ME 262 BASIC FLUID MECHANICS

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Lecture 5

(Pipe Flow, Reynolds Number, Forces in Pipe Flow)

18. WATER FLOW IN PIPES

18.1. Important Definitions

Pressure Pipe Flow: Refers to full water flow in closed conduits of circular cross sections under a certain pressure gradient. For a given discharge (Q), pipe flow at any location can be described by the pipe cross section, the pipe elevation, the pressure, and the flow velocity in the pipe.

Elevation (h) of a particular section in the pipe is usually measured with respect to a horizontal reference datum such as mean sea level (MSL).

Pressure (P) in the pipe varies from one point to another, but a mean value is normally used at a given cross section.

Mean velocity (V) is defined as the discharge (Q) divided by the cross-sectional area (A)

$$V = \frac{Q}{A}$$

18.2. The Reynolds Number

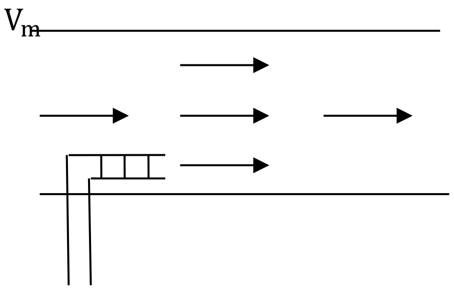
Reynold's Number: It is a dimensionless number to characterize the flow.

Reynolds Number

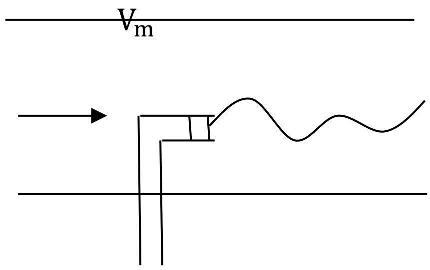
Laminar Flow | Turbulent Flow

Smooth Layers, highly ordered motion

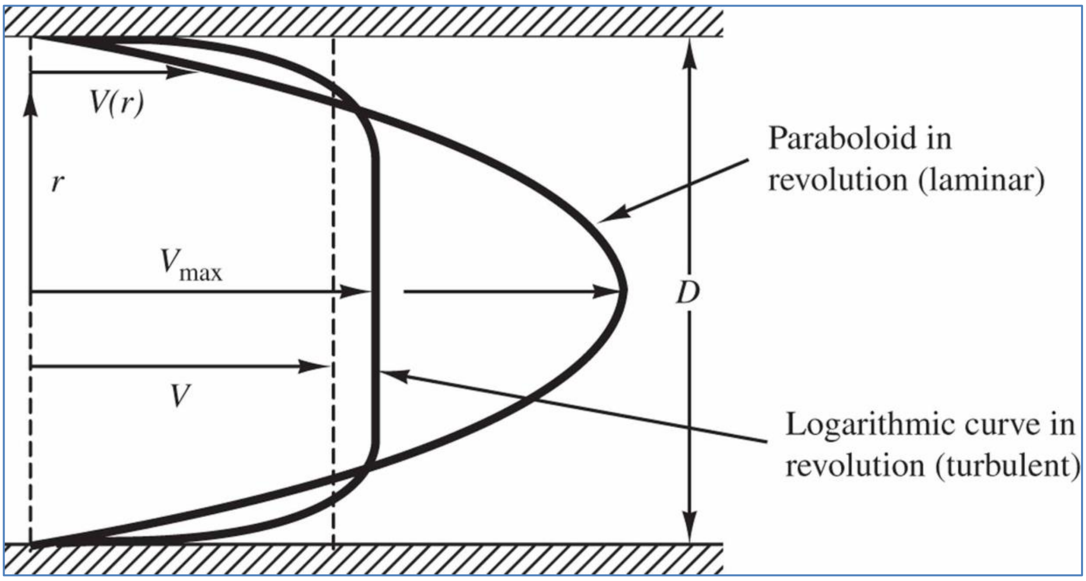
Velocity fluctuations highly disordered motion



