MATH 171 FINAL EXAM (18.05.2010)

Name:
Instructor:
Section:

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Total

ATTENTION: There are 8 questions on 4 pages. Solve all of them. Duration is one hour. Simply giving a final result is not sufficient to answer any question, so show all the steps you pursued to get any final result. Otherwise your answer will not be evaluated as a correct answer.

1. The Clark Company management would like to know the total sales units that are required for the company to earn a profit of $100,000. The following data are avaliable: unit selling price of $20; variable cost per unit of $15 ; total fixed cost of $600,000. From these data determine the required sales units. **(10 Points)**

q=quantity

Profit = Total Revenue – Total Cost (variable cost + variable cost)

100,000=20q - (600,000+15q)
700,000=5q
q= 700,000/5
q=140,000

2. Find the equation of the line perpendicular to $y = 3x - 5$ and passing through the point (3, 4). **(10 Points)**

$$\left.\begin{matrix} mn = -1 \\ m = 3 \end{matrix}\right\} \Rightarrow n = -1/3$$

$$y = n(x - x_0) + y_0$$

$$y = -\frac{1}{3}(x-3) + 4$$

$$y = -\frac{1}{3}x + 5$$

3. Solve the following inequality $\left|\frac{3x-8}{2}\right| \geq 4$ **(5 Points)**

$$\frac{3x-8}{2} \leq -4 \qquad \text{or} \qquad \frac{3x-8}{2} \geq 4$$

$$3x-8 \leq -8 \qquad \text{or} \qquad 3x-8 \geq 8$$

$$x \leq 0 \qquad \qquad x \geq 16/3$$

Solution: $(-\infty, 0] \cup [16/3, \infty)$

4. Find the following limits if they exist. **(15 Points)**

(a) $\lim_{x\to 1} \frac{x^3-1}{x^2-1}$

$$\lim_{x\to 1} \frac{x^3-1}{x^2-1} = \lim_{x\to 1} \frac{(x-1)(x^2+x+1)}{(x-1)(x+1)} = \lim_{x\to 1} \frac{(x^2+x+1)}{(x+1)} = \frac{3}{2}$$

(b) $\lim_{x\to\infty} \sqrt{x^2+x} - x$

$$\lim_{x\to\infty} \left(\frac{(\sqrt{x^2+x}-x)(\sqrt{x^2+x}+x)}{\sqrt{x^2+x}+x} \right) = \lim_{x\to\infty} \left(\frac{(x^2+x-x^2)}{\sqrt{x^2+x}+x} \right) = \lim_{x\to\infty} \left(\frac{x}{\sqrt{x^2+x}+x} \right)$$

$$\lim_{x\to\infty} \left(\frac{x}{\sqrt{x^2(1+1/x)}+x} \right) = \lim_{x\to\infty} \left(\frac{x}{x\sqrt{(1+1/x)}+x} \right) = \lim_{x\to\infty} \left(\frac{1}{\sqrt{(1+1/x)}+1} \right) = \frac{1}{2}$$

(c) Given $f(x) = \begin{cases} x^2+1, & if \ x \geq 1 \\ 3, & if \ x < 1 \end{cases}$ find the points where f is discontinues**.**

$$\left.\begin{array}{l} \lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} x^2+1 = 2 \\ \lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} 3 = 3 \end{array}\right\} \Rightarrow \lim_{x\to 1^-} f(x) \neq \lim_{x\to 1^+} f(x)$$

$f(x)$ is not continuous at $x = 1$

5. Sketch the graph of $y = \dfrac{x}{x-1}$ **(20 Points)**

<u>Intercepts:</u> $x = 0 \quad \Rightarrow \quad y = 0 \quad (0, 0)$

<u>Asymptotes:</u>

$$\lim_{x \to 1^-} \frac{x}{x-1} = -\infty; \lim_{x \to 1^+} \frac{x}{x-1} = \infty \qquad x = 1 \text{ is a vertical asymptote}$$

$$\lim_{x \to \pm\infty} \frac{x}{x-1} = 1 \qquad y = 1 \text{ is a horizontal asymptote.}$$

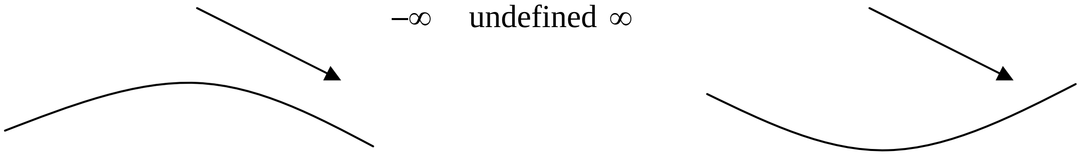
<u>First Derivative Test for Increasing and Decreasing Range of the Function:</u>

