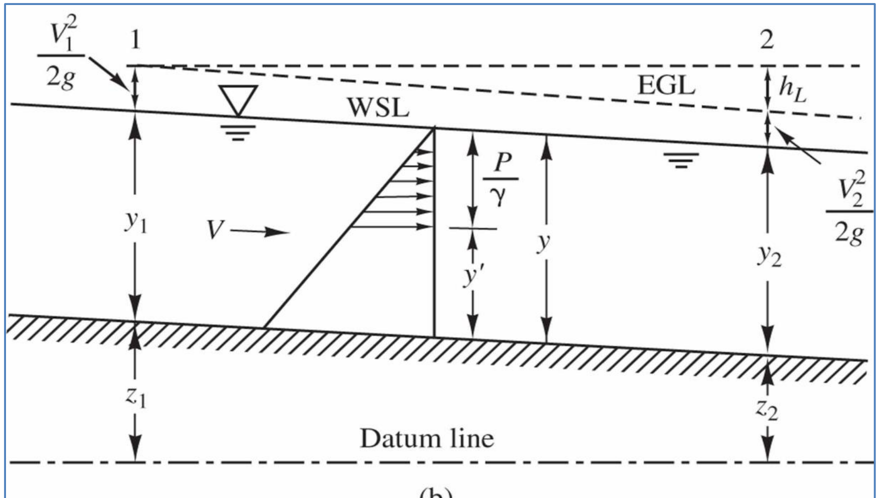
UNIFORM FLOW IN OPEN CHANNEL

Uniform Flow in Open Channels

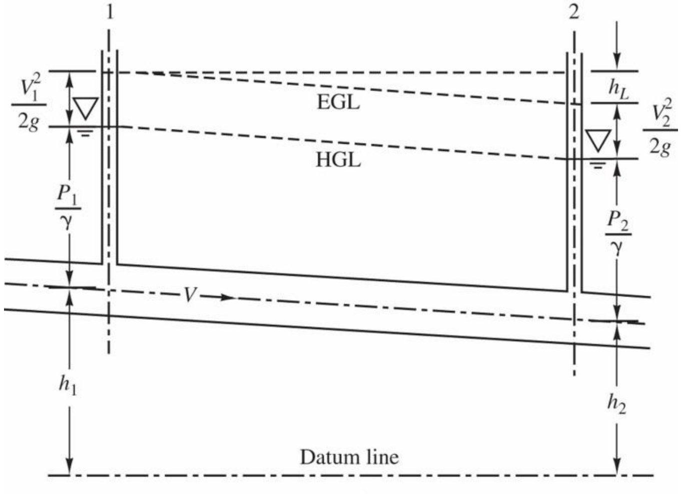
1. Water depth, flow area, discharge and velocity must remain unchanged in all sections of the entire channel.
2. EGL, the water surface, the channel bottom must be parellel to each other

$$S_e = S_{w.s.} = S_o$$

Slope of EGL — Slope water surface — Slope of Channel



(b)



(a)

ENERGY PRINCIPLES IN OPEN CHANNEL FLOW

Energygy contained in a unit weight of water flowing in an open channel may also be measured in three basic forms:

