ME 262 BASIC FLUID MECHANICS

Assistant Professor Neslihan Semerci

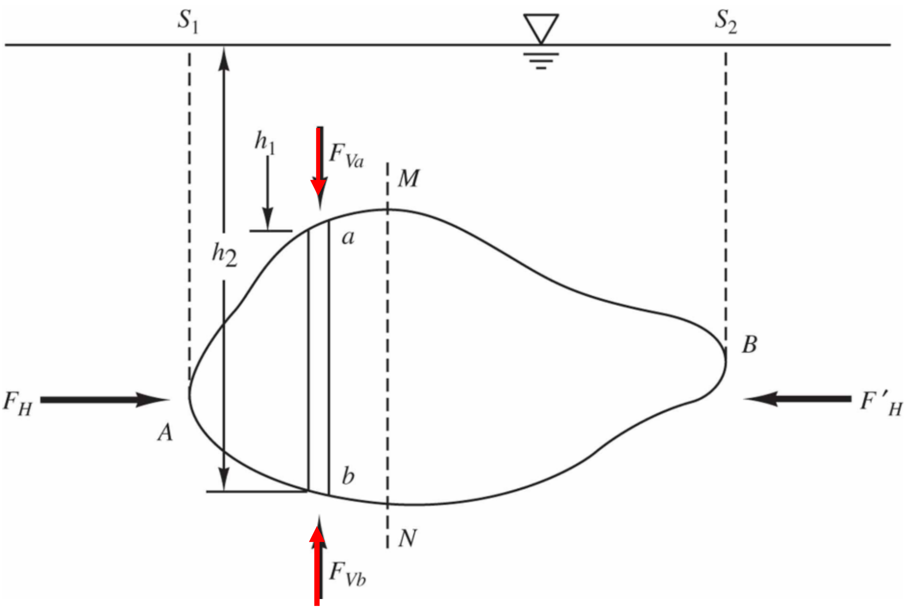
Lecture 4

(Buoyancy and Viscosity of water)

16. BUOYANCY

Whenever an object is floating in a fluid or when it is completely submerged in the fluid, it is subjected to a force called the "buoyant" force. Buoyancy is the tendency of a fluid to exert a supporting force on a body placed in the fluid.

Archimedes discovered that the weight of a submerged body is reduced by an amount equal to the weight of the liquid displaced by the body.



$F_H = F'_H$ ➔ Both forces are calculated using the same vertical projection area MN.

Horizontal pressure force components in the direction normal to the page must be also equal for the same reason. They share the same projection in the plane of the page.

Vertical pressure force component:

Vertical pressure force on the top of the strip = $\gamma_{water} h_1 dA$ (downward)

Vertical pressure force on the bottom of the strip = $\gamma_{water} h_2 dA$ (upward)

Resultant vertical force component = $F_V = \gamma h_2 dA - \gamma h_1 dA$

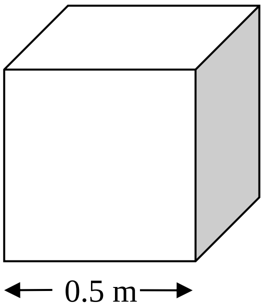
$$= \gamma(h_2 - h_1)dA \uparrow$$

F_V = Weight of the water column "ab" replaced by the prism.

$$F_b = \gamma_{liquid} \forall_{displaced}$$

$$F_b = F_V$$

Example16.1

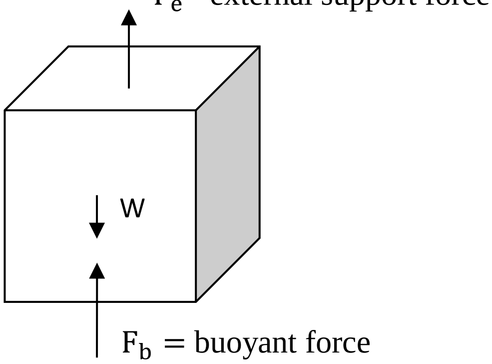


A cube, 0.50 m on a side, is made of bronze having a specific weight of 86.9 kN/m^3 . Determine the magnitude and direction of the force required to hold the cube in equilibrium completely.

a) in water

b) in mercury, $(s.g)_{mercury}$=13.54

Assume that the bronze cube will not stay in equilibrium itself, some external force is required



$\sum F_V = 0$ in equilibrium

$F_b + F_e - W = 0$

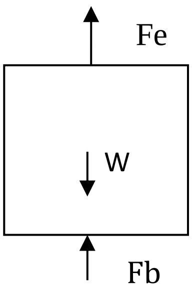
$F_e = W - F_b$

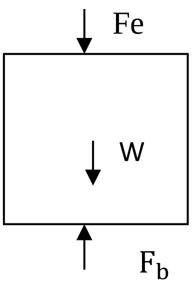
$$W_{cube} = \gamma_B \forall = (86.9\ kN/m^3)(0.5)^3 = 10.86\ \ kN$$

$$F_b = \gamma_f \forall_d = (9.81\ kN/m^3)(0.125) = 1.23\ \ kN$$

$$F_e = 10.86 - 1.23 = 9.63\ kN$$

An upward force of 9.63 kN is required to hold the block in equilibrium under water.





