MATH 171 FINAL EXAM (30.12.2010)

Name: **Student No:**
Instructor: **Signature:**
Section:

Q1	Q2	Q3	Q4	Q5	Total

ATTENTION: There are 5 questions on 5 pages. Solve all of them. Duration is 90 minutes. Simply giving a final result is not sufficient to answer any question, so show all the steps you pursued to get any final result. Otherwise your answer will not be evaluated as a correct answer.

1. *a.* A debt of 1500 TL due in four years is to be repaid by a single payment at the end of the second year. How much is the payment if the interest rate of 20% compounded semiannually is assumed. ($1.1^2 = 1.21$; $1.1^4 \approx 1.5$) **(10 Points)**

Solution:

$$P = 1500\left(1+\frac{0.20}{2}\right)^{-2\times(4-2)} = 1500\times 1.1^{-4} = \frac{1500}{1.5} = 1000\,TL$$

b. If $y=\sqrt{\dfrac{(x-1)^3}{e^{x^2-1}}}$ **,** find dy/dx by using logarithmic differentiation. **(10 points)**

Solution:

$$y=\sqrt{\frac{(x-1)^3}{e^{x^2-1}}}$$

$$\ln y = \ln\left(\frac{(x-1)^3}{e^{x^2-1}}\right)^{\frac{1}{2}} = \frac{1}{2}\left\{\ln(x-1)^3 - \ln e^{x^2-1}\right\}$$

$$\ln y = \frac{1}{2}\left\{3\ln(x-1) - (x^2-1)\right\}$$

$$\frac{y'}{y} = \frac{1}{2}\left\{3\frac{1}{x-1} - 2x\right\}$$

$$y' = \frac{y}{2}\left\{\frac{3-2x^2+2x}{x-1}\right\} = -\frac{1}{2}\sqrt{\frac{(x-1)^3}{e^{x^2-1}}}\,\frac{2x^2-2x-3}{x-1}$$

2. *a.* Find the following limit if it exists. **(10 points)**

$$\lim_{x\to 4}\frac{\sqrt{x}-2}{x-4}=?$$

Solution:

1st Method

$$\lim_{x\to 4}\frac{\sqrt{x}-2}{x-4}=\lim_{x\to 4}\frac{\left(\sqrt{x}-2\right)\left(\sqrt{x}+2\right)}{(x-4)\left(\sqrt{x}+2\right)}=\lim_{x\to 4}\frac{\cancel{(x-4)}}{\cancel{(x-4)}\left(\sqrt{x}+2\right)}=\frac{1}{4}$$

2nd Method

$$\lim_{x\to 4}\frac{\sqrt{x}-2}{x-4}=\lim_{x\to 4}\frac{\cancel{\left(\sqrt{x}-2\right)}}{\cancel{\left(\sqrt{x}-2\right)}\left(\sqrt{x}+2\right)}=\lim_{x\to 4}\frac{1}{\sqrt{x}+2}=\frac{1}{4}$$

b. Find the limit $\lim_{x\to\pm\infty}\frac{-3x^3+2x-5}{-4-3x+4x^3}$. **(10 points)**

Solution:

Bu sorunun son hali elimde değil ama yapı olarak böyle bir şeydi. Cevap pay ve paydadaki en büyük dereceli terimlerin limitine bakmak yetiyor..

$$\lim_{x\to\pm\infty}\frac{-3x^3+\cancel{2x-5}}{\cancel{-4-3x}+4x^3}=\lim_{x\to\pm\infty}\frac{-3x^3}{4x^3}=\frac{-3}{4}$$

3. *a*. Find the slope of the tangent line to the curve $y = x^e + e^x + 3^e$ when $x = 1$. **(10 points)**