1. Kinetic Energy
2. Pressure Energy
3. Elevation (Potential) Energy above a certain datum line.

Total energy in open channel

$$H=z_{\square} + \frac{V_{\square}^{2}}{2g} + y_{\square}$$

Specific Energy (E) ➔ Energy with respect to channel bottom

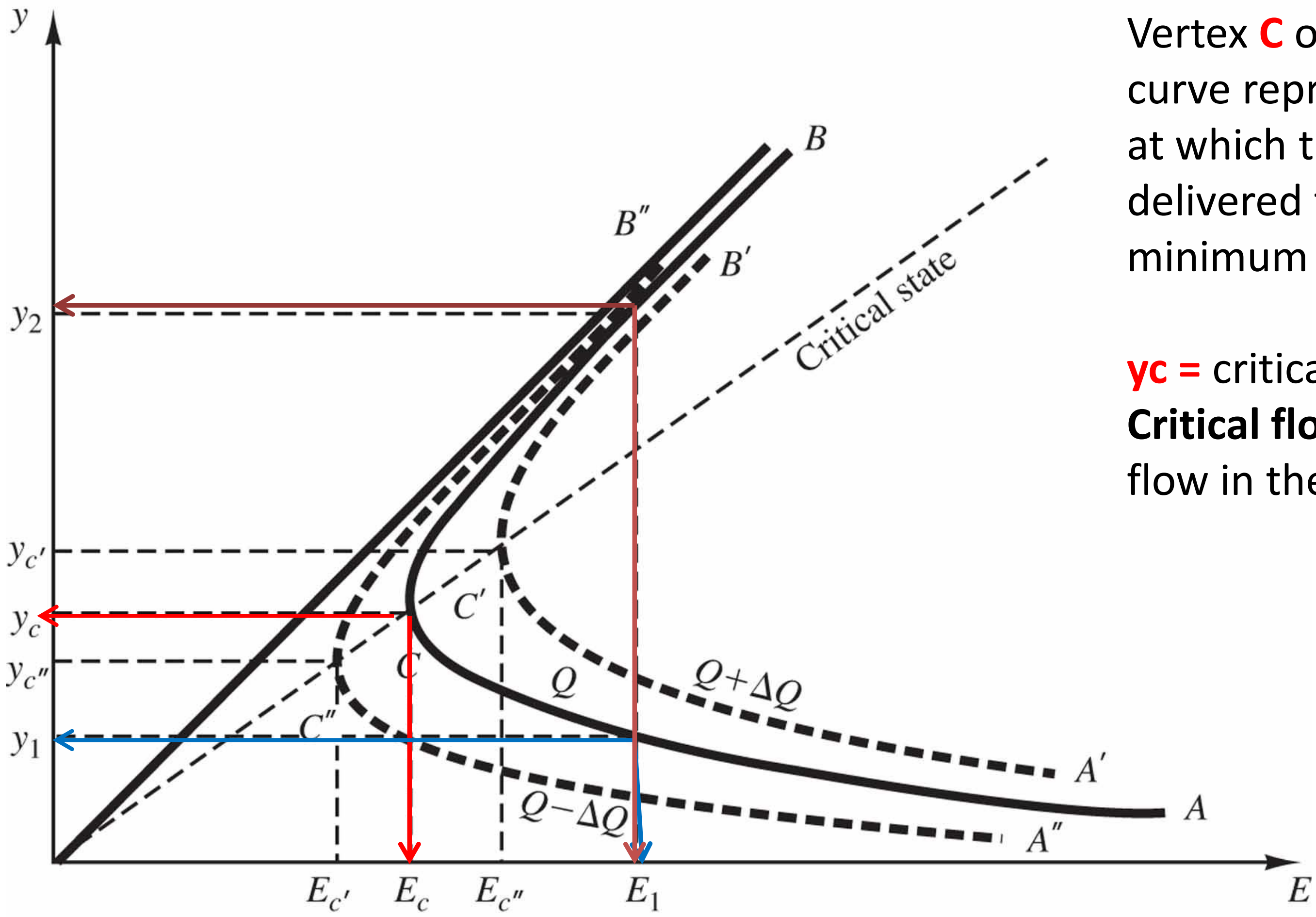
$$E = y_1 + \frac{V_1^{2}}{2g}$$

$$E = \frac{Q^2}{2.g.A^2} + y$$

For a given water area and discharge

For a given discharge, Q, specific energy (E) at any section is a function of depth of flow only.

