MATH 171   MIDTERM EXAM   (26.10.2010)

Name:
Instructor:
Section:

Q1	Q2	Q3	Q4	Total
25	25	25	25	100

ATTENTION: There are 4 questions on 4 pages. Solve all of them. Duration is ONE hour. Since simply giving a final result is not sufficient to answer any question, you are responsible to show up all the steps you pursued to get any final result. Otherwise your answer will not be evaluated as a correct answer.

1. Solve $\left|2x-3\right|-8\leq 0$ inequality. (25 Points)

<u>Solution:</u>

$$-8\leq 2x-3\leq 8 \qquad \Rightarrow \qquad -8+3\leq 2x\leq 8+3$$

$$\Rightarrow \qquad -5\leq 2x\leq 11$$

$$\Rightarrow \qquad -\frac{5}{2}\leq x\leq \frac{11}{2}$$

Solution: $-\dfrac{5}{2}\leq x\leq \dfrac{11}{2}$ or $\left[-\dfrac{5}{2},\dfrac{11}{2}\right]$

2. Determine an equation ($y = mx + b$) of the straight line that passes through (0,-3) and is perpendicular to the line $y = \dfrac{3}{2}x - 2$. (25 Points)

Solution:

The perpendicular equation, $y = \dfrac{3}{2}x - 2$, has a slope of 3/2. The line in the question perpendicular to the line with a slope 3/2 has a slope:

$$m = -\frac{1}{m_\perp} = -\frac{1}{\dfrac{3}{2}} = -\frac{2}{3}$$

Therefore, the line equation with a $(x_1,y_1) = (0,-3)$ point and a slope of $m = -2/3$ can be written as

$$y - y_1 = m(x - x_1) \quad \Rightarrow \quad y - (-3) = -\frac{2}{3}(x - 0)$$

$$\Rightarrow \quad y = -\frac{2}{3}x - 3$$

3. Find the *x*- and *y*-intercepts and the vertex of the $y = 4 - (2-3x)^2$ function and then plot it. Find the domain and range of the function. (25 Points)
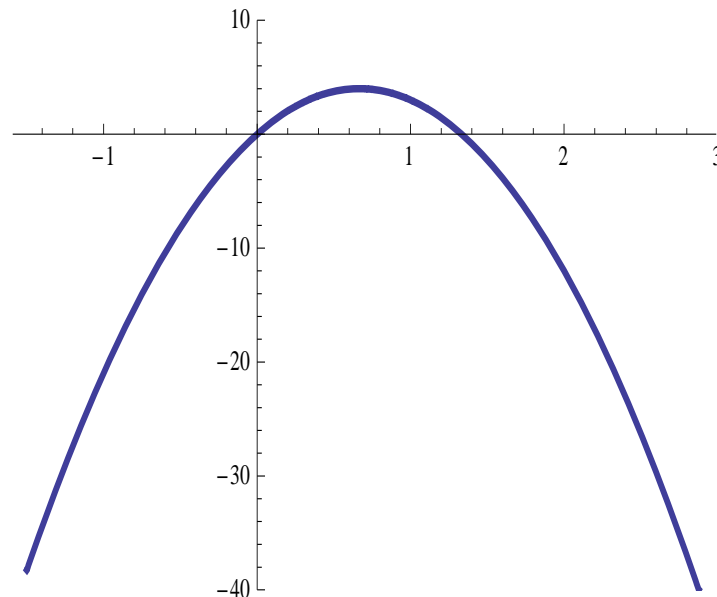
Solution:

x-intercept: we can obtain it just putting $y = 0$ such that

$$y = 0 = 4 - (2-3x)^2 = \cancel{4} - 9x^2 + 12x - \cancel{4} = 3x(-3x+4) \implies \begin{cases} x = 0 \\ x = 4/3 \end{cases}$$

y-intercept: we can obtain it just putting $x = 0$ such that $y = 4 - (2-3x)^2\big|_{x=0} = 0$

x_{vertex} of $y = -9x^2 + 12x$:

$$x_{vertex} = -\frac{b}{2a} = -\frac{12}{2(-9)} = \frac{2}{3} \implies y_{vertex} = -9x^2 + 12x\big|_{x=2/3} = -9\frac{4}{9} + 12\frac{2}{3} = 4$$

Domain: $-\infty \le x < \infty$ or $(-\infty, \infty)$

Range: $-\infty \le f(x) < 4$ or $(-\infty, 4]$

4. Solve the below equation for x: (25 Points)
$$\log(2 - x^2 + x) = 1 + \log(x+1).$$

<u>Solution:</u>

$$\log(2 + x - x^2) = 1 + \log(x+1)$$

$$\log(2 + x - x^2) - \log(x+1) = 1$$

$$\log\left(\frac{2 + x - x^2}{x+1}\right) = 1$$

$$2 + x - x^2 = 10(x+1)$$

$$8 + 9x + x^2 = (x+1)(x+8) = 0 \qquad \Rightarrow \qquad x = -1 \quad \text{and} \quad x = -8$$

The original equation is not defined at $x = -1$ and $x = -8$; therefore, {-1, -8} is NOT a solution of the original equation.

Hence there is no solution or { } .