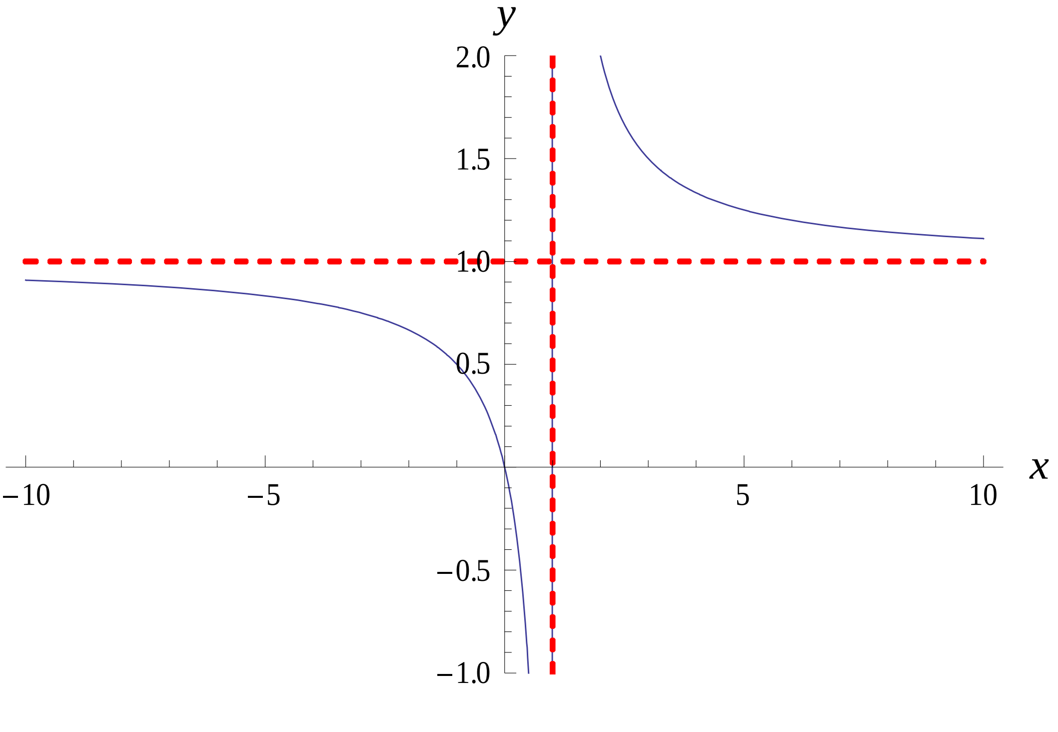
$$y' = -\frac{1}{(x-1)^2} \quad \Rightarrow \quad y' < 0 \text{ for all } x \ (x \neq 1)$$

<u>Second Derivative Test for the Curvature of the Function (Concave up or down?):</u>

$$y'' = \frac{2}{(x-1)^3} \quad \Rightarrow \quad \begin{cases} y'' < 0, & x < 1 \\ y'' > 0, & x > 1 \end{cases}$$

x	$-\infty$		1		∞
y'		-	undefined	-	
y''		-	undefined	+	
y			$-\infty$ undefined ∞		





6. If the total-cost function for a manufacturer is given by $c = \frac{5q^2}{q+3} + 5000$ find the marginal-cost function. **(10 points)**

$$\frac{dc}{dq} = \frac{10q(q+3) - 5q^2}{(q+3)^2} = \frac{10q^2 + 30q - 5q^2}{(q+3)^2} = \frac{5q^2 + 30q}{(q+3)^2}$$

7. Find following the derivatives. **(30 Points)**

(a). $y = x^e + e^3$

$$y' = ex^{e-1}$$

(b). $xe^y + y^2 = x^3$

$$xe^y + y^2 = x^3$$
$$(xe^y + y^2)' = (x^3)'$$
$$e^y + y'xe^y + 2yy' = 3x^2$$
$$y' = \frac{3x^2 - e^y}{xe^y + 2y}$$

(c). $y = \frac{(2x-5)^3}{x^2(x^2+1)^{1/4}}$

$$y = \frac{(2x-5)^3}{x^2(x^2+1)^{1/4}}$$
$$\ln y = \ln\left(\frac{(2x-5)^3}{x^2(x^2+1)^{1/4}}\right) = \ln(2x-5)^3 - \ln x^2(x^2+1)^{1/4}$$
$$\ln y = 3\ln(2x-5) - 2\ln x - \frac{1}{4}(x^2+1)$$
$$\frac{y'}{y} = \frac{6}{(2x-5)} - \frac{2}{x} - \frac{2x}{4(x^2+1)}$$
$$y' = y\left(\frac{6}{(2x-5)} - \frac{2}{x} - \frac{2x}{4(x^2+1)}\right)$$
$$y' = \frac{(2x-5)^3}{x^2(x^2+1)^{1/4}}\left(\frac{6}{(2x-5)} - \frac{2}{x} - \frac{x}{2(x^2+1)}\right)$$