Specific energy curves of different discharges at a given channel section



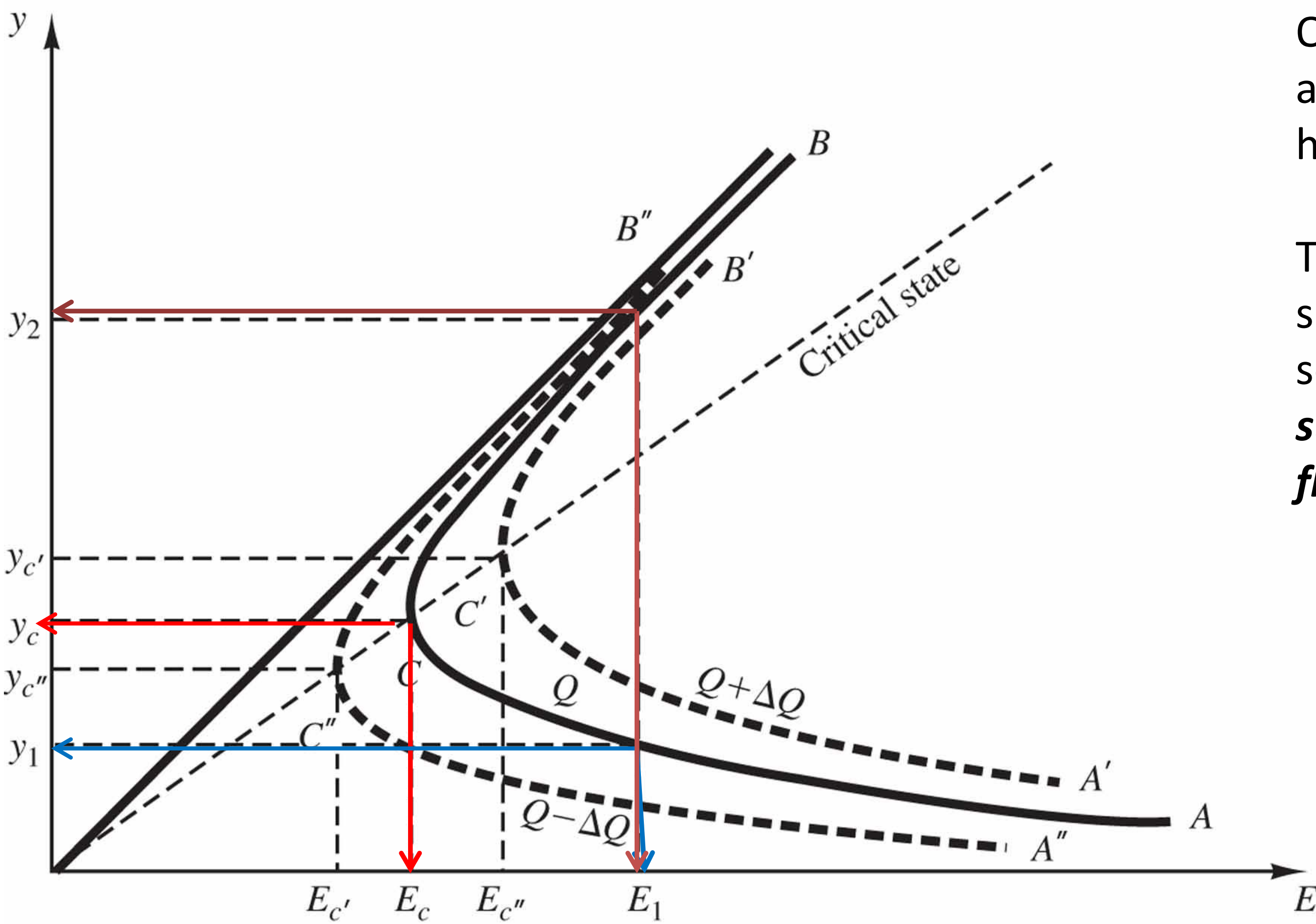
Vertex **C** on a specific energy curve represents the depth **(yc)** at which the discharge Q may be delivered through the section at minimum energy, $\mathbf{E_c}$.

yc = critical depth for the **Critical flow**= The corresponding flow in the section

supercritical flow or rapid flow

subcritical flow.

Specific energy curves of different discharges at a given channel section



At a smaller depth, y1 Q can be delivered only by a higher velocity and, a higher specific energy.

The state of rapid and shallow flow through a section is known as ***supercritical flow or rapid flow.***

At a larger depth the same discharge may be delivered through the section with a smaller velocity and a higher specific energy than a critical depth. It is known as ***subcritical flow.***

For a given value of specific energy, E1, the discharge may pass through the channel section at either depth d1 (supercritical flow) or d2 (subcritical flow). These two depths known as alternate depths.

FROUDE NUMBER

.

$$\frac{dE}{dy} = \frac{d}{dy}.\left(\frac{Q^2}{A^2.g} + y\right)$$

$$\frac{-Q^2}{g.A^3}.\frac{dA}{dy} + 1 = 0$$

$\frac{dA}{dY} = T$

$$\frac{-Q^2}{g.A^3}.T + 1 = 0$$

A/T = D ➔Hydraulic Depth for rectangular sections

D = y for rectangular cross sections

$$\frac{dE}{dy} = 1 - \frac{Q^2}{g.DA^2} = 1 - \frac{V^2}{g.D} = 0$$

$$\text{or } \frac{V}{\sqrt{g.D}} = 1 = \frac{\text{inertial force}}{gravity\ force}$$

$$\text{Froude number, } N_F = \frac{V}{\sqrt{g.D}}$$

FROUDE NUMBER

F = 1 Flow is in critical state

F < 1 Subcritical state

F > 1 Supercritical state

$$\frac{Q^2}{g} = \frac{A^3}{T} = D.A^2 \qquad D = y \qquad A = b.y$$

$$\frac{Q^2}{g} = \frac{(b.y)^3}{T} = y.A^2 \quad \frac{Q^2}{g} = y^3.b^2$$

Unit discharge

$$q = \frac{Q}{b} = \frac{\text{Flowrate}}{\text{width ofchannel}}$$

$$y_c = \sqrt[3]{\frac{Q^2}{b^2.g}} \qquad\qquad y_c = \sqrt[3]{\frac{q^2}{g=9.8}}$$

