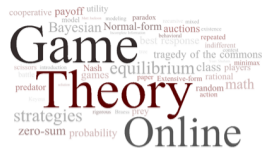




Mechanism Design as an Optimization Problem

Game Theory Course:
Jackson, Leyton-Brown & Shoham



Mechanism Design as an Optimization Problem



- We can understand mechanism design as the problem of finding the best possible mechanism, given constraints about how it operates
- We'll now consider some typical choices for
 - these constraints
 - this notion of “best”

Truthfulness



Definition (Truthfulness)

A transferrable utility mechanism is **truthful** if it is direct and $\forall i \forall v_i$, agent i 's equilibrium strategy is to adopt the strategy $\hat{v}_i = v_i$.

[illegible]
$$\forall v \forall x', \sum_i v_i(x) \geq \sum_i v_i(x').$$

- An efficient mechanism selects the choice that maximizes the sum of agents' utilities, disregarding monetary payments.
- How is this related to the standard game-theoretic definition of Pareto efficiency?

Efficiency

Definition (Efficiency)

A transferrable utility mechanism is **strictly Pareto efficient**, or just **efficient**, if in equilibrium it selects a choice x such that

$$\forall v \forall x', \sum_i v_i(x) \geq \sum_i v_i(x').$$

- An efficient mechanism selects the choice that maximizes the sum of agents' utilities, disregarding monetary payments.
- How is this related to the standard game-theoretic definition of Pareto efficiency?
 - if we include the mechanism as an agent, all Pareto-efficient outcomes involve the same choice (and different payments)
 - any outcome involving another choice is Pareto dominated: some agents could pay others such that all would prefer the swap



Efficiency



Definition (Efficiency)

A transferrable utility mechanism is **strictly Pareto efficient**, or just **efficient**, if in equilibrium it selects a choice x such that

$$\forall v \forall x', \sum_i v_i(x) \geq \sum_i v_i(x').$$

- Called **economic efficiency** to distinguish from other (e.g., computational) notions
- Also called **social-welfare maximization**
- Note: defined in terms of true (not declared) valuations.

cooperative payoff utility
 model decision making paradoxes
 Bayesian Normal-form auctions
 Game Nash equilibrium class players
 Theory
 predator strategies
 zero-sum probability Online
 repeated indifferent the commons
 rational math action
 extensive-form rational
 optimal choice policy

$$\forall v, \sum_i p_i(s(v)) = 0,$$

- regardless of the agents' types, the mechanism collects and disburses the same amount of money from and to the agents

[illegible]

A transferrable utility mechanism is **budget balanced** when

where s is the equilibrium strategy profile.

- regardless of the agents' types, the mechanism collects and disburses the same amount of money from and to the agents
- relaxed version: **weak budget balance**:

$$\forall v, \sum_i p_i(s(v)) \geq 0$$

- the mechanism never takes a loss, but it may make a profit

Budget Balance

Definition (Budget balance)

A transferrable utility mechanism is **budget balanced** when

$$\forall v, \sum_i p_i(s(v)) = 0,$$

where s is the equilibrium strategy profile.

- regardless of the agents' types, the mechanism collects and disburses the same amount of money from and to the agents
- Budget balance can be required to hold **ex ante**:

$$\mathbb{E}_v \sum_i p_i(s(v)) = 0$$

- the mechanism must break even only on expectation



Individual-Rationality

Definition (*Ex interim* individual rationality)

A mechanism is **ex interim individual rational** when

$\forall i \forall v_i, \mathbb{E}_{v_{-i}|v_i} v_i(\chi(s_i(v_i), s_{-i}(v_{-i}))) - p_i(s_i(v_i), s_{-i}(v_{-i})) \geq 0$,
where s is the equilibrium strategy profile.

- no agent loses by participating in the mechanism.
- *ex interim* because it holds for every possible valuation for agent i , but averages over the possible valuations of the other agents.



[illegible]

Mechanism Design as an Optimization Problem



Definition (Tractability)

A mechanism is **tractable** when $\forall \hat{v}$, $\chi(\hat{v})$ and $p(\hat{v})$ can be computed in polynomial time.

- The mechanism is (guaranteed to be) computationally feasible.

Revenue Maximization

We can also add an objective function to our mechanism. One example: revenue maximization.

Definition (Revenue maximization)

A mechanism is **revenue maximizing** when, among the set of functions χ and p that satisfy the other constraints, the mechanism selects the χ and p that maximize $\mathbb{E}_{\theta} \sum_i p_i(s(\theta))$, where $s(\theta)$ denotes the agents' equilibrium strategy profile.

- The mechanism designer can choose among mechanisms that satisfy the desired constraints by adding an objective function such as revenue maximization.



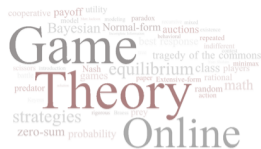
[illegible]

- ## Definition (Revenue minimization)

- Note: this considers the **worst case** over valuations; we could consider average case instead.

Fairness

- Fairness is hard to define. What is fairer:
 - charging all agents \$100 and making a choice they all hate equally?
 - charging all agents \$0 and making a choice that some hate and some like?



Fairness

- Fairness is hard to define. What is fairer:
 - charging all agents \$100 and making a choice they all hate equally?
 - charging all agents \$0 and making a choice that some hate and some like?
- **Maxmin fairness**: make the least-happy agent the happiest.



Definition (Maxmin fairness)

A transferrable utility mechanism is **maxmin fair** when, among the set of functions χ and p that satisfy the other constraints, the mechanism selects the χ and p that maximize

$$\mathbb{E}_v \left[\min_{i \in N} v_i(\chi(s(v))) - p_i(s(v)) \right],$$

where $s(v)$ denotes the agents' equilibrium strategy profile.

Price of Anarchy Minimization

- When efficiency is impossible, we can try to get close
- Minimize the **worst-case ratio** between optimal social welfare and the social welfare achieved by the given mechanism.



Definition (Price-of-anarchy minimization)

A transferrable utility mechanism **minimizes the price of anarchy** when, among the set of functions χ and p that satisfy the other constraints, the mechanism selects the χ and p that minimize

$$\max_{v \in V} \frac{\max_{x \in X} \sum_{i \in N} v_i(x)}{\sum_{i \in N} v_i(\chi(s(v)))},$$

where $s(v)$ denotes the agents' equilibrium strategy profile in the worst equilibrium of the mechanism—i.e., the equilibrium in which $\sum_{i \in N} v_i(\chi(s(v)))$ is the smallest.