GROUP A Math 171 Final Exam 06.01.2009

1 (10p)	2 (10p)	3 (10p)	4 (10p)	5 (10p)	6 (10p)	7 (5p+5p)		8 (15p)	9 (5p)	10 (5p+5p)		Total

Name:
Surname:
ID:
Section:

Show all your work clearly. Answers without justifications and calculations will get zero point. Calculators and mobile phones are strictly prohibited.

1. Determine concavity and x-values where the points of inflection occur for $y = -2x^3 + 2x^2 - 7x + 9$. (do not sketch the graph)

Answer:

$$y' = -6x^2 + 4x - 7$$

$$y'' = -12x + 4 = 0 \qquad \Rightarrow \qquad x = \frac{1}{3} \text{ is the inflection point.}$$

x	$x < 1/3$	$1/3$	$x > 1/3$
y''	$y'' > 0$	0	$y'' < 0$
y	Concave Up	Inflection point	Concave Down

2. For a manufacturer's product, the revenue function is given by $r = 240q + 57q^2 - q^3$. Determine the output for maximum revenue.

Answer:

$$r' = 240 + 114q - 3q^2 = -(3q^2 - 114q - 240)$$

(3q, q; 6, −40)

$$= -(3q + 6)(q - 40) = 0 \qquad \Rightarrow \qquad q = -2 \qquad or \qquad q = 40$$

q is a quantity so that it cannot be negative. So the extremum point is just 40 corresponding to the maximum from the first derivative test as below:

x		40	
y'	$-+-=+$	0	$-++=-$
y	↗		↘
	increasing	max	decreasing

Therefore the output for maximum revenue is

$$r\big|_{q=40} = 240 \times 40 + 57 \times 40^2 - 40^3 = 9,600 + 91,200 - 64,000 = 36,800$$

3. Find $\frac{dy}{dx}$ by using logarithmic differentiation for $y = \frac{x(1+x^2)^2}{\sqrt{2+x^2}}$

Answer:

$$\ln y = \ln x + 2\ln\left(1+x^2\right) - \frac{1}{2}\ln\left(2+x^2\right)$$

$$\frac{y'}{y} = \frac{1}{x} + 2\frac{2x}{1+x^2} - \frac{1}{2}\frac{2x}{2+x^2}$$

$$y' = \frac{x\left(1+x^2\right)^2}{\sqrt{2+x^2}}\left\{\frac{1}{x} + \frac{4x}{1+x^2} - \frac{x}{2+x^2}\right\}$$

4. Find $\frac{dy}{dx}$ by implicit differentiation for $x^2e^y + y = 13$

Answer:

$$2xe^y + x^2y'e^y + y' = 0 \Rightarrow \quad y'\left(1+x^2e^y\right) = -2xe^y \quad \Rightarrow \quad y' = -\frac{2xe^y}{1+x^2e^y}$$

5. Differentiate $y = x^{5e} - 5^x$

Answer:

$$y' = 5e\,x^{5e-1} - \ln 5 \cdot 5^x$$

6. Differentiate $y = \ln^2(2x+11)$

Answer:

$$y' = 2\ln(2x+11)\frac{2}{2x+11} = \frac{4\ln(2x+11)}{2x+11}$$

7. Find the following limits if they exist.

a) $\lim_{x\to-\infty}\frac{-1+x^2}{(2-3x)^2} = \lim_{x\to-\infty}\frac{x^2}{(-3x)^2} = \lim_{x\to-\infty}\frac{x^2}{9x^2} = \lim_{x\to-\infty}\frac{1}{9} = \frac{1}{9}$

b) $\lim_{t\to-2}\frac{t^2-4}{t+2} = \lim_{t\to-2}\frac{(t-2)(t+2)}{t+2} = \lim_{t\to-2}(t-2) = -4$

8. Find the points of discontinuity for $f(x) = \begin{cases} x+4 & \text{if } x > -2 \\ 3x+6 & \text{if } x \le -2 \end{cases}$

Answer:

i. $f(x)$ is defined for all the real numbers.

ii. $\left.\begin{array}{l} \lim_{x\to-2^+} f(x) = \lim_{x\to-2^+}(x+4) = 2 \\ \lim_{x\to-2^-} f(x) = \lim_{x\to-2^-}(3x+6) = 0 \end{array}\right\} \Rightarrow \lim_{x\to-2^+} f(x) \neq \lim_{x\to-2^-} f(x)$

So there is no limit at $x = -2$ which is a point of discontinuity.

9. Find the present value of $4000 due in 12 years at 7% compounded semi-annually. (write equation only)

Answer:

$$P = S\left(1+\frac{r}{n}\right)^{-t\times n} = \$4000\left(1+\frac{0.07}{2}\right)^{-12\times 2} = \$4000\left(1.035\right)^{-24}$$

10. a) $2+(2)7^{2x+3}=30$: Find x in terms of ln2 and ln7.

Answer:

$$7^{2x+3}=\frac{30-2}{2}=14 \quad \Rightarrow \quad (2x+3)\ln 7 = \ln 14 = \ln 7 + \ln 2$$

$$(2x+3-1)\ln 7 = \ln 2 \quad \Rightarrow \quad x = \frac{1}{2}\left(-2+\frac{\ln 2}{\ln 7}\right) = -1+\frac{\ln 2}{2\ln 7}$$

b) Find x: $2\log x = \log 4 + \log(x-1)$

Answer:

$$\log x^2 = \log\left(4(x-1)\right) \Rightarrow x^2 = 4x-4$$
$$\Rightarrow x^2-4x+4=(x-2)^2=0$$
$$\Rightarrow x=2$$