# MATH 171 MIDTERM EXAM 05/11/2013

**Name:** **Student ID # :** **Section #:**

| Q1 (10) | Q2(15) | Q3(15) | Q4(20) | Q5(20) | Q6(20) | Total (100) |
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**ATTENTION: Please show all your work in details. DO NOT USE calculators and cellphones. There are 6 questions on 4 pages. Solve all of them. Duration is ONE hour.**

**1-** (10) Solve the following inequality for $x$

$$|-3| < \left|\frac{5x-7}{-11}\right|$$

**Solution:** This inequality is equivalent to

$$3 < \frac{|5x-7|}{11} \text{ implies that } 33 < |5x-7| .$$

Therefore we have
$$33 < 5x - 7 \ \ or - 33 > 5x - 7$$
$$40 < 5x \ \ or \ -26 > 5x$$
$$8 < x \ \ or \ \frac{-26}{5} > x$$

Hence, the solution of the inequality is $\left(-\infty, \frac{-26}{5}\right) \cup (8, +\infty)$.

**2-** (15) Let $f(x) = x^2 - 4x + 4$ , for x≥2 and $g(x) = x + 1$. Then find $(fog)(x),\ and\ (fog)^{-1}(x)$.

**Solution:** Let $f(x) = x^2 - 4x + 4 = (x-2)^2$ and $g(x) = x + 1$. Then

$$h(x) = (fog)(x) = f\big(g(x)\big) = f(x+1) = (x+1-2)^2 = (x-1)^2.$$

Let $h^{-1}(x) = (fog)^{-1}(x)$. Then $(hoh^{-1})(x) = h\big(h^{-1}(x)\big) = x$ ,

$$(h^{-1}(x) - 1)^2 = x \ \ for\ x \geq 2\ or\ \ h^{-1}(x) = \sqrt{x} + 1\ for\ x \geq 2 .$$

Or, $h^{-1}(x) = (fog)^{-1}(x)$. $h^{-1}(x) = (fog)^{-1}(x) = ((g)^{-1}o(f)^{-1})(x)$

where $f^{-1}(x) = \sqrt{x} + 2\ for\ x \geq 0$ and $g^{-1}(x) = x - 1$. Hence we have obtained the same result;

$$h^{-1}(x) = \Big((g)^{-1}\big(((f)^{-1})(x)\big)\Big) = \sqrt{x} + 2 - 1 = \sqrt{x} + 1\ for\ x \geq 0 .$$

**3-**(15) The Car care company manufactures a product that has a selling price of \$40 and a unit cost of \$30. If fixed costs are \$1,200,000, determine the least number of units that must be sold for the company to have a profit.

**Solution:** Let q be the number of units that must be sold. Then variable cost is 30q and so the total cost is 30q+1,200,000. The total revenue is 40q. Since we want to profit > 0, we have

$$\text{Total revenue-Total cost} > 0$$

$$40q-(30q + 1{,}200{,}000)>0$$

$$10q>1{,}200{,}000$$

$$q>120{,}000.$$

So, the least number of units that must be sold is $q = 120001$.

**4-**(20) Find the equations of the lines that passes through the point $(-1,-2)$ one is parallel and the other one is perpendicular to the line $-4y + 8x + 5 = 0$. (do not sketch it)

**Solution:** The line is parallel to $y = 2x + 5/4$ also has slope 2 ( $m_1 = m_2$ =2).

Using point-slope form we get

$$y-(-2) = 2\big(x-(-1)\big),\ y+2 = 2(x+1),$$

or

$$y = 2x.$$

Slope of the perpendicular line to $y = 2x + 5/4$ must be

$$m_1 \quad = -\frac{1}{m_2} = -\frac{1}{2} = 2.$$

Using point- slope form we get $y + 2 = -\frac{1}{2}(x+1)$, or

$$y = -\frac{1}{2}x - \frac{5}{2}.$$

The perpendicular line of the equation is found.

**5-**(20) For the equation $f(x) = -(2x-1)^2 - 8x + 9$ .

(a) Find the intercepts. (b) Find the vertex. (c) State the domain and the range. (d) Find the symetry axis (e) Sketch its graph.