Table 6.1 Cross-Sectional Relationships for Open-Channel Flow

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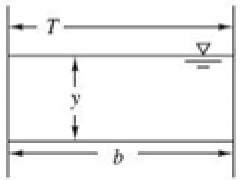
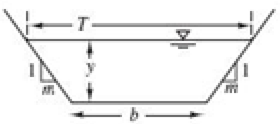
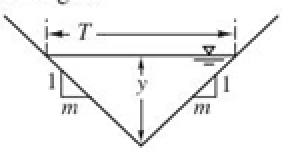
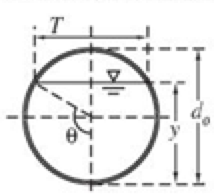
Section Type	Area (A)	Wetted perimeter (P)	Hydraulic Radius (R_h)	Top Width (T)	Hydraulic Depth (D)
Rectangular 	by	$b + 2y$	$\frac{by}{b + 2y}$	b	y
Trapezoidal 	$(b + my)y$	$b + 2y\sqrt{1 + m^2}$	$\frac{(b + my)y}{b + 2y\sqrt{1 + m^2}}$	$b + 2my$	$\frac{(b + my)y}{b + 2my}$
Triangular 	my^2	$2y\sqrt{1 + m^2}$	$\frac{my}{2\sqrt{1 + m^2}}$	$2my$	$\frac{y}{2}$
Circular (θ is in radians) 	$\frac{1}{8}(2\theta - \sin 2\theta)d_0^2$	θd_0	$\frac{1}{4}\left(1 - \frac{\sin 2\theta}{2\theta}\right)d_0$	$(\sin\theta)d_0$ or $2\sqrt{y(d_0 - y)}$	$\frac{1}{8}\left(\frac{2\theta - \sin 2\theta}{\sin\theta}\right)d_0$

Source: V. T. Chow, *Open Channel Hydraulics* (New York: McGraw-Hill, 1959).

Table 6.2 Typical Values of Manning's *n*

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Channel Surface	*n*
Glass, PVC, HDPE	0.010
Smooth steel, metals	0.012
Concrete	0.013
Asphalt	0.015
Corrugated metal	0.024
Earth excavation, clean	0.022–0.026
Earth excavation, gravel and cobbles	0.025–0.035
Earth excavation, some weeds	0.025–0.035
Natural channels, clean and straight	0.025–0.035
Natural channels, stones or weeds	0.030–0.040
Riprap lined channel	0.035–0.045
Natural channels, clean and winding	0.035–0.045
Natural channels, winding, pools, shoals	0.045–0.055
Natural channels, weeds, debris, deep pools	0.050–0.080
Mountain streams, gravel and cobbles	0.030–0.050
Mountain streams, cobbles and boulders	0.050–0.070

Figure 6.4 Normal depth solution procedure: (a) trapezoidal channels (*m* side slope)

