

[illegible]

Maxmin Strategies

Game Theory Course:
Jackson, Leyton-Brown & Shoham

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- ## Definition (Maxmin)

- Why would i want to play a maxmin strategy?

Maxmin Strategies

- Player i 's **maxmin strategy** is a strategy that maximizes i 's worst-case payoff, in the situation where all the other players (whom we denote $-i$) happen to play the strategies which cause the greatest harm to i .
- The **maxmin value** (or **safety level**) of the game for player i is that minimum payoff guaranteed by a maxmin strategy.

Definition (Maxmin)

The **maxmin strategy** for player i is $\arg \max_{s_i} \min_{s_{-i}} u_i(s_1, s_2)$, and the **maxmin value** for player i is $\max_{s_i} \min_{s_{-i}} u_i(s_1, s_2)$.

- Why would i want to play a maxmin strategy?
 - a conservative agent maximizing worst-case payoff
 - a paranoid agent who believes everyone is out to get him



Game Theory

Bayesian Normal-form auctions

equilibrium class players

strategies zero-sum probability Online

Nash equilibria rational math action

repeated tragedy of the commons

predator cooperative payoff utility

parades behavioral adulterous antimax

paper Extensive-form random math action

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Definition (Minmax, 2-player)

In a two-player game, the **minmax strategy** for player i against player $-i$ is $\arg \min_{s_i} \max_{s_{-i}} u_{-i}(s_i, s_{-i})$, and player $-i$'s **minmax value** is $\min_{s_i} \max_{s_{-i}} u_{-i}(s_i, s_{-i})$.

- Why would i want to play a minmax strategy?

-

Definition (Minmax, 2-player)

In a two-player game, the **minmax strategy** for player i against player $-i$ is $\arg \min_{s_i} \max_{s_{-i}} u_{-i}(s_i, s_{-i})$, and player $-i$'s **minmax value** is $\min_{s_i} \max_{s_{-i}} u_{-i}(s_i, s_{-i})$.

- Why would i want to play a minmax strategy?
 - to punish the other agent as much as possible

Minmax Theorem

Theorem (Minimax theorem (von Neumann, 1928))

In any finite, two-player, zero-sum game, in any Nash equilibrium each player receives a payoff that is equal to both his maxmin value and his minmax value.

Game Theory Online



Minmax Theorem

Theorem (Minimax theorem (von Neumann, 1928))

In any finite, two-player, zero-sum game, in any Nash equilibrium each player receives a payoff that is equal to both his maxmin value and his minmax value.

1. Each player's maxmin value is equal to his minmax value. The maxmin value for player I is called the **value of the game**.



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2. For both players, the set of maxmin strategies coincides with the set of minmax strategies.

Game Theory

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extensive-form paper

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paradox

cooperative payoff utility

added reason modeling

behavioral trust coincide

predator aggressive status PACV

aggressive status PACV

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strategies zero-sum probability Online

math random action

class rational

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Game Theory

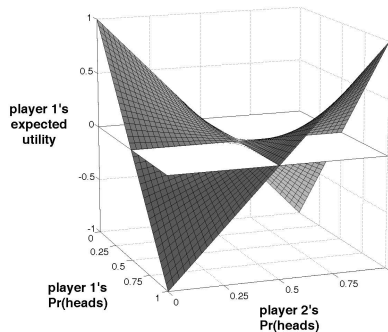
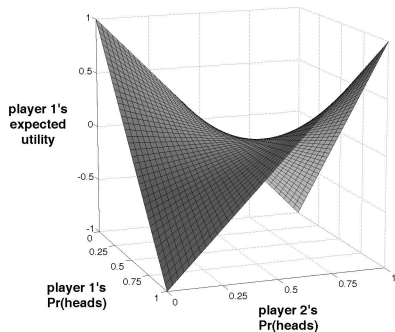
Equilibrium Nash Bayesian Normal-form Auctions Tragedy of the commons Class Paper Extensive-form Random Math Action Online Probability Zero-sum Strategies Predator Games Repeated Response Belief Adaptive Minimax Rational Signaling

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Saddle Point: Matching Pennies

Game Theory Online

cooperative payoff utility
Bayesian Normal-form auctions
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predator strategies
zero-sum probability
paradox
repeated
rational
math
action
random
maximise
different
behavior
model
evolution
stochastic
game
theory
online



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- ## Maxmin Strategies

Penalty Kick Game



		Goalie	
		L	R
Kicker	L	0.6, 0.4	0.8, 0.2
	R	0.9, 0.1	0.7, 0.3

How does the kicker maximize his minimum?

A word cloud featuring various terms related to game theory and online interactions. The most prominent words are "Game Theory" and "Online". Other significant words include "Bayesian Normal-form auctions", "Nash equilibria", "class projects", "rational action", "strategies", "zero-sum probability", "predator", "cooperative payoff utility", "added common modeling", "paradoxes", "repeated", "tragedy of the commons", "outcome", "selfishness", "social dilemmas", "behavioral", "past experience", "outcomes", "actions", "conditions", "timing", "paper Extensive-form rationality", "random action", "epistemic decision theory", and "math".

