

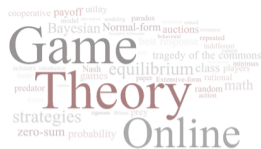
# The Myerson–Satterthwaite Theorem

Game Theory Course:  
Jackson, Leyton-Brown & Shoham

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- ### The Myerson–Satterthwaite Theorem

# Simple Exchange Setting



- Exchange of a single unit of an indivisible good
- Seller initially has the item and has a value for it of  $\theta_S \in [0, 1]$
- Buyer has need for the item and has a value for it of  $\theta_B \in [0, 1]$

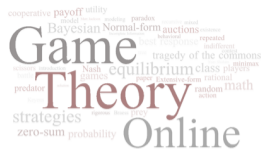
# An Example



- The buyer's value is equally likely to be either .1 or 1
- The seller's value for the good is equally likely to be 0 or .9
- Trade should take place for all combinations of values except (.1,.9)

# An Example of a Mechanism

- The seller announces a price in  $[0, 1]$
- The buyer either buys or not at that price.



# An Example of a Mechanism

- The seller announces a price in  $[0, 1]$
- The buyer either buys or not at that price.
- The seller should say a price of either  $.1$  or  $1$  (presume that buyer says yes when indifferent)
- When the seller's value is  $0$ :



Game Theory

Bayesian Normal-form auctions

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Online

strategies zero-sum probability

predator Nash equilibria

tragedy of the commons

repeated cooperative payoff utility

paradox prisoner's dilemma

behavioral adulterous antitrust

paper extensive-form random action

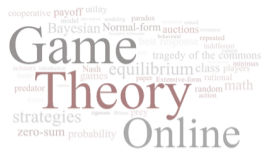
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- A word cloud visualization centered around the theme of Game Theory. The words are arranged in various sizes and orientations, with "Game Theory" being the most prominent. Other visible words include "Bayesian Normal-form auctions", "equilibrium class projects", "Nash equilibria", "predator-prey", "strategies", "zero-sum", "probability", "Online", "cooperative payoff utility", "added resource modeling", "paradox", "repeated", "tragedy of the commons", "antimax", "rational", "math", "action", "agame theory (G)".

# An Example of a Mechanism



- The seller announces a price in  $[0, 1]$
- The buyer either buys or not at that price.
- The seller should say a price of either  $.1$  or  $1$  (presume that buyer says yes when indifferent)
- When the seller's value is  $0$ :
  - price of  $.1$  leads to sale for sure: expected utility  $.1$ ,
  - price of  $1$  leads to sale with probability  $1/2$ , expected utility of  $1/2$ .
  - Better to set the high price.
  - *Inefficient trade*:  $(.1, 0)$  do not trade

[illegible]

- [illegible]

[illegible]

There exist distributions on the buyer's and seller's valuations such that:  
There does not exist any Bayesian incentive-compatible mechanism is  
simultaneously efficient, weakly budget balanced and interim individual  
rational.

[illegible]

- $$p_B(\theta_B) = v = -p_S(\theta_S).$$

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- $$p_B(\theta_B) = v = -p_S(\theta_S).$$

- Show the proof for *fully* budget balanced trade that is *ex post* individually rational. Extension of the proof is easy (you can do it!)
  - Trade should take place for all combinations of values except  $(\theta_B, \theta_S) = (.1, .9)$ .
  - Budget balance: we can write payments as a single price  $p(\theta_B, \theta_S)$  (payment made by buyer, received by the seller)
  - Weak budget balance: you can extend the proof - noting that the payment made by the buyer has to be at least that received by the seller.

# Proof

- (I)  $p(1, .9) \geq .9$  by individual rationality of the seller.



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# Proof

- (1)  $p(1, .9) \geq .9$  by individual rationality of the seller.
- (2)  $p(.1, 0) \leq .1$  by individual rationality of the buyer.
- (3)  $p(.1, .9) = 0$  by individual rationality of both the buyer and the seller.



# Proof

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- (4)  $p(1, 0) = ?$



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- (2)  $p(.1, 0) \leq .1$  by individual rationality of the buyer.
- (3)  $p(.1, .9) = 0$  by individual rationality of both the buyer and the seller.
- (4)  $p(1, 0) = ?$ 
  - incentive compatibility for seller of type  $\theta_S = 0$  not wanting to pretend to be  $\theta_S = .9$ :



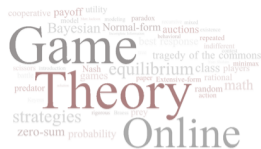
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- (3)  $p(.1, .9) = 0$  by individual rationality of both the buyer and the seller.
- (4)  $p(1, 0) = ?$ 
  - incentive compatibility for seller of type  $\theta_S = 0$  not wanting to pretend to be  $\theta_S = .9$ :
    - $\frac{p(1,0)}{2} + \frac{p(.1,0)}{2} \geq \frac{p(1,.9)}{2} + \frac{p(.1,.9)}{2},$



# Proof

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- (4)  $p(1, 0) = ?$ 
  - incentive compatibility for seller of type  $\theta_S = 0$  not wanting to pretend to be  $\theta_S = .9$ :
    - $\frac{p(1,0)}{2} + \frac{p(.1,0)}{2} \geq \frac{p(1,.9)}{2} + \frac{p(.1,.9)}{2}$ , which implies by (1), (2), (3) that
    - $p(1, 0) + .1 \geq .9 + 0$  or  $p(1, 0) \geq .8$ .



# Proof

- (1)  $p(1, .9) \geq .9$  by individual rationality of the seller.
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    - $p(1, 0) + .1 \geq .9 + 0$  or  $p(1, 0) \geq .8$ .
  - incentive compatibility for buyer of type  $\theta_B = 1$  not wanting to pretend to be  $\theta_B = .1$ :



Game Theory

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tragedy of the common mind repeated indifference behavioral decision theory paradox guided by reason leading to the prisoner's dilemma outcome is worse for both players than if they had cooperated instead of acting selfishly and the rational choice is to defect no matter what the other player does

- ## The Myerson–Satterthwaite Theorem

# Proof



- (1)  $p(1, .9) \geq .9$  by individual rationality of the seller.
- (2)  $p(.1, 0) \leq .1$  by individual rationality of the buyer.
- (3)  $p(.1, .9) = 0$  by individual rationality of both the buyer and the seller.
- (4)  $p(1, 0) = ?$ 
  - incentive compatibility for seller of type  $\theta_S = 0$  not wanting to pretend to be  $\theta_S = .9$ :
    - $\frac{p(1,0)}{2} + \frac{p(.1,0)}{2} \geq \frac{p(1,.9)}{2} + \frac{p(.1,.9)}{2}$ , which implies by (1), (2), (3) that
    - $p(1, 0) + .1 \geq .9 + 0$  or  $p(1, 0) \geq .8$ .
  - incentive compatibility for buyer of type  $\theta_B = 1$  not wanting to pretend to be  $\theta_B = .1$ :
    - $\frac{1-p(1,0)}{2} + \frac{1-p(1,.9)}{2} \geq \frac{1-p(.1,0)}{2} + \frac{1-p(.1,.9)}{2}$ , which implies by (1), (2), (3) that
    - $1 - p(1, 0) + .1 \geq .9 + 0$  or  $.2 \geq p(1, 0)$ .

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