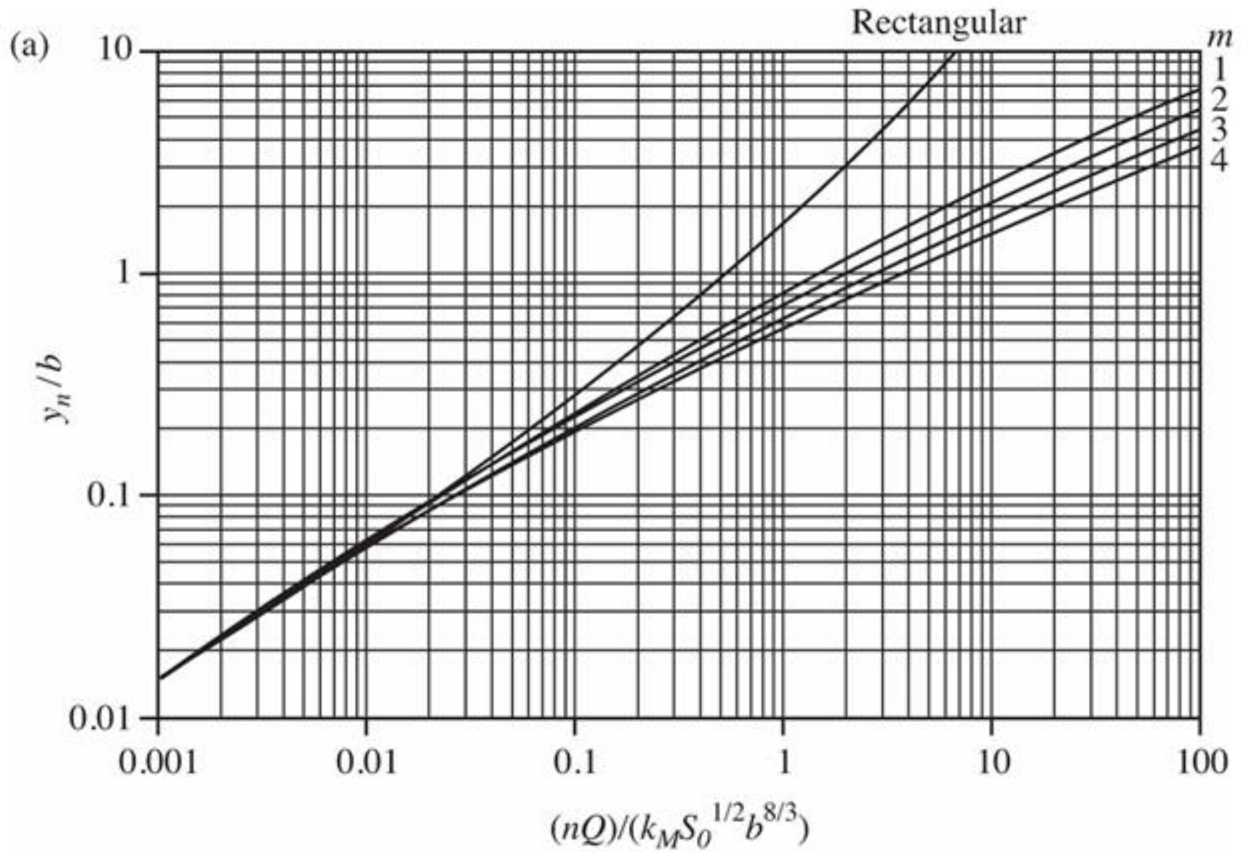


Figure 6.4 (continued) Normal depth solution procedure: (b) circular channels (d_0 diameter)