Dye injection



Dye injection



Velocity, m/sec

Diameter

$$R_e = \frac{V \times D}{v}$$

Kinematic viscosity

$$v = \mu/\rho$$ → density

↓

Dynamic viscosity

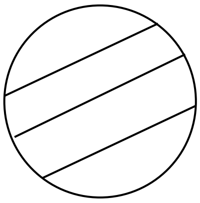
Re > 2000	Laminar Flow	Transition in between
Re < 4000	Turbulent Flow	

For non-circular pipes hydraulic diameter, D_h, is used.

c/s area

$$D_H = \frac{4 \times A_c}{P}$$

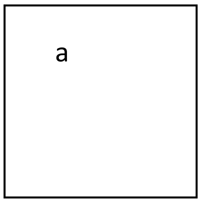
Perimeter



Circular pipe

$$D_H = \frac{4\times\pi D^2/4}{\pi D}$$

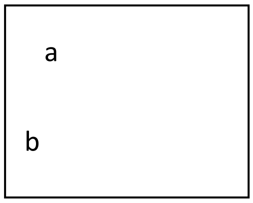
$$D_H = D$$



Square duct

$$D_H = \frac{4\times(a)^2}{4a}$$

$$D_H = a$$

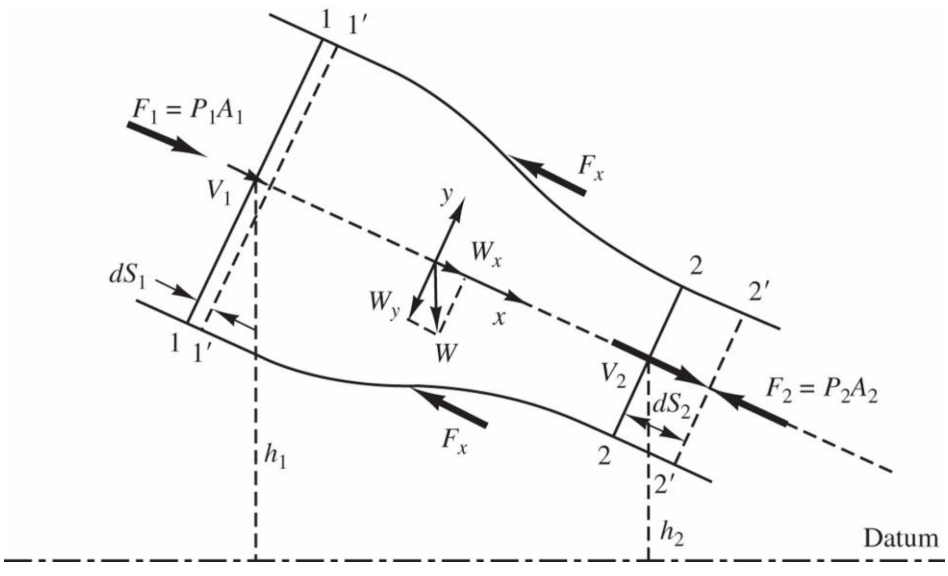


Rectangular duct

$$D_H = \frac{4\times(a\times b)}{2a+2b}$$

18.2. Forces in Pipe Flow

Below figure shows a section of water flowing in a circular pipe. A control volume is considered between section 1-1 and 2-2. After a short time interval, dt, the mass originally occupying the control volume, has a new position between 1’-1’ and 2’-2’.



F_x = Force exerted on the control volume by the wall of the pipe.

