**Problem Set 2** **Math-171** **Fall 2006**

1) Determine the domain and range of the exponential function $f(x) = 3^{x-1} + 4$ and sketch it.

2) (a) $\log 0.000001$=? (b) Simplify $e^{\ln x} + \ln e^{x} + \ln 1$ (c) If $e^5$=55 solve $2e^{2x} + 5 = 115$

(d) Solve log(logx)=1 (e) Solve $7 + (2)4^{x-1} = 12$ (f) Solve 2logx=log4+log(x-1)

(g) log(2x+4)-2=log3 x=? (g) $2 \cdot 3^{5x} = 100x$=? (h) $e^{\ln x}+\ln e^{x}+\ln 1$=? (i) $e^{\ln 6}$=?

3) Write the following expresions in terms of lnx, ln(x+1), and/or ln(x+2)

(a) $\ln \dfrac{x}{(x+1)(x+2)}$ (b) $\ln\left[\dfrac{1}{x+2}\sqrt[5]{\dfrac{x^2}{x+1}}\right]$

4) Graph the function $f(x) = 0.3^{x} - 1$. Give its x-intercepts, y-intercepts, domain and range.

5) Graph the functions f(x)= ln(x+1) and g(x)=$\log_{10}(-x)$. Determine their domain and range.

6) Suppose \$9000 is placed in a savings account that earns interest at a rate of 6%
a) compounded annually. What is the value of the account at the end of five years?
b) compounded quarterly. What is the value of the account at the end of five years?
(Equations are enough. You do not need to calculate)

7) Due to ineffective advertising. İstek Vakfı finds that its annual revenues have been cut sharply. Moreover, the revenue, $r$, at the end of $t$ years of the business satisfies the equation $r = 200{,}000e^{-0.2t}$ Find the annual revenue at the end of two years and at the end of three years.

8) Find the compound amount of an investment of \$4000 for five years at the rate of 10% compounded semiannually. (Give the equation; you do not need to estimate)

9) A dept of \$1000 due in five years is to be repaid by a single payment now. Find how much the payment is if an interest rate of 8% compounded quarterly is assumed. (Give the equation)

10) Find the effective rate of interest if a nominal rate of 8%
a) compounded annually is assumed. b) compounded quarterly is assumed.

11) Find the following limits if they exist:

(a) $\lim\limits_{x\to 1}\dfrac{x^3-1}{x-1}$ (b) $\lim\limits_{x\to 1}\dfrac{x^3-1}{x^2-1}$ (c) $\lim\limits_{x\to\infty}\dfrac{x^2-1}{x^3+1}$ (d) $\lim\limits_{x\to-\infty}\dfrac{3-2x-7x^3}{7+2x^2-5x^3}$

(e) $\lim\limits_{r\to\infty}\dfrac{3r^3}{r^2+1}$ (f) $\lim\limits_{t\to\infty}(t-1)^3$ (g) $\lim\limits_{x\to-\infty}\dfrac{2}{(4x-1)^3}$ (h) $\lim\limits_{t\to 2}\dfrac{t^2-4}{t+2}$

(i) $\lim\limits_{x\to 2^+}\dfrac{5x}{4-x^2}$ (j) $\lim\limits_{x\to 0^+}\dfrac{5}{x+x^2}$ (k) $\lim\limits_{t\to -2}\dfrac{t^2-4}{t+2}$ (l) $\lim\limits_{x\to-\infty}\dfrac{ex^2-x^4}{30x-x^3}$

12) Find all points of discontinuity for the function $f(x) = \dfrac{x-7}{x^3-x}$

13) Given $f(x) = \begin{cases} 2^{x-1} & \text{if } x > 1 \\ 2^{x} & \text{if } x \le 1 \end{cases}$, find the points where f is discontinues.

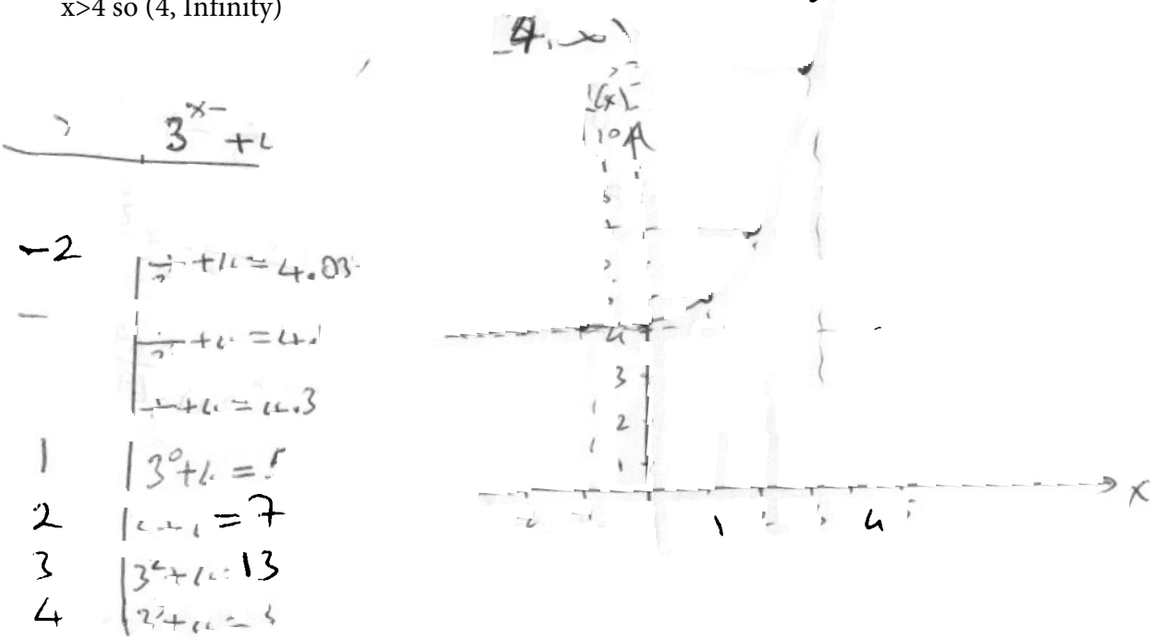
# — Problem Set 2 —

(1)