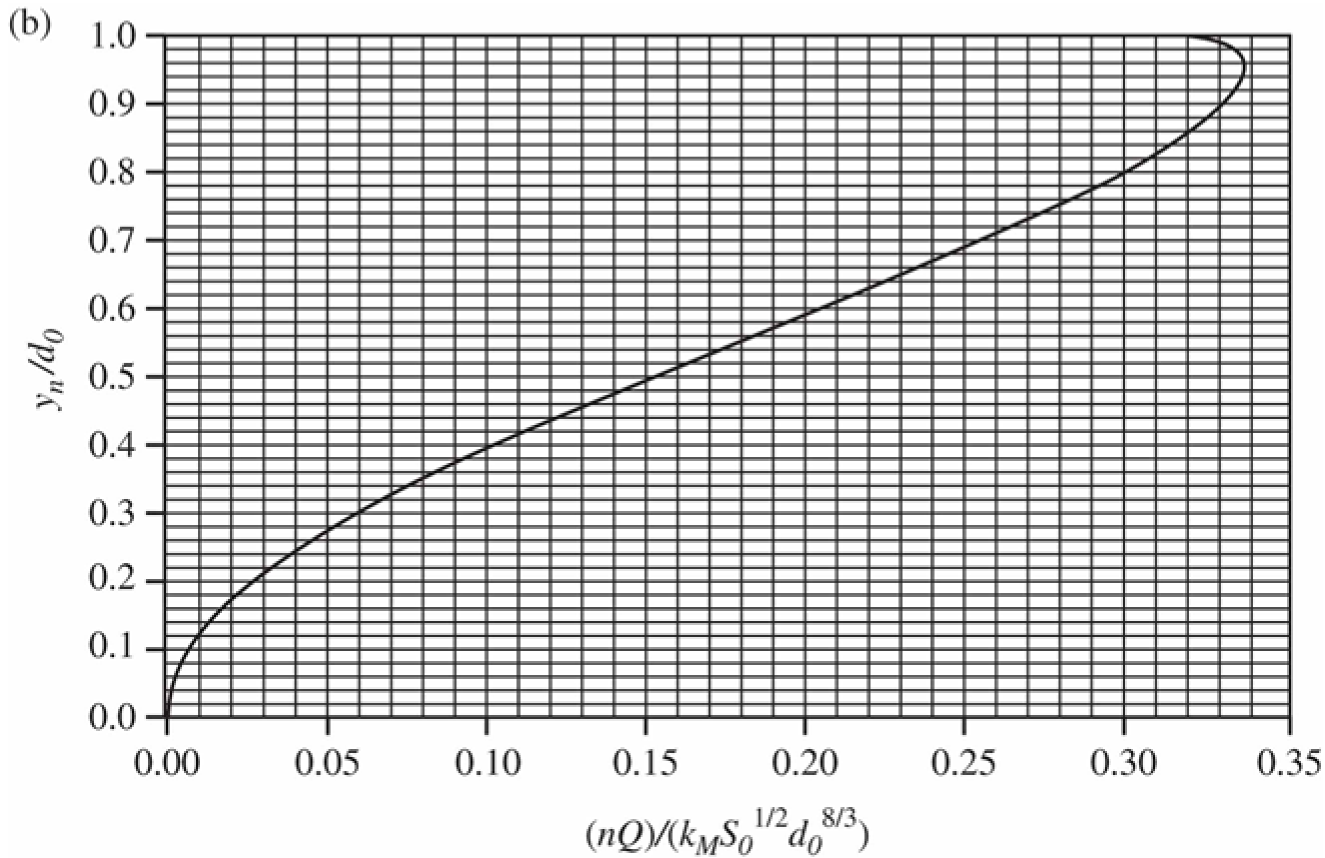


Figure 6.9 Critical depth solution procedure: (a) trapezoidal channels

