MATH 171 MIDTERM EXAM (FALL 2012) 01/11/2012

Name: **Student ID # :**

Q1 (15)	Q2(15)	Q3(15)	Q4(20)	Q5(20)	Q6(15)	Total (100)

ATTENTION: There are 6 questions on 4 pages. Solve all of them. Duration is ONE hour.

1- (15) Solve the following inequality

$$|3-2x| \geq 4$$

Solution:

$$3-2x \geq 4 \text{ or } 3-2x \leq -4$$

$$-2x \geq 1 \text{ or } -2x \leq -7$$

$$x \leq -\tfrac{1}{2} \text{ or } x \geq \tfrac{7}{2}$$

The solution is $\left(-\infty, -\tfrac{1}{2}\right] \cup \left[\tfrac{7}{2}, \infty\right)$

2- (15) Solve the following equation for *x*:

$$\log_2 x + \log_4 x = 3$$

Solution:

$$\log_2 x + \frac{\log_2 x}{\log_2 4} = 3$$

$$\log_2 x + \frac{\log_2 x}{2} = 3 \qquad (\log_2 2^2 = 2\log_2 2 = 2(1))$$

$$\frac{3}{2}\log_2 x = 3 \quad => \quad \log_2 x = 2 \quad => \quad \log_2 x = 2 \quad => \quad x = 2^2 => \quad x = 4$$

3-(15) The T-Shirt manufacturer produces N shirts at a total labor cost (in dollars) of 1.1 N and a total material cost of 0.4N. The fixed cost for the plant is $7200. If each shirt sells for $3.5, how many must be sold by the company to realize a profit.

Solution: Let N=required number of shirts. Then,

Total revenue=3.5N, and Total Cost=1.1N+0.4N+7200

Profit>0

3.5N-(1.1N+0.4N+7200) >0

2N-7200>0

2N>7200

N>3600 (At least 3601shirt must be sold)

4-(20) Two lines passing through (-3,2). a) One is parallel to the line $2y+4x-2=0$ and b) the other perpendicular to it. Find equations of these lines. (do not sketch it)

Solution: a) line parallel to $y=-2x+1$ also has slope -2 ($m_1=m_2$). Using point-slope form we get

$$y-(2)=-2(x-(-3)) \text{ or}$$

$$y=-2x-4$$

b)Slope of perpendicular line to $y=-2x+1$ must be $\frac{1}{2}$ ($m_1=-\frac{1}{m_2}$). Using point-slope form we get

$$y-(2)=\frac{1}{2}(x-(-3)) \text{ or,}$$

$$y=\frac{1}{2}x+\frac{3}{2}+2 \text{ or,}$$

$$y=\frac{1}{2}x+\frac{7}{2}$$

5-(20) For the equation $y=(2x-2)^2+8x-8$, (a) find the intercepts, (b) find the vertex, (c) state the domain and the range (d) and then sketch it.

Solution:

$$y=(2x-2)^2+8x-8 \qquad \text{or} \qquad y=4x^2-8x+4+8x-8$$

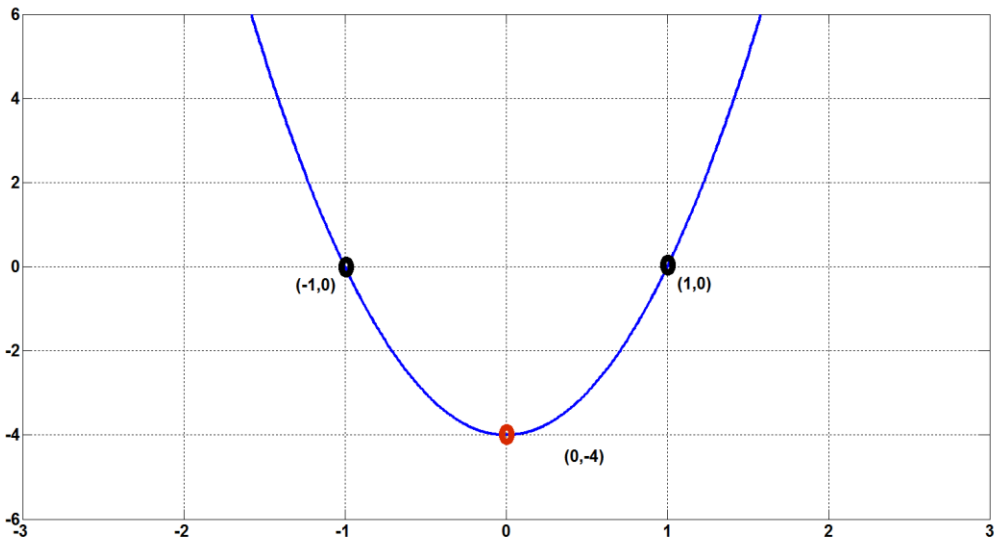
$$y=4x^2-4 \qquad \text{where,} \quad a=4, b=0, c=-4$$

a) $y-\text{intercept}: c=-4$ and $x-\text{intercept}: 4x^2-4=0 \Rightarrow x=\pm 1$

b) $x_{vertex}=-\dfrac{b}{2a}=-\dfrac{0}{8}=0$ and $y_{vertex}(x_{vertex}=0)=4(0)^2-4=-4$

c) Domain: $-\infty \le x < \infty$ or $(-\infty, \infty)$

d) Range: all $y \ge -4$ *or* $-4 \ge f(x) \ge \infty$ *or* $[-4, \infty)$



6-(15) A trust fund for 10-years old child is being set up by a single payment so that at age 25 the child will receive $60,000. Find how much the payment is if an interest rate of 6% compounded semiannually. (For your convenience use the following table)

$(1.06)^{-15} = 0.417$	$(1.06)^{15} = 2.396$	$(1.03)^{-15} = 0.641$	$(1.03)^{15} = 1.558$
$(1.06)^{-30} = 0.174$	$(1.06)^{30} = 5.743$	$(1.03)^{-30} = 0.412$	$(1.03)^{30} = 2.427$

Solution: Present value of payment: $P = S(1+r)^{-n}$ where

$$S = 60000, \quad r = 0.06/2 = 0.03, \quad \text{and} \quad n = 15 \times 2 = 30.$$

Then the payment can be calculated as:

$$P = 60000(1+0.03)^{-30} = 60000(0.412) = 24720$$