

MATH 171 MIDTERM EXAM (01.04.2010)

Name:

Instructor:

Section:

| Q1 | Q2 | Q3 | Q4 | Q5 | Q6 | Q7 | Q8 | Total |
|----|----|----|----|----|----|----|----|-------|
| | | | | | | | | |
| 10 | 10 | 10 | 15 | 15 | 15 | 10 | 15 | 100 |

ATTENTION: There are 8 questions on 4 pages. Solve all of them. Duration is one hour. Simply giving a final result is not sufficient to answer any question, so show all the steps you pursued to get any final result. Otherwise your answer will not be evaluated as a correct answer.

1. Solve $\left|x - \frac{1}{2}\right| \geq \frac{1}{2}$ inequality. (10 Points)

Solution:

$$\text{If } x - \frac{1}{2} \geq 0$$

$$x - \frac{1}{2} \geq \frac{1}{2} \quad \Rightarrow \quad x \geq 1$$

$$\text{If } x - \frac{1}{2} < 0$$

$$-\left(x - \frac{1}{2}\right) \geq \frac{1}{2} \quad \Rightarrow \quad x - \frac{1}{2} \leq -\frac{1}{2} \quad \Rightarrow \quad x \leq 0$$

$$\text{Solution: } x \leq 0 \text{ and } x \geq 1 \quad \text{or} \quad (-\infty, 0] \cup [1, \infty)$$

2. Find the domain and range of the function: $f(x) = \begin{cases} 10 & \text{if } x = 3 \\ x^2 & \text{if } 1 \leq x < 3 \end{cases}$. (10 Points)

Solution:

$$\text{Domain: } 1 \leq x \leq 3$$

$$\text{Range: } 1 \leq f(x) < 9 \text{ and } \{10\}$$

$$[1, 9) \cup \{10\}$$

3. $f(p) = \frac{4}{p}$ and $g(p) = \frac{p-2}{3}$, find $(f \circ g)(p)$ and $(g \circ g)(p)$. (10 Points)

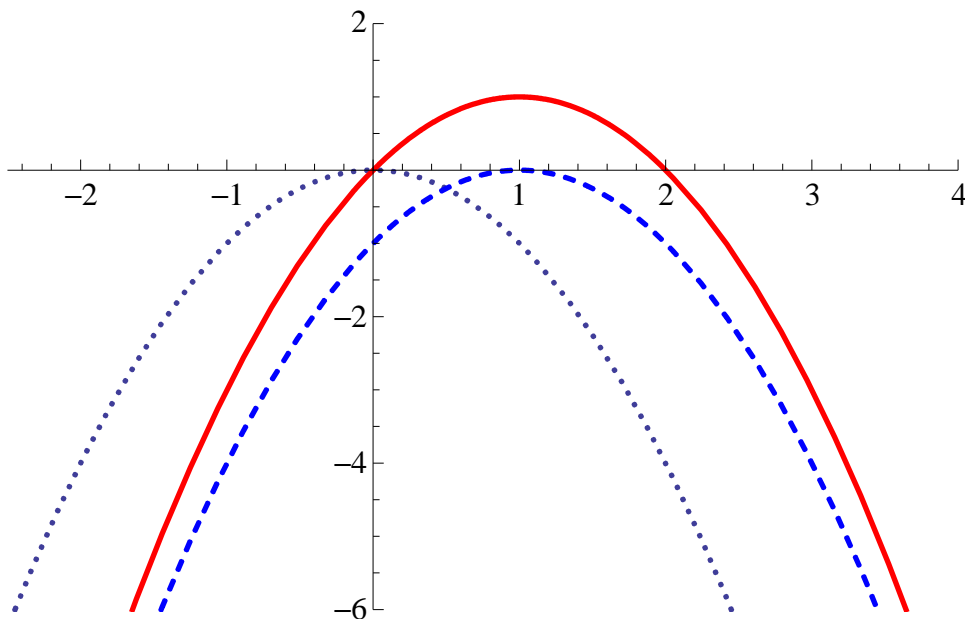
Solution:

$$(f \circ g)(p) = f(g(p)) = f\left(\frac{p-2}{3}\right) = \frac{4}{\frac{p-2}{3}} = \frac{12}{p-2}$$

$$(g \circ g)(p) = g(g(p)) = g\left(\frac{p-2}{3}\right) = \frac{\frac{p-2}{3} - 2}{3} = \frac{p-8}{9}$$

4. a. Starting from $y = -x^2$, plot the graph of $y = 1 - (x-1)^2$ by using transformation techniques.
 b. Find the x - and y -intercepts and the vertex of the given function (10+5 Points)

Solution: a.



