CSE 817 Fall 2016 Assignment #1

Due: October 24^{th}, 2016, 14:30

1 2x2 Normal-Form Game

Consider the following game:

		Player 2 C	D
Player 1	A	-16,0	12,12
	B	8,8	0,-16

Where the first number in each square is the payoff of player 1 and the second number is player 2's payoff.

1. **[3 pts]** Find all Pareto optimal pure strategy profiles.
2. **[3 pts]** Find the pure strategy Nash equilibria.
3. **[3 pts]** Which of the above equilibria do you prefer? Suppose player 2 has decided to play according to one of the equilibria from question 2 (but you do not know which.) What would you play as player 1?
4. **[6 pts]** Find all mixed-strategy Nash equilibria.

2 Nash Equilibrium in 3-Player Games

	D	E
A	$0, 1, 0$	$3, 1, 2$
B	$0, 3, 1$	$2, 3, 1$
C	$2, 3, 0$	$3, 2, 1$

	D	E
A	$3, 1, 1$	$2, 2, 3$
B	$1, 2, 3$	$2, 3, 2$
C	$0, 2, 1$	$3, 2, 2$

Figure 1: A three-player normal form game. Player one chooses the row, player two chooses the column. Player three's choice of L or R corresponds to the left and right tables.

1. **[6 pts]** List all of the pure-strategy Nash equilibria of the game in Figure 1.
2. **[9 pts]** Characterize the set of Nash equilibria in which player two does not play a pure strategy.

3 Bertrand Duopoly

Two firms produce identical goods, with a production cost of c per unit. Each firm sets a nonnegative price (p_1 and p_2). All consumers buy from the firm with the lower price, if $p_i \neq p_j$. Half of the consumers buy from each firm if $p_i = p_j$. D is the total demand.

Profit of firm i is:

- 0 if $p_i > p_j$ (no one buys from firm i);
- $D(p_i - c)/2$ if $p_i = p_j$ (half of customers buy from firm i);
- $D(p_i - c)$ if $p_i < p_j$ (all customers buy from firm i).

1. [**6 pts**] Identify a *strictly* dominated strategy in this game, and explain why the strategy is strictly dominated.
2. [**6 pts**] Identify a *weakly* dominated strategy in this game that is not strictly dominated, and explain why the strategy is only weakly dominated.
3. [**8 pts**] Find a Nash equilibrium in this game.

4 n player game

In a college there are n students. They are simultaneously sending data over the colleges data network. Let $x_i \geq 0$ be the size of data sent by student i. Each student i chooses x_i, simultaneously. The speed of network is inversely proportional to the total size of the data, so that it takes $x_1 + ... + x_n$ minutes to send the message. The payoff of student i is $u_i(x_i, x_{-i}) = x_i - x_i(x_1 + ... + x_n)$.

1. [**8 pts**] Find the pure-strategy Nash equilibrium of this game, and compute the equilibrium payoffs. (*Hint: First take derivative to find best response for a single player, then apply symmetry, $x_i = x$.*)

5 Linear Programming & Nash Equilibrium

Consider the following game:

	D	E
A	3, 1	1, 2
B	0, 3	3, 1
C	2, 3	2, 1

1. [**8 points**] Construct the linear program that the support enumeration method would use to find a full-support Nash equilibrium of this game (i.e., an equilibrium where every action is played with positive probability).
2. [**5 points**] Show that this game has no full-support Nash equilibrium, by showing that your linear program is infeasible.

6 Iterated Elimination

1. [**3 points**] What strategies survive iterated elimination of strictly-dominated strategies in this normal-form game?

	L	C	R
U	$2,0$	$1,1$	$4,2$
M	$3,4$	$1,2$	$2,3$
D	$1,3$	$0,2$	$3,0$

2. [**7 points**] What are the pure strategy Nash equilibria of the game? Find a mixed-strategy NE.

3. [**7 points**] Prove that iterated elimination of strictly-dominated actions never removes an action that is in the support of any mixed-strategy Nash equilibrium.

4. [**2 points**] Give an example of a game where no action can be eliminated by iterated elimination of strictly-dominated actions.

5. [**8 points**] Explain why the time complexity of iterated elimination of strictly-dominated actions is $O(m^{n+2}n^2)$ for n players, each with m actions.

7 Correlated Equilibrium

Consider the following two-player game:

		Player 2		
		D	E	F
Player 1	A	9,10	2,5	3,4
	B	1,6	17,9	7,5
	C	0,3	2,4	6,13

1. [**12 pts**] Find a correlated equilibrium of the game where player 1 achieves an expected payoff of 14. As randomizing devices, you have three publicly observable, fair coins: a 10Kr, a 25Kr and a 50Kr coin. You may not use any other randomizing devices.