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CSE 246 Analysis of Algorithms

Spring 2012 Final

07.06.2012 Thursday, Duration: 120 minutes

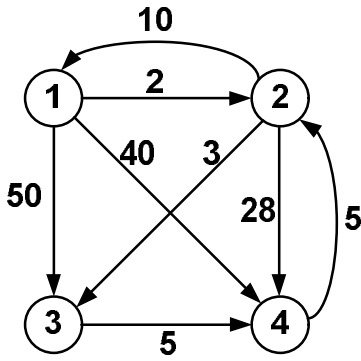
Q1	Q2	Q3	Q4	Q5	Q6	Q7	SUM
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Q-1. (16 pts) Let $X[1 \dots n]$ and $Y[1 \dots n]$ be two arrays, each containing n numbers already in sorted order. Give an $O(\log n)$ time algorithm to compute the median of all $2n$ elements in arrays X and Y . (Hint: Decrease-by-half). (Description is enough. No need to give a pseudocode.)

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Q-2. Consider the following weighted digraph.



(b) (13 pts) Solve the all-pairs shortest path problem by applying Floyd's algorithm.

(a) (3 pts) What is the transitive closure of the above digraph?

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Q-3. Construct the five-symbol alphabet {A, B, C, D, _} with the following occurrence frequencies in a text made up of these symbols:

Symbol	A	B	C	D	_
Frequency	0.4	0.1	0.2	0.15	0.15

(a) (7 pts) Construct the Huffman code for the above data.

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(b) (2 pts) Encode ABACADAB using this coding.

(c) (2 pts) Decode 10001110110001010 using this coding.

(d) (5 pts) Find the compression ratio of this coding.

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Q-4. (a) (6 pts) What is the trivial lower bound of finding the largest element in an array? Why? Is this lower bound tight? Why?

(b) (10 pts) Find a tight lower bound class for the problem of finding two closest numbers among n real numbers x_1, x_2, \dots, x_n . Prove that the lower bound you find is tight. (Hint: Tight lower bound of element uniqueness problem is known to be $\Omega(n \log n)$.)

Q-5. (a) (1 pts) What is the long form of NP?

(b) (3 pts) Does the element uniqueness problem in P? Why?

(c) (3 pts) Does the element uniqueness problem in NP? Why?

(d) (3 pts) Give an example problem that is neither in P nor in NP.

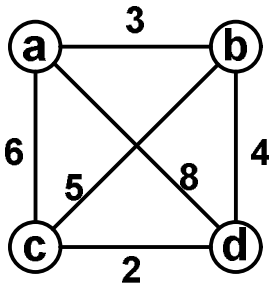
(e) (4 pts) Given a problem, how would you prove that it is NP-complete? Describe all steps briefly.

(f) (3 pts) What happens if you can find a polynomial time algorithm for an NP-complete problem? Why?

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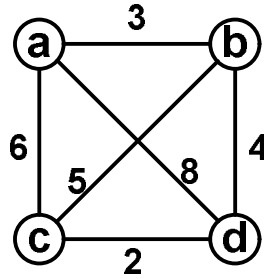
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Q-6. (a) (14 pts) Apply the branch-and-bound algorithm to solve the traveling salesman problem for the following graph:



(b) (3 pts) State whether the above is a best-case instance for the branch-and-bound algorithm or not. (In other words, can the branch-and-bound algorithm perform faster for any other input of fully-connected graph with four nodes?) Why?

Q-7. Consider the same graph as Q-6.



(a) (7 pts) Apply the multifragment-heuristic algorithm in order to solve the traveling salesman problem approximately.

(Remember that multifragment-heuristic algorithm can be described as follows:

Step 1: Sort the edges in increasing order of their weights. Initialize the set of tour edges to be constructed to empty set.

Step 2: Add next edge on the sorted list to the tour, skipping those whose addition would've created a vertex of degree 3 or a cycle of length less than n . Repeat this step until a tour of length n is obtained.)

(b) (4 pts) What is the accuracy ratio of this approximation algorithm?

(c) (6 pts) Prove that the performance ratio of this algorithm is unbounded above, i.e. $R_A = \infty$. (Hint: Carefully select one of the edges, and try to change its weight to an arbitrarily large number.)