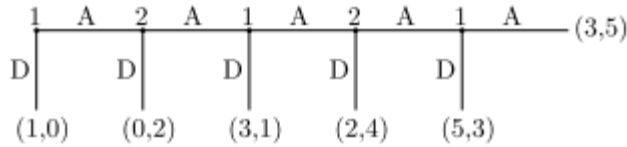# CSE 817 Homework 2

Due: November 21th, 2016, Monday

Q-1. (15 pts) Consider the following centipede game:

Suppose that Player 2 is not always rational, such that: **(i)** With probability p player 2 is a rational player who follows the unique subgame perfect equilibrium strategy. **(ii)** With probability $1 - p$ player 2 is "irrational" and simply flips a fair coin at each of his choice points. For every possible value of p, find a best response (possibly mixed) strategy for player 1 to this player 2. (Obviously there will be ranges of p for which a strategy is always a best response.) Show your work.

Q-2. (25 pts) Consider the following extensive form zero-sum game with imperfect information:
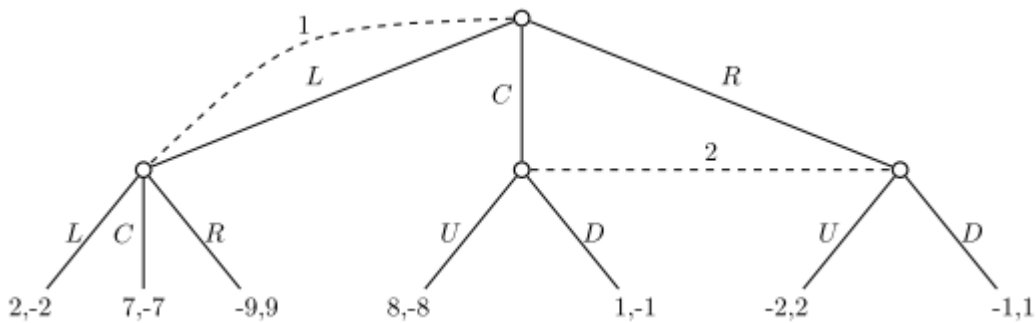

(a) What are the pure strategies for each player? List all Nash equilibrium pure strategy profiles ("none exists" is a possible answer).

(b) Show that there are infinitely many mixed Nash equilibria in this game. (Hints: (i) Try to find a simple characterization for mixed Nash equilibria, i.e. using a variable p and giving a range of p for which the equilibria holds. (ii) As an initial step, you can eliminate some strategy profiles. (iii) Try to make player 2 indifferent. But player 1 should not be indifferent. If player 1 is indifferent, then player 2 cannot be indifferent. Do you see why?)

(c) Find an equilibrium in behavioural strategies (that is different than equilibria in mixed strategies). Can Player 1 get higher payoff by adopting behavioural strategy?

Q-3. (12 pts) In the following imperfect information extensive form game, find the set of subgame perfect equilibria, in pure strategies only.
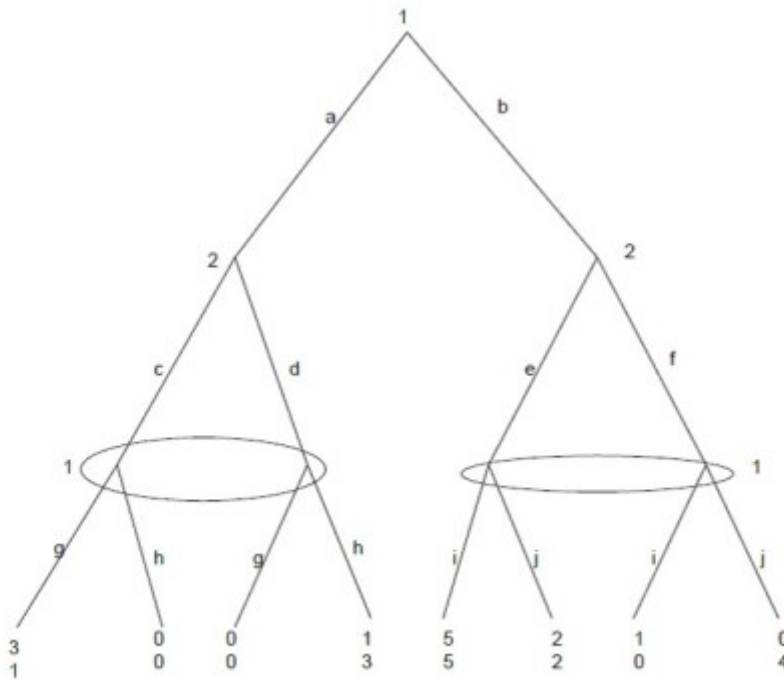
Q-4. (23 pts) Consider the following repeated Prisoners' Dilemma with discount factor $0<\delta<1$:

	Cooperate	Defect
Cooperate	4,4	-1,6
Defect	6,-1	0,0

(a) What is the grim trigger strategy? For what values of δ does there exist a SPE where both players play the grim trigger strategy?

(b) What is the tit-for-tat strategy? For what values of δ does there exist a SPE where both players play the tit-for-tat strategy?

(c) Consider the strategy: Begin by cooperating. Cooperate after any history in which there has been no more than one period in which a player defected. In all other histories, defect. Call this strategy "Forgive Once."

     i. Find a best response to the strategy Forgive Once. Is your strategy a best response to Forgive Once in every subgame?

     ii. What is the outcome of the game if player plays Forgive Once and player two plays the best response identified above. Do the strategies Forgive Once and the best response from (i) constitute a Nash Equilibrium for the repeated Prisoner's Dilemma? If so, prove it. If not, explain why not.

Q-5. (20 pts) Consider a Minimum Spanning Tree game defined as follows:

A group of customers must be connected to a critical service provided by some central facility, such as a power plant or an emergency switchboard. In order to be served, a customer must either be directly connected to the facility or be connected to some other connected customer. Let us model the customers and the facility as nodes on a graph, and the possible connections as edges with associated costs. This situation can be modeled as a coalitional game (N, v). N is the set of customers, and $v(S)$ is the cost of connecting all customers in S directly to the facility minus the cost of the minimum spanning tree that spans both the customers in S and the facility.

Now, consider the following scenario. There are three customers (1, 2 and 3) and numbers on the edges are the connection costs.
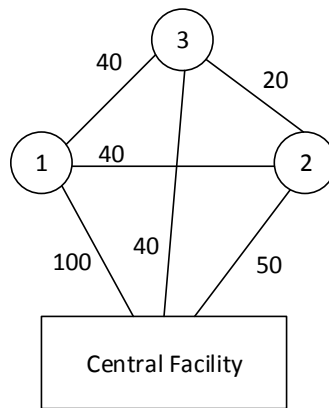
(a) Give $v(S)$ value for every possible coalition S.

(b) Calculate the Shapley value.

(c) Is the Shapley value in the core?

Q-6. (20 pts) Investigate the literature, and find an example application of **coalitional games** or **repeated games** in one of the following areas: (1) Wireless networks, (2) Network security, (3) Cloud computing, (4) Computer Vision, (5) Robotics, (6) Network Economics.

For the selected example, clearly describe the game model, and briefly explain how the game is analyzed. (One A4-page answer is enough.)