For incompressible and steady flows; mass flux (fluid mass) enters to the control volume

$$= \rho \times d \times Vol_{1-1'}$$

Mass flux leaves the control volume

$$= \rho \times d \times Vol_{2-2'}$$

According to principle of conservation of mass;

$$\frac{\rho \times d \times Vol_{1-1'}}{dt} = \frac{\rho \times d \times Vol_{2-2'}}{dt}$$

$$\rho \times A_1 \times V_1 = \rho \times A_2 \times V_2 = \rho Q$$

c/s area section 2

c/s section 1 velocity

or

$$A_1 \times V_1 = A_2 \times V_2 \text{ (Continuity Equation)}$$

Applying Newton' s second law to moving mass in the control volume yields

$$\sum \vec{F} = m\vec{a} = m\frac{d\vec{V}}{dt} = \frac{m\vec{V}_2 - m\vec{V}_1}{\Delta t}$$

Along the axial direction of the flow, the external forces exerted on the control volume may be expressed as

$$\sum F_X = P_1 \times A_1 - P_2 \times A_2 - F_X + W_X$$

V_1, V_2, P_1, P_2 are the velocities and pressures at sections 1-1 and 2-2, respectively.

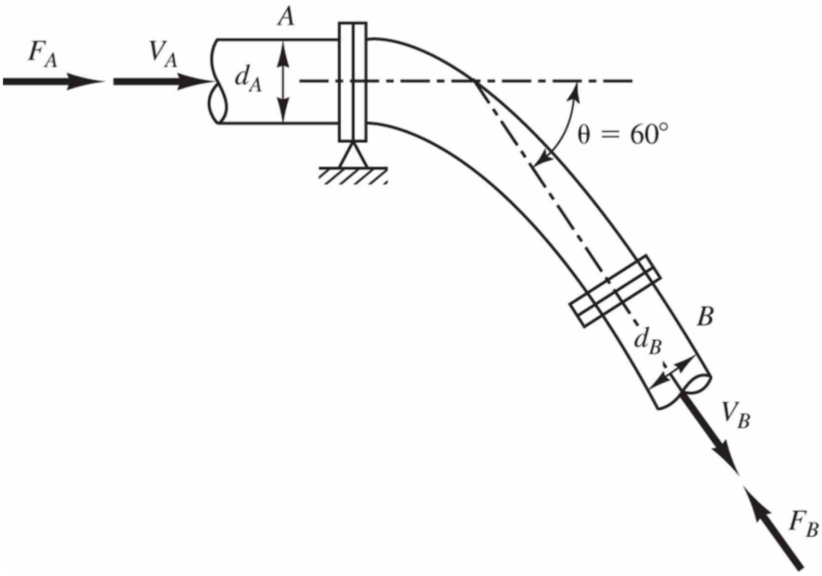
Fx= Axial direction force exerted on the control volume by the wall of the pipe,
W_x= Axial component of the weight of the liquid on the control volume.

$$\underbrace{\begin{aligned} \sum F_X &= \rho \times Q(V_{X_2} - V_{X_1}) \\ \sum F_y &= \rho \times (V_{y_2} - V_{y_1}) \\ \sum F_z &= \rho \times Q(V_{z_2} - V_{z_1}) \end{aligned}}_{\text{Principle of conservation of momentum}}$$

General writing in vector quantities;

$$\sum \vec{F} = \rho Q(\vec{V}_2 - \vec{V}_1)$$

Example 18.1 (Example 3.2, Hwang, 4th Ed.): A horizontal nozzle discharges 0.01 m^3/sec of water at 4^0C into the air. The suplu pipe's diameter (dA=40 mm) is twice as large as the nozzle diamater (dB=20 mm). The nozzle is held in place by a hinge mechanism. Determine the magnitıde and direction of the reaction force at the hinge, if the gauge pressure at A is 500,000 N/m^2. (assume weigth of the hinge is negligible).



$$F_{X,A} = P \times A_A = (500,000\ \mathrm{N/m^2})\underbrace{\left(\frac{40\ \mathrm{mm}}{1000}\right)^2 \times \frac{\pi}{4}}_{\text{c /s area of section A}} = 628\ \mathrm{N}$$

