

MATH 171 MIDTERM EXAM (3.11.2011)

Name:
Section:

No:

Q1	15	Q2	15	Q3	15	Q4	20	Q5	15	Q6	20	Total	100

ATTENTION: There are 6 questions on 4 pages. Solve all of them. Duration is ONE hour. Since simply giving a final result is not sufficient to answer any question, you are responsible to show up all the steps you pursued to get any final result. Otherwise your answer will not be evaluated as a correct answer.

1. Solve $|2-3x|-8 \geq 0$ inequality. (15 Points)

Solution:

$$\begin{aligned} |2-3x| \geq 8 &\Rightarrow 2-3x \geq 8 \Rightarrow -3x \geq 6 \Rightarrow x \leq -2 \\ &\Rightarrow 2-3x \leq -8 \Rightarrow -3x \leq -10 \Rightarrow x \geq 10/3 \\ \text{Solution: } &x \leq -2 \text{ and } x \geq 10/3 \text{ or } (-\infty, -2] \cup [10/3, \infty) \end{aligned}$$

2. Determine the equation of the straight line that passes through (-1, 1) and is perpendicular to the line $2y - 5x + 3 = 0$. (15 Points)

Solution:

The perpendicular equation, $y = \frac{5}{2}x - \frac{3}{2}$, has a slope of $5/2$. The line in the question perpendicular to the line with a slope $3/2$ has a slope:

$$m = -\frac{1}{m_{\perp}} = -\frac{1}{\frac{5}{2}} = -\frac{2}{5}$$

Therefore, the line equation with a $(x_1, y_1) = (-1, 1)$ point and a slope of $m = -2/5$ can be written as

$$\begin{aligned} y - y_1 &= m(x - x_1) &\Rightarrow & y - 1 = -\frac{2}{5}(x - (-1)) \\ & &\Rightarrow & y = -\frac{2}{5}x - \frac{2}{5} + 1 = -\frac{2}{5}x + \frac{3}{5} \end{aligned}$$

General Form: $5y + 2x - 3 = 0$ (actually one form of many of them)

3. The demand per week for a product is 1500 units when the price is 100 TL each, and 500 units when the price is 200 TL each. Find the demand equation for the product, assuming that it is linear. (15 Points)

Solution:

The data are as follows:

$$\begin{aligned} q_1 = 26,000 & \Rightarrow p_1 = 16TL \\ q_2 = 10,000 & \Rightarrow p_2 = 24TL \end{aligned}$$

From these data we may calculate the slope of the demand curve that is assumed linear:

$$m = \frac{p_2 - p_1}{q_2 - q_1} = \frac{200 - 100}{500 - 1500} = -\frac{100}{1000} = -0.1$$

Hence we can find the linear demand equation as

$$\begin{aligned} p - p_1 &= m(q - q_1) \\ p - 100 &= -0.1(q - 1500) \Rightarrow p = -0.1q + 250 \end{aligned}$$

4. Find the x - and y -intercepts and the vertex of the $y = 10 - 12x - (3 - 2x)^2$ function and then sketch it. Find the domain and range of the function. (20 Points)

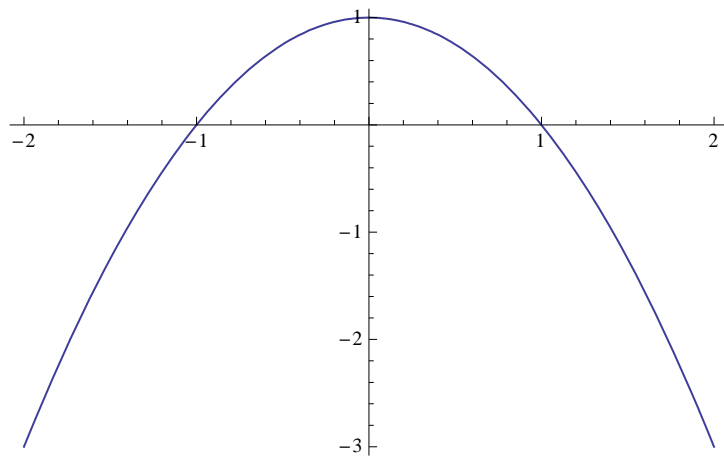
Solution:

x -intercept: we can obtain it just putting $y = 0$ such that

$$y = 0 = y = 10 - 12x - (3 - 2x)^2 = 10 - 12x - 9 + 12x - 4x^2 = 1 - 4x^2 \Rightarrow x = \pm \frac{1}{2}$$

y -intercept: we can obtain it just putting $x = 0$ such that $y = 1 - 4x^2 \Big|_{x=0} = 1$

$$x_{\text{vertex}} \text{ of } y = 1 - 4x^2 : x_{\text{vertex}} = -\frac{b}{2a} = -\frac{0}{2(-4)} = 0 \Rightarrow y_{\text{vertex}} = 1$$



Domain: $-\infty \leq x < \infty$ or $(-\infty, \infty)$

Range: $-\infty \leq f(x) \leq 1$ or $(-\infty, 1]$

5. Solve the below equation for x : (15 Points)

$$\log_2(4-x) + \log_2 2 - 2\log_2 x = 0.$$

Solution:

$$\log_2(4-x) + \log_2 2 - 2\log_2 x = 0$$

$$\log_2[2(4-x)] - \log_2 x^2 = 0$$

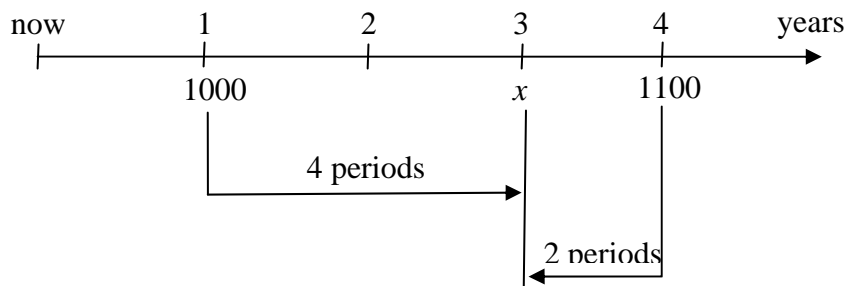
$$\log_2\left(\frac{2(4-x)}{x^2}\right) = 0 \Rightarrow \frac{2(4-x)}{x^2} = 2^0 = 1$$

$$x^2 + 2x - 8 = 0$$

$$(x-2)(x+4) = 0 \Rightarrow x = 2 \quad \text{and} \quad \cancel{x = -4}$$

The original equation is not defined at $x = -4$; therefore, the solution is just $\{ 2 \}$.

6. A debt of 1000 TL due in a year and 1100 TL due in four years is to be repaid by a single payment three years from now. If the interest rate is 10% compounded semiannually, how much is the payment? ($1.05^2 \approx 1.10$; $1.05^4 \approx 1.22$; $1.10^2 \approx 1.21$; $1.10^4 \approx 1.46$) (20 Points)



Annual interest rate is 10% and it is compounded semiannually so we have a rate of $0.10/2=0.05$ semiannually.

Let say the single payment amount x three years from now in the question.

We can write the equation of value of the debt at the third year:

$$x = 1000(1.05)^4 + 1100(1.05)^{-2} = 1000(1.22) + \frac{1100}{1.1} = 1220 + 1000 = 2220\text{TL}$$