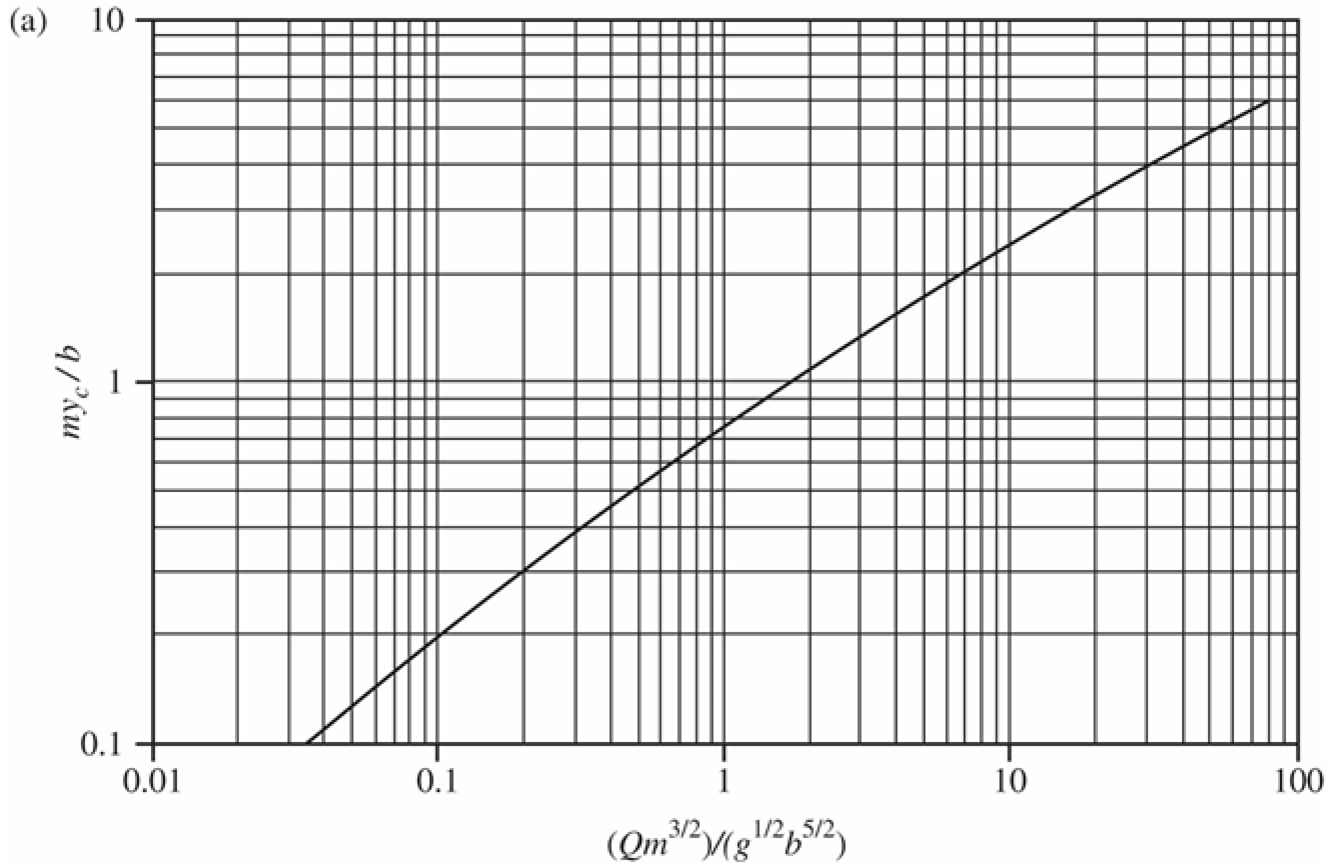
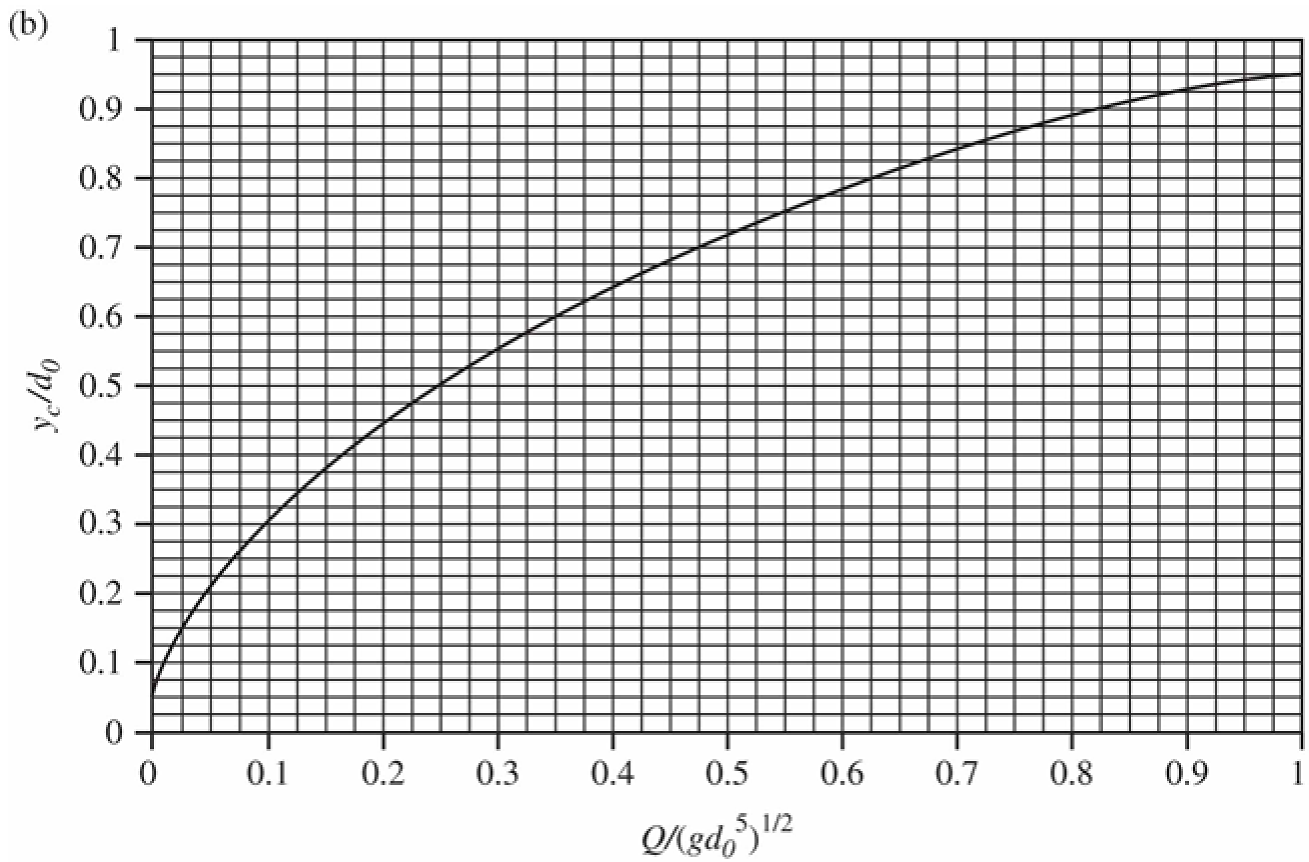
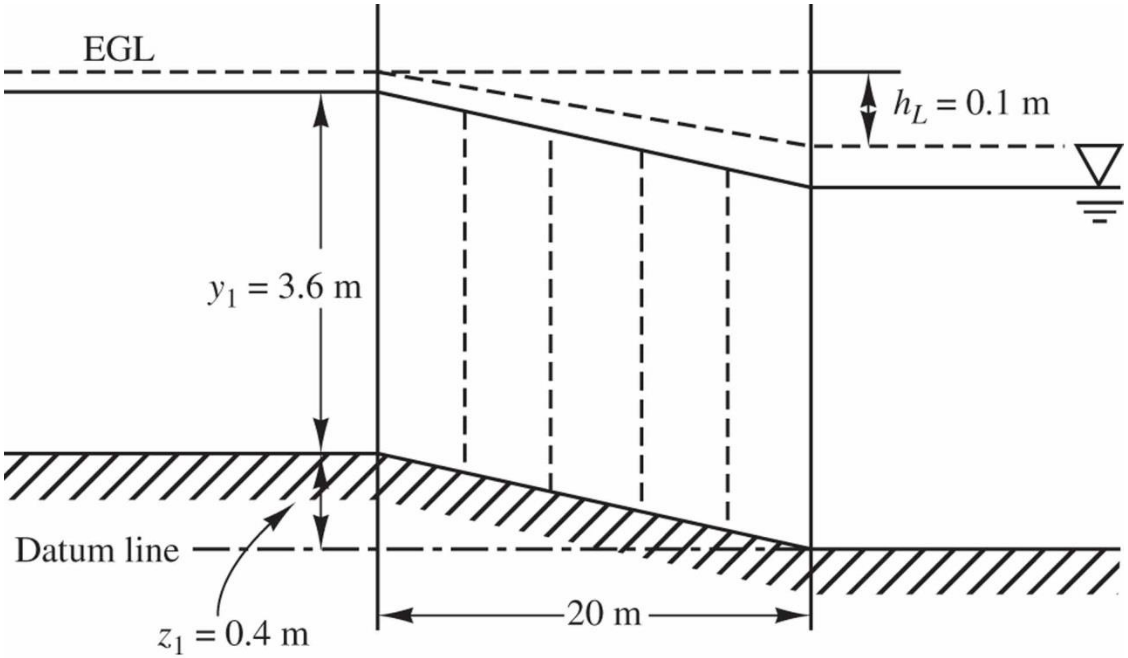


