CSE 817 Assignment #3

Due: Final Exam, January 16^{th}, 2017

1 Bayesian Games

Consider a problem where two Turkish students should prepare a presentation on the same research topic for the Game Theory course. They studied and did a literature survey together, but leave the preparation of slides to the last night. At the last night before the presentation day, since they were not able to get in touch with each other, they went to their home directly. At the same night, there is also an important Champions League match of a Turkish club. Both students must simultaneously decide between working on the preparation of slides or watching the football match, but they cannot contact with each other. Each student has a preference for one of the two choices. The students dont know each others preferences, but know that they are drawn from commonly known joint distribution. This distribution is given in Table 1 (It is known that Student 2 is a fan of the Turkish club. Student 1 also likes watching football matches, but he is a supporter of another team.) Starting from a baseline utility of zero, a student gains 3 units of utility if he does the action he prefers. But if both of them watches football match and lacks to prepare the presesentation slides, they will both lose 5 units of utility. Thus, for example if they both prefer watching the match, and both watch the match, they each get a utility of $0 + 3 - 5 = -2$.

Student 1	Student 2	Probability
w_1	w_2	0.1
w_1	$\neg w_2$	0.6
$\neg w_1$	w_2	0.2
$\neg w_1$	$\neg w_2$	0.1

Table 1: The common prior joint distribution on student preferences. w_i means that student i prefers to watch the match, $\neg w_i$ means he prefers to prepare the presentation slides.

1. **[5pts]** Model the setting as a Bayesian game. Denote by W and S, the action of watching the match, and working on preparation of slides, respectively. Your answer can be a figure similar to Figure 6.7 or Figure 6.9 in the textbook (Shoham et al).
2. **[4pts]** Draw the payoff matrix of the induced normal form of the game. Explicitly state the meaning of an action in the induced normal form game. Find all Bayes-Nash equilibria of the game.
3. **[2pts]** What is the *ex-ante* expected utility to player 1 of the strategy profile (SW, WS)? ("not enough information" is a potential answer)
4. **[2pts]** What is the *ex-interim* utility to player 1 of the strategy profile (SW, WS) if player 1 has type w_1? ("not enough information" is a potential answer)
5. **[2pts]** What is the *ex-post* utility to player 1 of the strategy profile (SW, WS) if player 1 has type w_1? ("not enough information" is a potential answer)

2 Bayesian Games & Auctions

Two bidders participate in an auction to buy a painting. Their values for the painting are either 20 or 100, both being equally likely. Each individual knows her valuation but does not the valuation of the other player. If the acceptable bids are 10, 30, 50 and bidder cannot employ weakly dominated strategies.

1. **[5pts]** Find the Bayes equilibria of the first-price auction.
2. **[5pts]** Find the Bayes equilibria of the second-price auction.
3. **[5pts]** Compare the auctioneer's revenue in each auction.

3 Social Choice - 1

Each agent submits a total preference ordering, along with additional information of that agents utility for each outcome. (Obviously, the preference ordering reflects the ordering of the outcomes by utility, with the highest utility outcome being most preferred.) Let the utility for each outcome be an element of [0, 100]. The social welfare function orders each outcome by the sum of the utilities of that outcome for each agent. In the case of ties, the outcome with the earlier lexicographic ordering is preferred.

1. **[4pts]** Is this Pareto efficient? Justify.
2. **[4pts]** Is this independent of irrelevant utilities? (In this setting, define IIU as the proposition that the social ordering of o and o' does not change as long as agents do not change their utilities for o and o'.) Justify.
3. **[4pts]** Is this non dictatorial? Justify.
4. **[4pts]** Does this voting scheme contradict Arrows theorem? If yes, explain why; if not, demonstrate that Arrows theorem is not violated.

