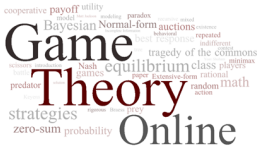
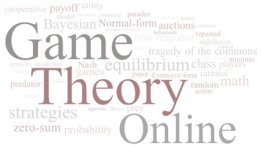


Impossibility of Non-paradoxical Social Choice Functions

Game Theory Course:
Jackson, Leyton-Brown & Shoham

Social Choice Functions



- Maybe Arrow's theorem held because we required a whole preference ordering.
- Idea: social choice functions might be easier to find
- We'll need to redefine our criteria for the social choice function setting; PE and IIA discussed the ordering

Weak Pareto Efficiency

Definition (Weak Pareto Efficiency)

A social choice function C is weakly Pareto efficient if it never selects an outcome o_2 when there exists another outcome o_1 such that $\forall i \in N$, $o_1 \succ_i o_2$.

- A dominated outcome can't be chosen.

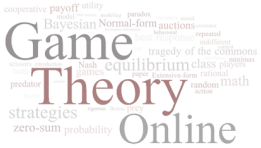
Monotonicity

Definition (Monotonicity)

C is monotonic if, for any $o \in O$ and any preference profile $[\succ] \in L^n$ with $C([\succ]) = o$, then for any other preference profile $[\succ']$ with the property that $\forall i \in N, \forall o' \in O$, $o \succ'_i o'$ if $o \succ_i o'$, it must be that $C([\succ']) = o$.

- an outcome o must remain the winner whenever the support for it is increased in a preference profile under which o was already winning

Dictatorship



Definition (Dictatorship)

C is dictatorial if there exists an agent j such that C always selects the top choice in j's preference ordering.

The bad news

Theorem (Muller-Satterthwaite, 1977)

Any social choice function that is weakly Pareto efficient and monotonic is dictatorial.

- Perhaps contrary to intuition, social choice functions are no simpler than social welfare functions after all.
- The proof repeatedly "probes" a social choice function to determine the relative social ordering between given pairs of outcomes.
- Because the function must be defined for all inputs, we can use this technique to construct a full social welfare ordering.

But... Isn't Plurality Monotonic?

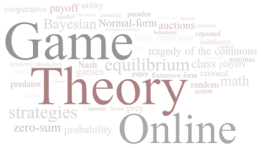
Plurality satisfies weak PE and ND, so it must not be monotonic.

Consider the following preferences:

3 agents:	$a \succ b \succ c$
2 agents:	$b \succ c \succ a$
2 agents:	$c \succ b \succ a$

Plurality chooses a.

But... Isn't Plurality Monotonic?



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Consider the following preferences:

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Plurality chooses a.

Increase support for a by moving c to the bottom:

3 agents: $a \succ b \succ c$
2 agents: $b \succ c \succ a$
2 agents: $b \succ a \succ c$

Now plurality chooses b.