

Social Choice: Paradoxical Outcomes

Game Theory Course:
Jackson, Leyton-Brown & Shoham

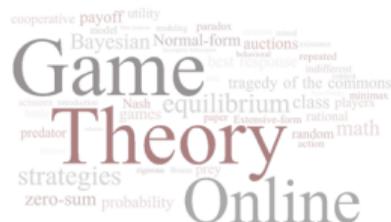
Condorcet example

499 agents: $A \succ B \succ C$

3 agents: $B \succ C \succ A$

498 agents: $C \succ B \succ A$

- What is the Condorcet winner? B
- What would win under plurality voting?



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- What would win under plurality with elimination?



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- What would win under plurality voting? A
- What would win under plurality with elimination? C



Sensitivity to Losing Candidate

35 agents: $A \succ C \succ B$

33 agents: $B \succ A \succ C$

32 agents: $C \succ B \succ A$

- What candidate wins under plurality voting?



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- What candidate wins under plurality voting? A
- What candidate wins under Borda voting? A
- Now consider dropping C . Now what happens under both Borda and plurality?

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- What candidate wins under plurality voting? A
- What candidate wins under Borda voting? A
- Now consider dropping C . Now what happens under both Borda and plurality? B wins.

Sensitivity to Agenda Setter

35 agents: $A \succ C \succ B$

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- Who wins pairwise elimination, with the ordering A, B, C ?



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Sensitivity to Agenda Setter



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- Who wins pairwise elimination, with the ordering A, B, C ? C
- Who wins with the ordering A, C, B ? B
- Who wins with the ordering B, C, A ? A

Another Pairwise Elimination Problem

I agent: $B \succ D \succ C \succ A$

I agent: $A \succ B \succ D \succ C$

I agent: $C \succ A \succ B \succ D$

- Who wins under pairwise elimination with the ordering A, B, C, D ?



Another Pairwise Elimination Problem

I agent: $B \succ D \succ C \succ A$

I agent: $A \succ B \succ D \succ C$

I agent: $C \succ A \succ B \succ D$

- Who wins under pairwise elimination with the ordering A, B, C, D ? D .
- What is the problem with this?
 - *all* of the agents prefer B to D —the selected candidate is Pareto-dominated!

