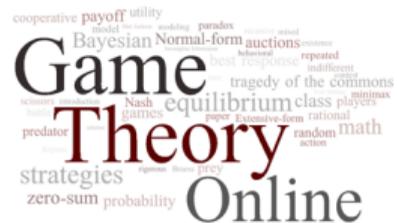




Vickrey-Clarke-Groves Mechanisms: Definitions

Game Theory Course:
Jackson, Leyton-Brown & Shoham



A positive result

- Recall that in the quasilinear utility setting, a direct mechanism consists of a **choice rule** and a **payment rule**.
- A **VCG mechanism**:
 - has truth as a dominant strategy (satisfies truthfulness, is strategy-proof)
 - makes efficient choices (*not including payments*)



A positive result

- Recall that in the quasilinear utility setting, a direct mechanism consists of a **choice rule** and a **payment rule**.
- A **VCG mechanism**:
 - has truth as a dominant strategy (satisfies truthfulness, is strategy-proof)
 - makes efficient choices (*not including payments*)
- And, under additional assumptions about the setting, can satisfy:
 - weak budget balance
 - *interim* individual rationality



Groves Mechanisms



Definition (Groves mechanisms)

Direct mechanisms, (χ, p) , such that

$$\chi(\hat{v}) \in \arg \max_x \sum_i \hat{v}_i(x)$$
$$p_i(\hat{v}) = h_i(\hat{v}_{-i}) - \sum_{j \neq i} \hat{v}_j(\chi(\hat{v}))$$

Some people refer to these as VCG mechanisms, although that name has more recently started to be used to refer to a specific mechanism within this class.

VCG discussion



$$\chi(\hat{v}) = \arg \max_x \sum_i \hat{v}_i(x)$$

$$p_i(\hat{v}) = \sum_{j \neq i} \hat{v}_j(\chi(\hat{v}_{-i})) - \sum_{j \neq i} \hat{v}_j(\chi(\hat{v}))$$

Questions:

- who pays 0?

VCG and Groves Mechanisms: Truthfulness



Theorem

Truth telling is a dominant strategy under any Groves mechanism including the pivotal mechanism (a VCG mechanism).

Consider agent i 's problem of choosing the best strategy \hat{v}_i . A best strategy for i is solves

$$\max_{\hat{v}_i} (v_i(\chi(\hat{v}_i, \hat{v}_{-i})) - p(\hat{v}_i, \hat{v}_{-i}))$$

Substituting in the payment function for a Groves mechanism this becomes:

$$\max_{\hat{v}_i} \left(v_i(\chi(\hat{v})) - h_i(\hat{v}_{-i}) + \sum_{j \neq i} \hat{v}_j(\chi(\hat{v})) \right)$$

Since $h_i(\hat{v}_{-i})$ does not depend on \hat{v}_i , it is sufficient to solve

$$\max_{\hat{v}_i} \left(v_i(\chi(\hat{v})) + \sum_{j \neq i} \hat{v}_j(\chi(\hat{v})) \right).$$

VCG Truthfulness



So, i would like to pick a declaration \hat{v}_i that will lead the mechanism to pick an $x \in X$ which solves

$$\max_x \left(v_i(x) + \sum_{j \neq i} \hat{v}_j(x) \right). \quad (1)$$

Under a Groves mechanism,

$$\chi(\hat{v}) \in \arg \max_x \left(\hat{v}_i(x) + \sum_{j \neq i} \hat{v}_j(x) \right).$$

A Groves mechanism will choose x in a way that solves the maximization problem in Equation (1) when $\hat{v}_i = v_i$. Thus, truth-telling is a dominant strategy for agent i . □

Summary



- Groves mechanisms, and VCG mechanisms in particular, have nice dominant strategy properties
- Agents' payments include the impact of their announcements on other agents
- Internalize the externalities and lead to efficient decisions (x 's)
- But may burn payments to do so!