Solution:

$$y' = ex^{e-1} + e^x$$

The slope is $y'\big|_{x=1} = e(1^{e-1}) + e^1 = 2e$

***b*.** If $y + 3 = xe^y - x^3$**,** find *dy*/*dx* by using implicit differentiation. **(10 points)**

Solution:

$$(y+3)' = (xe^y - x^3)'$$

$$y' = e^y + xe^y y' - 3x^2$$

$$y'(1 - xe^y) = e^y - 3x^2$$

$$y' = \frac{e^y - 3x^2}{1 - xe^y}$$

4. *a.* If $y=\sqrt[3]{z}$ and $z=x^4-x^3+2$, find $\dfrac{dy}{dx}$ by using chain rule and express it in terms of x. **(10 points)**

Solution:

$$y=\sqrt[3]{z}=z^{\frac{1}{3}}$$

$$\frac{dy}{dx}=\frac{dy}{dz}\frac{dz}{dx}=\frac{1}{3}z^{\frac{1}{3}-1}\left(4x^3-3x^2\right)=\frac{4x^3-3x^2}{3z^{\frac{2}{3}}}=\frac{4x^3-3x^2}{3\left(x^4-x^3+2\right)^{\frac{2}{3}}}$$

b. Find the second derivative ($y''=d^2y/dx^2$) of $y=e^{y+x}$ and express it in terms of y. **(10 points)**

Solution:

1st method for y':

$$y'=\left(y'+1\right)e^{y+x} \qquad \Rightarrow \qquad y'\left(1-e^{y+x}\right)=e^{y+x} \qquad \Rightarrow \qquad y'=\frac{e^{y+x}}{1-e^{y+x}}=\frac{y}{1-y}$$

2nd method for y':

$$\ln y=\ln e^{y+x}=\left(y+x\right)\ln e=y+x$$
$$\ln y=y+x$$
$$\frac{y'}{y}=y'+1 \qquad \Rightarrow \qquad y'\left(\frac{1}{y}-1\right)=1 \quad \Rightarrow \qquad y'=\frac{y}{1-y}$$

Second Derivative of y:

$$y''=\frac{d^2y}{dx^2}=\frac{1-y-(-1)y}{\left(1-y\right)^2}y'=\frac{1}{\left(1-y\right)^2}\frac{y}{1-y}=\frac{y}{\left(1-y\right)^3}$$

5. Answer the questions below for the curve $y = 4x^2 - x^4$ (DO NOT SKETCH IT):

a. Find the x-intercepts and y-intercepts. **(5 points)**

Solution:

$y = 4x^2 - x^4 = x^2(4 - x^2)$

x-intercepts: $y = 0 = x^2(4 - x^2) \quad \Rightarrow \quad x = 0 \quad \& \quad x = \pm 2$

$\{(-2,0), (0,0), (2,0)\}$

y-intercept: $x = 0 \quad \Rightarrow \quad y = 0 \qquad (0, 0)$

b. Use the first derivative test to find where increasing and decreasing intervals of the function occur. **(5 points)**

Solution:

The derivative of y is $y' = 8x - 4x^3 = 4x(2 - x^2)$.

The critical values are as follows:

$$y' = 4x(2 - x^2) \quad \Rightarrow \quad x = 0 \quad \therefore \quad x = \pm\sqrt{2}.$$

x		$-\sqrt{2}$		0		$+\sqrt{2}$	
y'	+	0	-	0	+	0	-
y	↗ increasing	Max	↘ decreasing	min	↗ increasing	max	↘ decreasing

c. Determine where relative extrema occur. **(5 points)**

Solution:

$x = 0$ minimum

$x = \pm\sqrt{2}$ maximum

d. Determine where the given function is concave up and where it is concave down, and where inflection point(s) occur. **(5 points)**

Solution:

The second derivative of y is $y'' = 8 - 12x^2 = 4(2 - 3x^2)$.

The inflection points are as follows:

$$y'' = 4(2 - 3x^2) = 0 \quad \Rightarrow \quad x = \pm\sqrt{2/3}.$$

x		$-\sqrt{2/3}$		$+\sqrt{2/3}$	
y''	-	0	+	0	-
y	Concave down		Concave up		Concave down