Assume cube would sink

$F_b + F_e - W = 0$

$F_e = W - F_b$

Assume cube would float

$F_b - F_e - W = 0$

$F_e = F_b - W$

Part b) in mercury; $W = \gamma_B \forall = (86.9\ \mathrm{kN/m^3})(0.125\ \mathrm{m^3}) = 10.86\ \mathrm{kN}$

$$F_b = \gamma_m \forall = (s.g)_m \gamma_w \forall = (13.54)(9.81\ \mathrm{kN/m^3})(0.125\ \mathrm{m^3})$$

$$F_b = 16.60\ \mathrm{kN}$$

$$F_e = W - F_b \qquad\qquad F_e = F_b - W$$

$$F_e = 10.86\ \mathrm{kN} - 16.60 \qquad\qquad Fe = 16.60 - 10.86$$

$$= -5.74\ \mathrm{kN} \qquad\qquad = +5.74\ \mathrm{kN}$$

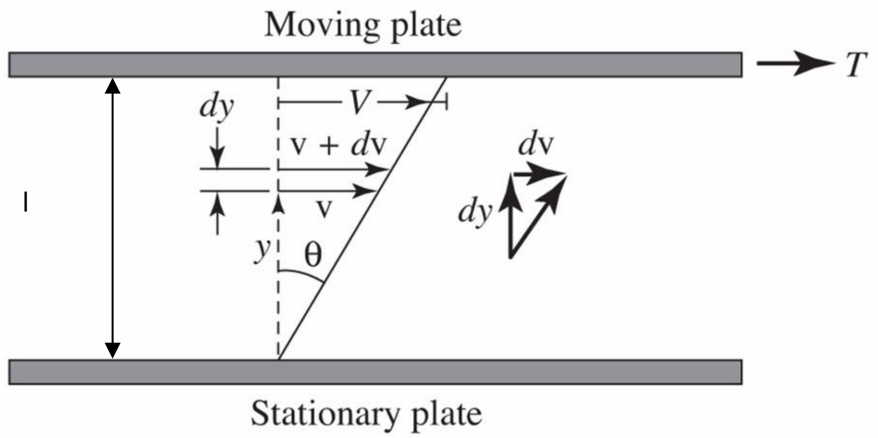
The direction of the force should be ↓ downward

Note: Both solutions yield the same numerical value, but they have opposite signs. The negative sign for the solution on the left means the assumed direction for F_e in the case of cube sink was wrong. Therefore both approaches give the same result.

17. VISCOSITY

When two solid bodies in contact move relative to each other, a friction force develops at the contact surface in the direction opposite to motion. The situation is similar when a fluid moves relative to a solid or when two fluids move relative to each other.

Viscosity is the property that represents the internal resistance of fluid to motion.



In steady operation, the fluid velocity between plates varies linearly between 0 and V.

Velocity Profile

$$u(y) = \frac{y}{l} \times V$$

Velocity Gradient

$$\frac{du}{dy} = \frac{V}{l}$$

y = vertical distance from the lower plate

After a differential time interval, dt

$d\beta$ = differential angle

da = differential distance

$$da = V \times dt$$

$$d\beta \cong \tan\beta = \frac{da}{l} = \frac{Vdt}{l} = \frac{du}{dy}\,dt$$

$d\beta$ → Angular deformation

$\frac{du}{dy}$ → Velocity gradient

$$\frac{d\beta}{dt} = \frac{du}{dy}$$

$\frac{d\beta}{dt}$ → rate of deformation

$\frac{du}{dy}$ → velocity gradient

It’s verified experimentally that for most fluids, rate of deformation is directly proportional to shear stress.

$\tau \propto \frac{d\beta}{dt}$ or $\tau \propto \frac{du}{dy}$

Newtonian Fluids

The rate of deformation is proportional to the shear stress are called Newtonian fluids.

