MATH 171 MIDTERM EXAM 11/07/2012

Name: **Student ID # :**

Q1 (15)	Q2(15)	Q3(15)	Q4(20)	Q5(20)	Q6(15)	Total (100)

ATTENTION: There are 6 questions on 4 pages. Solve all of them. Duration is ONE hour.

1- (15) Solve the following inequality

$$\left|\frac{3x-8}{2}\right| \geq 4$$

Solution:

$$\frac{3x-8}{2} \leq -4 \quad \text{or,} \quad \frac{3x-8}{2} \geq 4$$

$$3x-8 \leq -8 \quad \text{or,} \quad 3x-8 \geq 8$$

$$3x \leq 0 \quad \text{or,} \quad 3x \geq 16$$

$$x \leq 0 \quad \text{or,} \quad x \geq \frac{16}{3}$$

The solution is $(-\infty, 0] \cup \left[\frac{16}{3}, \infty\right)$

2- (15) Solve the following equation for *x*:

$$\ln(x+3) + \ln 4 = 2\ln x$$

Solution:

$$\ln\left(4(x+3)\right) = 2\ln x \quad \text{or,} \quad \ln\left(4(x+3)\right) = \ln x^2$$

$$4x+12 = x^2 \quad \text{or} \quad x^2 - 4x - 12 = 0$$

$$(x-6)(x+2) = 0$$

Only $x = 6$ satisfies ($x > 6$)

3-(15) The ABC company manufactures a product that has a selling price of $40 and a unit cost of $30. If fixed costs are $100,000, determine the least number of units that must be sold for the company to have a profit.

Solution: Let q be the number of units that must be sold. Then variable cost is $30q$ and so the total cost is $30q+100000$. The total revenue is $40q$. Since we want to profit>0, we have

total revenue-total cost >0

$$40q-(30q+100000)>0$$

$$10q>100000$$

$$q>10000$$

So, the least number of units that must be sold is $q=10001$

4-(20) Two lines passing through (-2,1). a) One is parallel to the line $y=\frac{1}{2}x+1$ and b) the other perpendicular to it. Find equations of these lines. (do not sketch it)

Solution: a) line parallel to $y=\frac{1}{2}x+1$ also has slope $\frac{1}{2}$ ($m_1=m_2$). Using point-slope form we get

$$y-(1)=\tfrac{1}{2}(x-(-2)) \text{ or,}$$

$$y=\tfrac{1}{2}x+2$$

b)Slope of perpendicular line to $y=\frac{1}{2}x+1$ must be -2 ($m_1=-\frac{1}{m_2}$). Using point-slope form we get

$$y-(1)=-2(x-(-2)) \text{ or,}$$

$$y=-2x-3$$

5-(20) For the equation $y = x^2 - 8x + 14$, (a) find the intercepts, (b) find the vertex, (c) state the domain and the range (d) and then sketch it within the grid area.

Solution:

$$y = x^2 - 8x + 14 \qquad a = 1, b = -8, c = 14$$

$$Vertex: -\frac{b}{2a} = -\frac{-8}{2\square 1} = 4 \qquad f(4) = 4^2 - 8(4) + 14 = -2 \quad \text{Vertex=(4,-2)}$$

Y-intercept: $c = 14 \Rightarrow (0,14)$

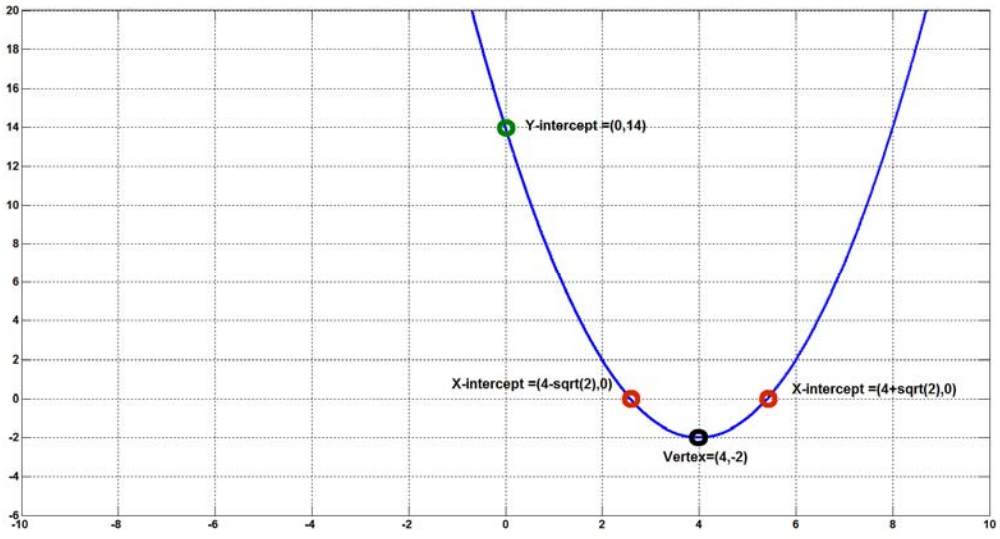
X-intercepts: solving $x^2 - 8x + 14 = 0$ by the quadratic formula,

$$x = \frac{(-8) \pm \sqrt{(-8)^2 - 4(1)(14)}}{2(1)} = \frac{8 \pm \sqrt{8}}{2} = \frac{8 \pm 2\sqrt{2}}{2} = 4 \pm \sqrt{2}$$

$$\Rightarrow (4+\sqrt{2}, 0), (4-\sqrt{2}, 0)$$

$$Rang: all \quad y \geq -2 \quad or \quad -2 \geq f(x) \geq \infty \quad or \quad [-2, \infty)$$

$$Domain: \text{ - } -\infty \leq x < \infty \qquad \text{or} \qquad (-\infty, \infty)$$



6- (15) How long will it take for 1000TL to amount to 1500TL at an annual rate of 12% compounded monthly? (Use table if necessary)

Table

ln(1.12)/log(1.5)=0.28	ln(1.01)/ln(1.5)=0.02	ln(1.5)/ln(1.12)=3.58	ln(1.5)/ln(1.01)=40.75

Solution: Compound amount : $S = P(1+r)^n$ where

$S = 1500$, $P = 1000$ TL and the periodic rate $r = 0.12/12 = 0.01$. The number of interest periods is found as

$$1500 = 1000(1+0.01)^n$$

$$\ln 1.5 = n\ln(1.01) \text{ or } n = \ln 1.5/\ln(1.01) \approx 40.75 \quad (month)$$

The number of years is then given by $40.75/12 \approx 3.4$