$F_{y,A} = 0$ (all flow in the x-direction), $F_{x,B} = F_{y,B} = 0$ (atmospheric pressure)

$$V_A = \frac{Q}{A_A} = \frac{0.01\ \mathrm{m^3/s}}{\left(\frac{40}{1000}\right)^2 \times \frac{\pi}{4}} = 7.96\ \mathrm{m/sec} = V_{x,A},\ V_{y,A} = 0$$

$$A_B \times V_B = Q = 0.01\ \mathrm{m^3/sec}$$

$$\left(\frac{20\ \mathrm{mm}}{1000}\right)^2 \times \frac{\pi}{4} \times V_B = 0.01\ \mathrm{m^3/sec}$$

$$V_B = 31.8\ \mathrm{m/sec}$$

$$V_{X,B} = (31.8\ \mathrm{m/sec}) \times (\cos 60^0) = 15.9\ \mathrm{m/sec}$$

$$V_{y,B} = (31.8\ \mathrm{m/sec}) \times (\mathrm{Sin}\ 30^0) = 27.5\ \mathrm{m/sec}$$

$$\sum F_X = \rho \times Q(V_{XB} - V_{XA}) \quad \rightarrow (+)$$

$$628\ \mathrm{N} - F_X = (998\ \mathrm{kg/m^3}) \times (0.01) \times (15.9 - 7.96)$$

$$F_X = 549\ \mathrm{N} \quad \leftarrow (-)$$

(Both forces and velocities are vector quantities that must adhere to the sign convention. Because the sign of F_x ended up positive, our assumed direction was correct).

$$\sum F_y = \rho \times Q\left(V_{yB} - V_{yA}\right) \quad \uparrow (+)$$

Assume F_y is negative

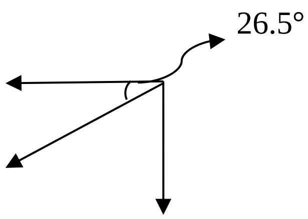
$$-F_y = (998\ \mathrm{kg/m^3}) \times (0.01) \times (-27.5 - 0)$$

$$F_y = 274\ \mathrm{N} \downarrow$$

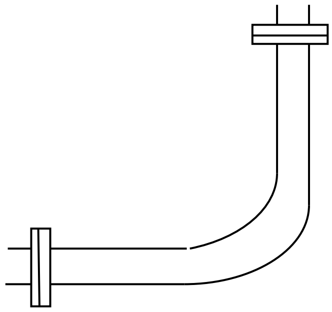
The resultant force is;

$$F = [(549\ \mathrm{N})^2 + (274\ \mathrm{N})^2]^{1/2} = 614\ \mathrm{N}$$

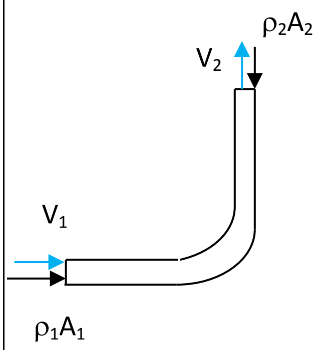
$$\theta = \tan^{-1}\left(\frac{F_y}{F_X}\right) = 26.5°$$



Example18.2:



Calculate the force that must be exerted on the pipe to hold it in equilibrium as shown in Figure above. The elbow is in a horizontal plane and connected to " two 4 in Schedule 40 pipes" carrying 3000L/min of water at 15^0C. The inlet pressure 550 kPa



$\rho_1 A_1$ and $\rho_2 A_2$ are the forces due to the fluid pressures.

