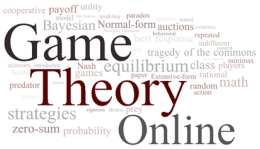


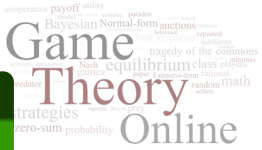
Limitations of VCG

Game Theory Course:
Jackson, Leyton-Brown & Shoham

1. Privacy

- VCG requires agents to fully reveal their private information
- This private information may have value to agents that extends beyond the current interaction
  - for example, the agents may know that they will compete with each other again in the future
- It is often preferable to elicit only as much information from agents as is required to determine the social welfare maximizing choice and compute the VCG payments.

2. Susceptibility to Collusion



Example

Agent	U(build road)	U(do not build road)	Payment
1	200	0	150
2	100	0	50
3	0	250	0

- What happens if agents 1 and 2 both increase their declared valuations by \$50?

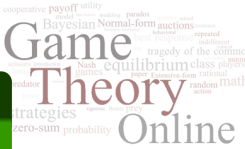
2. Susceptibility to Collusion

Example

Agent	U(build road)	U(do not build road)	Payment
1	250	0	
2	150	0	
3	0	250	0

- What happens if agents 1 and 2 both increase their declared valuations by \$50?

2. Susceptibility to Collusion

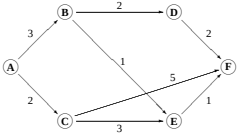


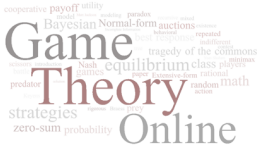
Example

Agent	U(build road)	U(do not build road)	Payment
1	250	0	100
2	150	0	0
3	0	250	0

- What happens if agents 1 and 2 both increase their declared valuations by \$50?
- The choice is unchanged, but both of their payments are reduced.
- Thus, while no agent can gain by changing his declaration, groups *can*.

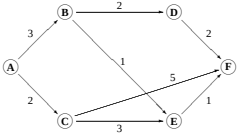
3. VCG is not Frugal

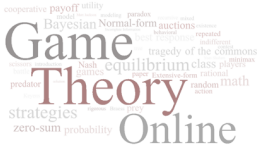




- VCG can end up paying arbitrarily more than an agent is willing to accept (or equivalently charging arbitrarily less than an agent is willing to pay)
- Consider the effect of AC's cost on the payment to AB.

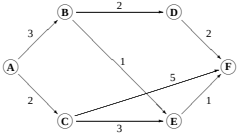
3. VCG is not Frugal





- VCG can end up paying arbitrarily more than an agent is willing to accept (or equivalently charging arbitrarily less than an agent is willing to pay)
- Consider the effect of AC's cost on the payment to AB.
  - If the cost of this edge increased to 8, our payment to AB would increase to $p_{AB} = (-12) - (-2) = -10$.

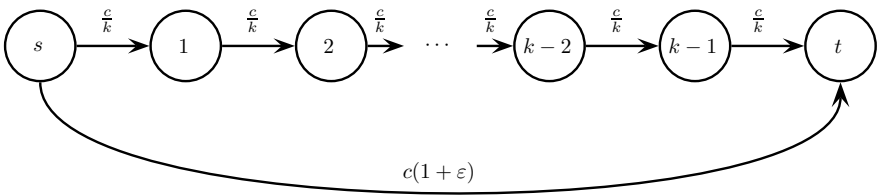
3. VCG is not Frugal



- VCG can end up paying arbitrarily more than an agent is willing to accept (or equivalently charging arbitrarily less than an agent is willing to pay)
- Consider the effect of AC's cost on the payment to AB.
  - If the cost of this edge increased to 8, our payment to AB would increase to $p_{AB} = (-12) - (-2) = -10$.
  - If the cost were any $x \geq 2$, we would select the path $ABEF$ and would have to make a payment to AB of $p_{AB} = (-4 - x) - (-2) = -(x + 2)$.
  - The gap between agents' true costs and the payments that they could receive under VCG is unbounded.

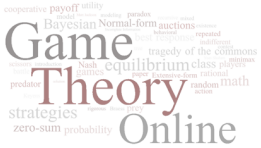
3. VCG is not Frugal

Are VCG's payments at least close to the cost of the *second* shortest disjoint path?



- The top path has a total cost of c.
- VCG picks it, pays each of the k agents $c(1+\varepsilon) - (k-1)\frac{c}{k}$.
- Hence VCG's total payment is $c(1+k\varepsilon)$.
- For fixed ε, VCG's payment is $\Theta(k)$ times (i.e., only a constant away from k times) the cost of the second shortest disjoint path.

4. Revenue Monotonicity Violated

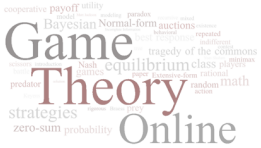


Revenue monotonicity: revenue always weakly increases as agents are added.

Example

Agent	U(build road)	U(do not build road)	Payment
1	0	90	0
2	100	0	90

4. Revenue Monotonicity Violated



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Example

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3	100	0	0

4. Revenue Monotonicity Violated

Revenue monotonicity: revenue always weakly increases as agents are added.

Example

Agent	U(build road)	U(do not build road)	Payment
1	0	90	0
2	100	0	0
3	100	0	0

- Adding agent 3 causes VCG to make the same choice but to collect zero revenue!
- Agent 2 could pretend to be two agents and eliminate his payment.

5. Cannot Return All Revenue to Agents

- We may want to use VCG to induce agents to report their valuations honestly, but may not want to make a profit by collecting money from the agents.
- Thus, we might want to find some way of returning the mechanism's profits back the agents.
- However, the possibility of receiving a rebate after the mechanism has been run changes the agents' incentives.
- In fact, even if profits are given to a charity that the agents care about, or spent in a way that benefits the local economy and hence benefits the agents, the VCG mechanism is undermined.
- It *is* possible to return at least *some* of the revenues to the agents, but it must be done very carefully, and in general not all the money can be returned.