4 Social Choice - 2

Let $d(\succ, \succ')$ denote the distance between two rankings $\succ$ and $\succ'$, defined as the number of pairs o, o' for which $o \succ o'$ and $o' \succ' o$. For example, $d(A \succ B \succ C \succ D, C \succ B \succ A \succ D) = 3$ because the rankings disagree on (A, B), (A, C) and (B, C). Let $\succ_i$ denote the preferences of agent i. Define the cost of a ranking $c(\succ \mid \succ_1, \succ_2, ...)$ as the total distance between that ranking and the preferences of the agents:

$$c(\succ \mid \succ_1, \succ_2, ...) = \sum_{i=1}^{n} d(\succ_i, \succ).$$

Now, we say that a social welfare function M is cost-minimizing if it chooses a ranking $\succ$ that minimizes the cost. That is,

$$M(\succ_1, \succ_2, ...) \in \arg\min_{\succ} c(\succ \mid \succ_1, \succ_2, ...).$$

1. **[3pts]** Consider the following preferences:

$$D \succ_1 C \succ_1 B \succ_1 A$$
$$D \succ_2 B \succ_2 C \succ_2 A$$
$$C \succ_3 A \succ_3 D \succ_3 B$$
$$C \succ_4 D \succ_4 B \succ_4 A$$
$$C \succ_5 D \succ_5 B \succ_5 A$$

   What ranking will M select?

2. **[3pts]** Give an example preferences for which the outcome selected by Borda is different than the outcome ranked first by M.

3. **[5pts]** Prove that if a Condorcet winner exists, it will be ranked first by M.

4. **[5pts]** Consider a mechanism such that agents provide their ranking (simultaneously) and the mechanism uses M to make a decision and the outcome ranked highest by M will be chosen. (When there are multiple cost-minimizing rankings, assume that M breaks ties alphabetically). Demonstrate (by providing a counterexample) that in this mechanism, it is not a (very weak) dominant strategy for the agents to report their true preferences, even for the case of three agents and three outcomes.

5 Mechanism Design - Auctions

1. **[5pts]** Consider a single-item auction with at least three bidders. Prove that awarding the item to the highest bidder, at a price equal to the third-highest bid, yields an auction that is not dominant-strategy incentive compatible (DSIC).

2. **[5pts]** Suppose there are k identical copies of a good and $n > k$ bidders. Suppose also that each bidder can receive at most one good. What is the analog of the second-price auction? Prove that your auction is DSIC.

3. **[5pts]** Use Myersons Lemma to prove that the Vickrey auction is the unique single-item auction that is DSIC, always awards the good to the highest bidder, and charges losers 0.

6 VCG Mechanism - 1

[10pts] Consider two goods (A and B) and three bidders. Bidder 1 has valuation 1 for A and B together (i.e., $v_1(AB) = 1$) and 0 otherwise. Bidder 2 has valuation 1 for A (i.e., $v_2(AB) = v2(A) = 1$) and 0 otherwise. Bidder 3 has valuation 1 for B and 0 otherwise. Compute the VCG allocation and payments when only the first two bidders are present. Do the same when all three bidders are present.

Can adding an extra bidder ever decrease the revenue of the Vickrey (single-item) auction? Give a brief explanation.

7 VCG Mechanism - 2

A mechanism designer wishes to find the lowest-cost route from node A to node F in the network of Figure 1. Each edge is controlled by a single agent.

1. **[5pts]** Compute the VCG outcome and each agent's payments if every agent declares the costs labelled on the edges.

2. **[2pts]** Which agent gets the highest net reward? Explain why that agent has higher reward than the others.

3. **[4pts]** How would the situation change if the edge DF had a cost of 5 instead of 2? Calculate all payments of all agents and discuss your result.

4. **[4pts]** Again consider the case where cost of DF is 5. Can any two agent increase their revenue by colluding? If yes, give an example. If no, give the reason.

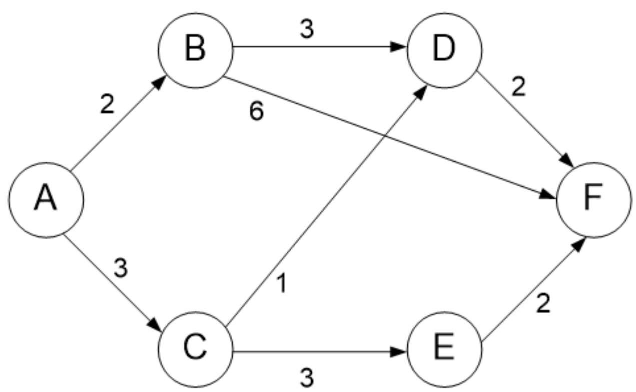

Figure 1: Reported costs for a routing network.