b. x -intercept: we can obtain it just putting $y = 0$ such that

$$y = 0 = 1 - (x-1)^2 = -x^2 + 2x = x(-x + 2) \Rightarrow \begin{cases} x = 0 \\ x = 2 \end{cases}$$

y -intercept: we can obtain it just putting $x = 0$ such that $y = 1 - (x-1)^2 \Big|_{x=0} = 0$

$$x_{\text{vertex}} \text{ of } y = -x^2 + 2x: x_{\text{vertex}} = -\frac{b}{2a} = -\frac{2}{2(-1)} = 1 \Rightarrow y_{\text{vertex}} = -x^2 + 2x \Big|_{x=1} = 1$$

5. Determine the slope-intercept form and a general linear form of an equation of the straight line that passes through (1, 2) and is perpendicular to the line $-3y + 5x = 7$. (15 Points)

Solution:

The perpendicular equation $-3y + 5x = 7$ can be written as $y = \frac{5}{3}x - \frac{7}{3}$ that has a slope of $5/3$. The line in the question perpendicular to the line with a slope $5/3$ has a slope:

$$m_{\perp} = -\frac{1}{m_{\perp}} = -\frac{1}{\frac{5}{3}} = -\frac{3}{5}$$

Therefore, the line equation with the point (1,2) and the slope of $-3/5$ can be written as

$$\begin{aligned} y - y_1 &= m(x - x_1) &\Rightarrow & y - 2 = -\frac{3}{5}(x - 1) \\ & &\Rightarrow & y = -\frac{3}{5}x + \frac{3}{5} + 2 = -\frac{3}{5}x + \frac{13}{5} \quad : \text{ the slope-intercept form} \\ & &\Rightarrow & 5y + 3x - 13 = 0 \quad : \text{ the general form} \end{aligned}$$

6. The demand per week for a best-selling book is 26,000 books when the price is 16 TL each, and 10,000 books when the price is 24 TL each. Find the demand equation for the book, assuming that it is linear. (15 Points)

Solution:

The data are as follows:

$$\begin{aligned} q_1 &= 26,000 &\Rightarrow & p_1 = 16TL \\ q_2 &= 10,000 &\Rightarrow & p_2 = 24TL \end{aligned}$$

From these data we may calculate the slope of the demand curve that is assumed linear:

$$m = \frac{p_2 - p_1}{q_2 - q_1} = \frac{24 - 16}{10,000 - 26,000} = -\frac{8}{16,000} = -0.0005 = -0.5 \times 10^{-3}$$

Hence we can find the linear demand equation as

$$\begin{aligned} p - p_1 &= m(q - q_1) \\ p - 24 &= -0.0005(q - 10,000) \quad \Rightarrow \quad p = -0.0005q + 29 \end{aligned}$$

7. Solve the below equation for x : (10 Points)

$$\log_2 \left(\frac{2}{x} \right) = 3 + \log_2 x.$$

Solution:

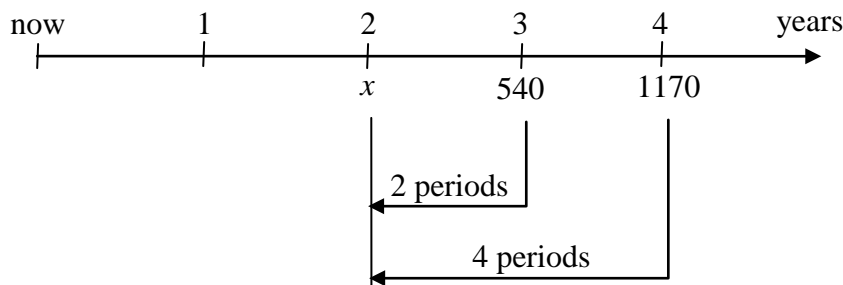
$$\log_2 \left(\frac{2}{x} \right) = 3 + \log_2 x$$

$$\underbrace{\log_2 2}_{=1} - \log_2 x - \log_2 x = 3$$

$$-2 \log_2 x = 2 \Rightarrow x = 2^{-1} = \frac{1}{2}$$

8. A debt of 540 TL due in three years and 1170 TL due in four years is to be repaid by a single payment two years from now. If the interest rate is 8% compounded semiannually, how much is the payment? ($1.02^2 \approx 1.04$; $1.02^4 \approx 1.08$; $1.04^2 \approx 1.08$; $1.04^4 \approx 1.17$) (15 Points)

Solution:



Annual interest rate is 8% and it is compounded semiannually so we have a rate of 0.04 semiannually.

Let say the single payment amount x two years from now in the question.

We can write the equation of value of the debt at year 2.

$$x = 540(1.04)^{-2} + 1170(1.04)^{-4} = \frac{540}{1.08} + \frac{1170}{1.17} = 500 + 1000 = 1500TL$$