Figure 6.9 (continued) Critical depth solution procedure: (b) circular channels



Example 6.5

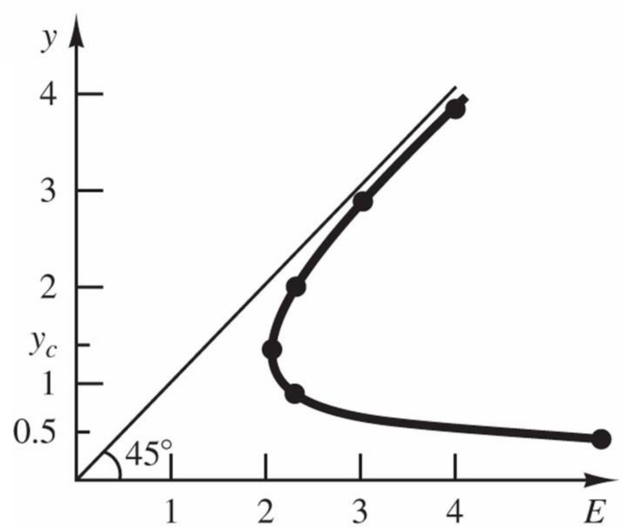
$$E = \frac{1.27}{y^2} + y$$



(a)

Y= 0.5→ $E = \frac{1.27}{0.5^2} + 0.5$=5.60

Y= 4→ $E = \frac{1.27}{4^2} + 4$=4.07



(b)

$y_c = \sqrt[3]{\frac{5^2}{g}} = 1.37$ m

E_c(m) = 2.05 m

y (m)	E (m)
0.5	5.60
1.0	2.27
2.0	2.32
3.0	2.32
3.0	3.14
4.0	4.07