



# Bayesian Game Setting

- Extend the social choice setting to a new setting where agents can't be relied upon to disclose their preferences honestly.
- Start with a set of agents in a Bayesian game setting (but no actions).



## Definition (Bayesian game setting)

A **Bayesian game setting** is a tuple  $(N, O, \Theta, p, u)$ , where

- $N$  is a finite set of  $n$  agents;
- $O$  is a set of outcomes;
- $\Theta = \Theta_1 \times \dots \times \Theta_n$  is a set of possible joint type vectors;
- $p$  is a (common prior) probability distribution on  $\Theta$ ; and
- $u = (u_1, \dots, u_n)$ , where  $u_i : O \times \Theta \mapsto \mathbb{R}$  is the utility function for each player  $i$ .



# What we're up to

- The problem is to pick a mechanism that will **cause rational agents to behave in a desired way**
  - each agent holds private information, in the Bayesian game sense
- Various **equivalent** ways of looking at this setting
  - perform an optimization problem, given that the values of (some of) the inputs are unknown
  - choose the Bayesian game out of a set of possible Bayesian games that maximizes some performance measure
  - design a game that *implements* a particular social choice function in equilibrium, given that the designer does not know agents' preferences and the agents might lie



# Implementation in Dominant Strategies

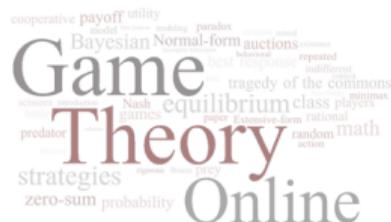


## Definition (Implementation in dominant strategies)

Given a Bayesian game setting  $(N, O, \Theta, p, u)$ , a mechanism  $(A, M)$  is an **implementation in dominant strategies** of a social choice function  $C$  (over  $N$  and  $O$ ) if for any vector of utility functions  $u$ , the game has an equilibrium in dominant strategies, and in any such equilibrium  $a^*$  we have  $M(a^*) = C(u)$ .



# Bayes–Nash Implementation Comments



## Bayes–Nash Equilibrium Problems:

- there could be more than one equilibrium
  - which one should I expect agents to play?
- agents could mis-coordinate and play none of the equilibria
- asymmetric equilibria are implausible

## Refinements:

- Symmetric Bayes–Nash implementation
- *Ex-post* implementation