Net external force in the x-direction;

$$F_X = \rho \times Q(V_{2X} - V_{1X}) \text{ and also}$$

$$F_X = R_X - P_1 \times A_1$$

$$V_{2X} = 0$$

$$V_{1X} = -V_1$$

$$R_X - P_1 \times A_1 = \rho \times [0 - (-V_1)]$$

$$R_X = \rho \times Q \times V_1 + P_1 \times A_1$$

From the given data;

$$Q = 3000 \text{ L/min} \times \frac{1 \text{ min}}{60 \text{ sec}} \times \frac{1 \text{ m}^3}{1000 \text{ L}} = 0.05 \text{ m}^3/\text{sec}$$

$$V_1 = \frac{Q}{A_1} = \frac{0.05}{8.213 \times 10^{-3} \text{ m}^2} = 6.09 \text{ m/sec}$$

$\rho = 1000 \text{ kg/m}^3$ at 15 °C

$\mathrm{R_X} = (1000\ \mathrm{kg/m^3}) \times (0.05\ \mathrm{m^3/sec}) \times (6.09\ \mathrm{m/sec}) + (550\ \mathrm{N/m^2}) \times 8.21 \times 10^{-3}\mathrm{m}^2$

$= 4517\ \mathrm{N}$

$$\mathrm{R_X} = 305\ \mathrm{N} + 4517\ \mathrm{N}$$

$$= 4822\ \mathrm{N}$$

In y-direction, the equation for the net external force;

$\mathrm{F_y} = \rho \times \mathrm{Q}\left(\vec{\mathrm{V}}_{2\mathrm{y}} - \vec{\mathrm{V}}_{1\mathrm{y}}\right)$

$$\mathrm{R_y} = \mathrm{F_y} + \mathrm{P_2} \times \mathrm{A_2}$$

$$\mathrm{F_y} = \mathrm{R_y} - \mathrm{P_2} \times \mathrm{A_2}$$

$$\mathrm{R_y} = \rho \times \mathrm{Q}\left(\vec{\mathrm{V}}_{2\mathrm{y}} - \vec{\mathrm{V}}_{1\mathrm{y}}\right) + \mathrm{P_2} \times \mathrm{A_2}$$

$$\mathrm{R_y} = (1000\ \mathrm{kg/m^3}) \times (\mathrm{V_2} - 0) + \mathrm{P_2} \times \mathrm{A_2}$$

$$\mathrm{V_{2y}} = \mathrm{V_2}\ \mathrm{V_{1y}} = 0\ ,$$

If energy losses in the elbow is neglected➔ $\mathrm{V_2} = \mathrm{V_1}$,$\rho_2 = \rho_1$ since the sizes of the inlet and oulet are equal.

$\rho \times \mathrm{Q} \times \mathrm{V_2} = 305\ \mathrm{N}$ $\qquad$ $\mathrm{P_2} \times \mathrm{A_2} = 4517\ \mathrm{N}$

$$\mathrm{R_y} = (305 + 4517) = 4822\ \mathrm{N}$$

The forces R_x and R_y are the reactions caused at the elbow as the fluid turns 90^0.

Example 18.3 (Pr. 3.3.1, Hwang 4th Edition):

A jet of water exits a nozzle heading in the positive x-direction and strikes a flat plate at a 90° angle. The water then sprays through a 360 ° arc (y- and z-directions) exiting the plate. If the nozzle has a 20-cm diameter and the flow has a 3.44 m/sec velocity, what is the force exerted on the plate by the water?

$$\sum \mathrm{F_x} = \rho \times \mathrm{Q}(\mathrm{v_x}\,, \mathrm{out}, - \mathrm{v_x}, \mathrm{in})$$

c /s area = $\frac{\pi \times D^2}{4} = \frac{\pi \times (20/100)^2}{4}$

$$\mathrm{Q} = \mathrm{v} \times \mathrm{A}$$

$$= (3.44\ \mathrm{m/sec}) \times \pi \times \frac{\left(\frac{20}{100}\right)^2}{4} = 0.108\ \mathrm{m^3/sec}$$

$$F_x = (998\ \mathrm{kg/m^3}) \times (0.108\ \mathrm{m^3/sec}) \times (0 - 3.44\ \mathrm{m/sec})$$

$$-F_x = -371\ \mathrm{N}$$

$F_y = 0$ $\quad\quad$ $F_z = 0$

$$F_x = 371\ \mathrm{N}$$

Since all flow in control volume is x-directed and the out flow is zero in the y and z direction since the spray is equal in all.