Water, air, gasoline and oils are Newtonian Fluids

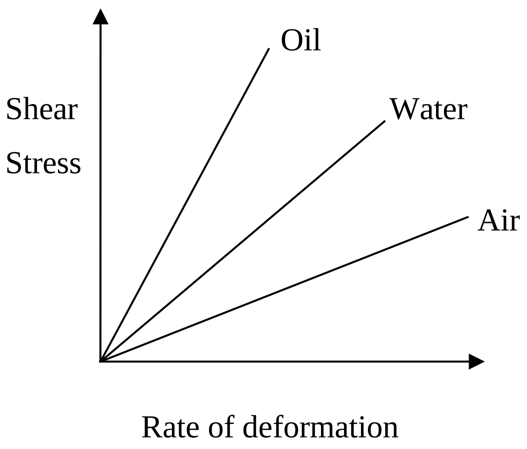
velocity gradient

$\tau = \mu \times \frac{du}{dy}$

dynamic viscosity of the fluid

$\mu \rightarrow kg/(m.s)$

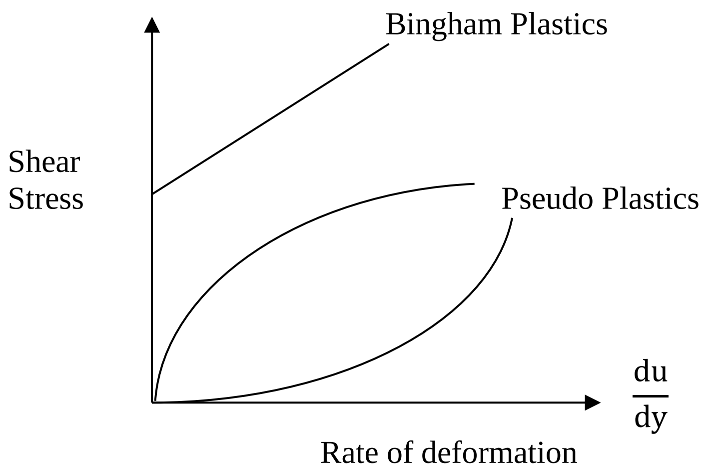
$\mu \rightarrow N.s/m^2$

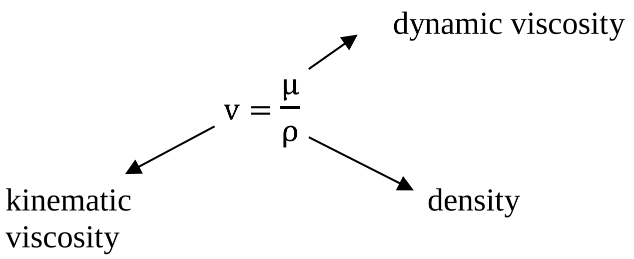


$$\mu = \frac{\tau}{du/dy} = \frac{y}{x}$$

Non-Newtonian Fluids

The relationship between rate of deformation and shear stress is not linear.



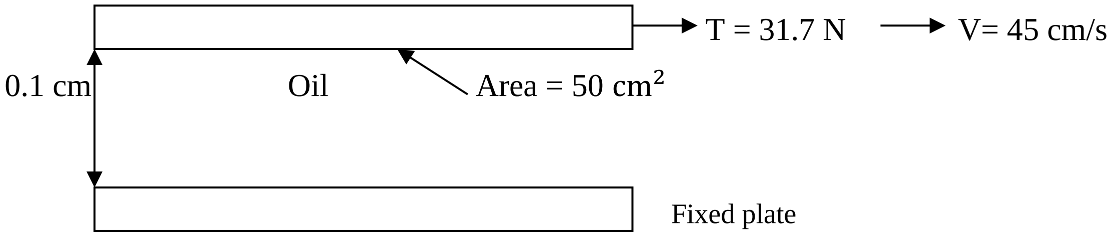


Viscosity depends on both temperature and pressure, for water viscosity changes with temperature.

The viscosity of liquids decreases with temperature,

The viscosity of gases increases with temperature

Example 17.1(Example 1.2, Hwang, 4th Edition) A flat plate of 50 cm^2 is being pulled over a fixed flat surface at a constant velocity of 45 cm/s. An oil film of unknown viscosity separates the plate and the fixed surface by a distance of 0.1 cm. The force (T) required to pull the plate is measured to be 31.7 N, and the viscosity of the fluid is constant. Determine the viscosity (absolute).



Solution:

$$\tau = \mu \times \frac{du}{dy} = \mu \times \frac{v}{l}$$

$$\text{Force} = \tau \times A$$

$$\text{Force} = (\mu \times \frac{v}{l}) \times (\text{Area})$$

$$31.7\ \text{N} = \mu \times \left(\frac{45\ \text{cm/sec}}{0.1\ \text{cm}}\right) \times (50\ \text{cm}^2)$$

$$\mu = \frac{31.7\ \text{N}}{\left(\frac{45\ \text{cm/sec}}{0.1\ \text{cm}}\right) \times (50\ \text{cm}^2) \times \frac{1\ \text{m}^2}{100\ \text{cm}^2}}$$

$$\mu = 141\ \text{N.sec/m}^2$$