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Penalty Kick Game

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What is his minimum?



Penalty Kick Game

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		L	R
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	R	0.9, 0.1	0.7, 0.3

What is his minimum?

$$\min_{s_2} [s_1(L)s_2(L) \cdot 0.6 + s_1(L)s_2(R) \cdot 0.8 + s_1(R)s_2(L) \cdot 0.9 + s_1(R)s_2(R) \cdot 0.7]$$

$$= \min_{s_2} \left[\begin{aligned} &s_1(L)s_2(L) \cdot 0.6 + s_1(L)(1 - s_2(L)) \cdot 0.8 + \\ &+ (1 - s_1(L))s_2(L) \cdot 0.9 + (1 - s_1(L))(1 - s_2(L)) \cdot 0.7 \end{aligned} \right]$$



Game Theory

cooperative payoff utility added common modeling paradoxes repeated rational Bayesian Normal-form auctions science behavioral Nash equilibrium class rational mathematics predator strategies zero-sum probability Online action random Extensive-form paper games tragedy of the commons outcome science

What is his minimum?

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What is his minimum?

$$= \min_{s_2} \left[s_1(L)s_2(L) \cdot 0.6 + s_1(L)(1 - s_2(L)) \cdot 0.8 + \right. \\ \left. + (1 - s_1(L))s_2(L) \cdot 0.9 + (1 - s_1(L))(1 - s_2(L)) \cdot 0.7 \right]$$

$$\Rightarrow 0.2 - s_1(L) \cdot 0.4 = 0$$

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Penalty Kick Game



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How does the goalie minimize the kicker's maximum?

Penalty Kick Game



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How does the goalie minimize the kicker's maximum?

$$\min_{s_2} \max_{s_1} [s_1(L)s_2(L) \cdot 0.6 + s_1(L)s_2(R) \cdot 0.8 + s_1(R)s_2(L) \cdot 0.9 + s_1(R)s_2(R) \cdot 0.7]$$

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What is the kicker's maximum?

Penalty Kick Game

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Kicker	L	0.6, 0.4	0.8, 0.2
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What is the kicker's maximum?

$$\max_{s_1} [s_1(L)s_2(L) \cdot 0.6 + s_1(L)s_2(R) \cdot 0.8 + s_1(R)s_2(L) \cdot 0.9 + s_1(R)s_2(R) \cdot 0.7]$$

$$= \max_{s_1} \left[\begin{aligned} &s_1(L)s_2(L) \cdot 0.6 + s_1(L)(1 - s_2(L)) \cdot 0.8 + \\ &+ (1 - s_1(L))s_2(L) \cdot 0.9 + (1 - s_1(L))(1 - s_2(L)) \cdot 0.7 \end{aligned} \right]$$



Penalty Kick Game



		Goalie	
		<i>L</i>	<i>R</i>
Kicker	<i>L</i>	0.6, 0.4	0.8, 0.2
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What is the kicker's maximum?

$$\begin{aligned} & \max_{s_1} [s_1(L)s_2(L) \cdot 0.6 + s_1(L)s_2(R) \cdot 0.8 + s_1(R)s_2(L) \cdot 0.9 + s_1(R)s_2(R) \cdot 0.7] \\ &= \max_{s_1} \left[\begin{aligned} & s_1(L)s_2(L) \cdot 0.6 + s_1(L)(1 - s_2(L)) \cdot 0.8 + \\ & + (1 - s_1(L))s_2(L) \cdot 0.9 + (1 - s_1(L))(1 - s_2(L)) \cdot 0.7 \end{aligned} \right] \\ &= \max_{s_1} [(0.1 + s_2(L) \cdot 0.4) \cdot s_1(L) + (0.7 + s_2(L) \cdot 0.2)] \end{aligned}$$

[illegible][illegible][illegible]

[illegible]

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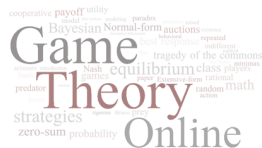
[illegible]

A word cloud visualization centered around the theme of Game Theory. The words are arranged in various sizes and orientations, with "Game Theory" being the largest and most prominent. Other significant words include "Bayesian Normal-form auctions", "equilibrium class players", "Nash equilibria", "predator", "strategies", "zero-sum probability", "Online", "cooperative payoff utility", "paradox", "repeated", "tragedy of the commons", "rational", "math", "action", "paper Extensive-form", "random", "game theory (1970)", "decision making", "behavioral", "self-interest", "adifferent", "antimax", "rational", "math", "action", "game theory (1970)".

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Computing Minmax

For 2 players minmax is solvable with LP (Linear Programming).



1. *Journal of the American Medical Association*, 1997; 277: 1039-1043.