**Solution** (a) x-intercept when $f(x) = 0 = x^2 + x - 2 = (x+2)(x-1) implies\ x = 1\ and\ x = -2.$

y-intercept when x=0 then $f(0) = -(2.0-1)^2 - 8.0 + 9 = c = 8.$

(b) Vertex

$f(x) = -(2x-1)^2 - 8x + 9$

$$f(x) = -(4x^2 - 4x + 1) \quad -8x + 9 = -4x^2 - 4x + 8 = -4(x^2 + x - 2)$$

$$a = -4, b = -4, c = 8$$

$$Vertex\ at\ x = \frac{-b}{2a} = \frac{4}{-8} = -\frac{1}{2}$$

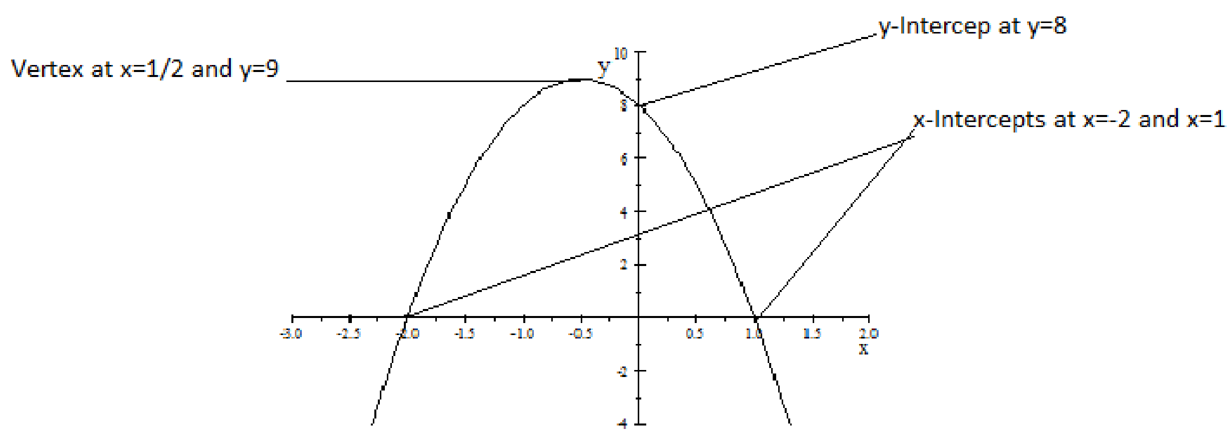
$$f\left(-\frac{1}{2}\right) = -\left(2\left(-\frac{1}{2}\right) - 1\right)^2 - 8\left(-\frac{1}{2}\right) + 9 = -(-2)^2 + 4 + 9 = 9.$$

© Range $-\infty < f(x) \le 9\ or\ (-\infty, 9]$ and Domain: $-\infty < x < +\infty\ or\ (-\infty, +\infty).$

(d) The symmetry axis is the vertex

$Vertex\ at\ x = \frac{-b}{2a} = \frac{4}{-8} = -\frac{1}{2}$

(e) The graph of the function $f(x) = -(2x-1)^2 - 8x + 9$ is



6) (20) Suppose consumers will demand 60 units of a product when the price $12.30 per unit and 36 units when the price $16.30 each. Find the demand equation, assuming that is linear. Find the price per unit when 48 units are demanded.

**Solutions:** The slope of the line passing through (60, 12.30) and (36, 16.30) is

$$m = \frac{16.30 - 12.30}{36 - 60} = \frac{4}{-24} = -\frac{1}{6}$$

An equation of the line-point slope form is

$$p - p_1 = m(q - q_1)$$
$$p - 12.30 = -\frac{1}{6}(q - 60)$$

Hence, the demand equation is

$$p = -\frac{1}{6}q + 22.30.$$

The price per unit when 48 units are demanded is

$$p = -\frac{1}{6}48 + 22.30 = -8 + 22.30 = 14.30\ \$.$$