MATH 171 FINAL EXAM SUMMER 2012 (06/08/2012)

Name: **Student ID #:**

Q1 (20)	Q2(15)	Q3(15)	Q4(15)	Q5(15)	Q6(20)	Total (100)

ATTENTION: There are 6 questions on 4 pages. Solve all of them. Duration is 90 minutes.

1- a (5) Find $\lim_{x\to -2}\left(\frac{x^2+2x}{x+2}\right)$. (Do not use L'Hospital)

Solution:

$$\lim_{x\to -2}\left(\frac{x^2+2x}{x+2}\right)=\lim_{x\to -2}\frac{x(x+2)}{x+2}=\lim_{x\to -2}x=-2$$

b (10) Find $\lim_{x\to\infty}\frac{x}{(3x-1)^2}$. (Do not use L'Hospital)

Solution:

$$\lim_{x\to\infty}\frac{x}{(3x-1)^2}=\lim_{x\to\infty}\frac{x}{9x^2-6x+1}$$

$$=\lim_{x\to\infty}\frac{x}{9x^2}=\lim_{x\to\infty}\frac{1}{9x}=\frac{1}{9}\lim_{x\to\infty}\frac{1}{x}=\frac{1}{9}(0)=0$$

2- (15) Find an equation of the tangent line to the curve $y = 3x^3 - 2x^2 + 1$ when $x_0 = 1$.

Solution:

$$y = 3x^3 - 2x^2 + 1$$

$$y' = 9x^2 - 4x \quad \Rightarrow \quad m = y'\big|_{x_0 = 1} = 9 - 4 = 5$$

$$y_0\big|_{x_0 = 1} = 3 - 2 + 1 = 2$$

$$y - y_0 = m(x - x_0)$$

$$y - 2 = 5(x - 1)$$

$$y = 5x$$

3- (15) If $y = 3u^3 - u^2 + 7u - 2$ and $u = 5x - 2$, find dy/dx when $x = 1$. (Hint: use the chain rule)

Solution:

$$y' = \frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx} = \left(9u^2 - 2u + 7\right)(5)$$

$$u\big|_{x=1} = 5 - 2 = 3$$

$$\left.\frac{dy}{dx}\right|_{x=1} = \left(9(3^2) - 2(3) + 7\right)(5) = (81 - 6 + 7)5 = 410$$

4- (15) Find the derivative of $y = x^{2x}$. (Use logarithmic differentiation)

Solution:

$$\ln y = \ln x^{2x}$$

$$\ln y = 2x \ln x$$

$$\frac{y'}{y} = 2\ln x + 2x\frac{1}{x}$$

$$y' = 2y(1 + \ln x)$$

$$y' = 2x^{2x}(1 + \ln x)$$

5- (15) If $x + xy + y^2 = 7$, find dy/dx at (1,2) (Use implicit differentiation).

Solution:

$$1 + (y + xy') + 2yy' = 0$$

$$xy' + 2yy' = -(y+1)$$

$$y'(x + 2y) = -(y+1)$$

$$y' = \frac{-(y+1)}{(x+2y)}$$

$$y'(1,2) = \frac{-(2+1)}{(1+4)} = -\frac{3}{5}$$

6- (20) For the function $f(x)=2+x-x^2$, determine; a) intercepts, b) intervals on which is increasing, decreasing, concave up, and concave down, c) relative maxima and minima. d) And then sketch the graph.

Solution:

(a) Intercepts $(2,0),(-1,0)$, and $(0,2)$

(b) $y'=1-2x$ CV: $x=\dfrac{1}{2}$. => Increasing on $\left(-\infty,\frac{1}{2}\right)$ and decreasing on $\left(\frac{1}{2},\infty\right)$.

$y''=-2$ => Concave Down on $(-\infty,\infty)$

(c) Relative maximum at $\left(\frac{1}{2}, f\left(\frac{1}{2}\right)=\frac{9}{4}\right)$

(d) Sketching the graph of function;