1.) $f(x) = 3^{x-1} + 4$ Domain? Range? sketch it

**Domain:** All the real numbers : $\mathbb{R}$

**Range:** $3^{x-1}$ cannot be negative. Lowest value it takes is zero when x goes to $-\infty$. So the range of the function $f(x)$ x>4 so (4, Infinity)




$3^{x-}+1$

-2 | $\frac{1}{9}+1 = 4.03$

| $\frac{1}{3}+1 = 4.3$

| $-1+1 = 1.3$

1 | $3^0+1 = 5$

2 | $3^1+1 = 7$

3 | $3^2+1 = 13$

4 | $3^3+1 = 4$

2) a) $\log 0.000001 = \log 10^{-6} = -6 \log 10 = -6$ ($\log 10 = 1$)

b) Simplify $e^{\ln x}$ ..... ....

$= x + x \ln e + 0 = 2x$ ($\ln e = 1$)

c) If $e^5 = 55$, solve $2e^{2x} + 5 = 115$

$\ln e^5 = \ln 55$

$5 \ln e = \ln 55$ ($\ln e = 1$)

$\ln 55 = 5$

$2e^{2x} = 115 - 5 = 110$

$e^{2x} = \frac{110}{2} = 55 \Rightarrow \ln e^{2x} = \ln 55$

$2x \ln e = \ln 55$ ($\ln e = 1$)

$x = \frac{\ln 55}{2} = \frac{5}{2} = 2.5$

d) Solve $\log(\log x)=1$

$10=\log x \Rightarrow \boxed{x=10^{10}}$

(e) Solve $7+2\times 4^{x-1}=12$

$2\times 4^{x-1}=12 \Rightarrow 4^{x-1}=\frac{12}{2}=6$

$\log 4^{x-1}=\log 6$

$(x-1)\log 4=\log 6$

$x-1=\frac{\log 6}{\log 4} \Rightarrow \boxed{x=\frac{\log 6}{\log 4}-1}$

$\ln 2\simeq 0.69$

$\ln 3\simeq 1.10$

$=\frac{\log 3+\log 2}{2\log 2}-1$

$=\frac{1.10+0.69}{2\times 0.69}-1=\frac{1.79}{1.38}-1$

$=0.28$

(f) Solve $2\log x=\log 4+\log(x-1)$

$\log x^2-\log(x-1)=\log 4$

$\log\frac{x^2}{x-1}=\log 4 \Rightarrow \frac{x^2}{x-1}=4 \Rightarrow x^2=4x-4$

$x^2-4x+4=0 \Rightarrow x=\frac{4\mp\sqrt{16-4\cdot 4}}{2}=\frac{4}{2}=2.$

(g) $\log(2x+4)-2=\log 3 \Rightarrow x=?$

$\log(2x+4)-\log 3=2$

$\log\frac{2x+4}{3}=2 \Rightarrow \frac{2x+4}{3}=10^2=100$

$2x+4=300 \Rightarrow 2x=296$

$\boxed{x=148}$

(g) $2 \cdot 3^{5x} = 100$, $x =$

$$3^{5x} = \frac{100}{2} = 50$$

$$\log 3^{5x} = \log 50$$

$$5x \log 3 = \log 50$$

$$x = \frac{\log 50}{5 \log 3} = \frac{\log 5 + \log 10}{5 \log 3} = \frac{1 + \log 5}{5 \log 3} \approx \frac{1 + 0.70}{5 \times 0.48} \approx 0.71$$

(h) $e^{\ln x} + \ln e^x + \ln 1 = x + x \underbrace{\ln e}_{=1} + 0 = 2x$

(i) $e^{\ln b} =$

(3) the following in terms of $\ln x$, ...

(a) $\ln \frac{x}{(x+1)(x+2)} = \ln x - \ln((x+1)(x+2))$

$= \ln x - \ln(x+1) - \ln(x+2)$

(b) $\ln \left[ \frac{1}{x+2} \sqrt[5]{\frac{x^2}{x+1}} \right] = \ln \left( \frac{1}{x+2} \right) + \ln \left( \frac{x^2}{x+1} \right)^{1/5}$

$= \ln (x+2)^{-1} + \frac{1}{5} \ln \left( \frac{x^2}{x+1} \right)$

$= \ln (x+2) + \frac{2}{5} \ln x - \frac{1}{5} \ln (x+1)$

4) $f(x) = 0.3^x - 1$

x-intercept? $\Rightarrow$ $y = 0 \Rightarrow 0.3^x - 1 = 0$

$0.3^x = 1 \Rightarrow \boxed{x = 0}$

y-intercept? $\Rightarrow$ $x = 0 \Rightarrow y = 0.3 - 1 = 1 - 1 = 0$

$\boxed{y = 0}$

Domain: All real numbers $\in \mathbb{R}$

Range: $x \to -\infty \Rightarrow 0.3^x \to 0$

Lowest value $0.3^x$ takes is zero

y takes $-1$ and greater than $-1$

Range $[-1, \infty)$

5) $f(x) = \ln(x+1)$ graph it?

Domain: $x > -1$ $(-1, \infty)$

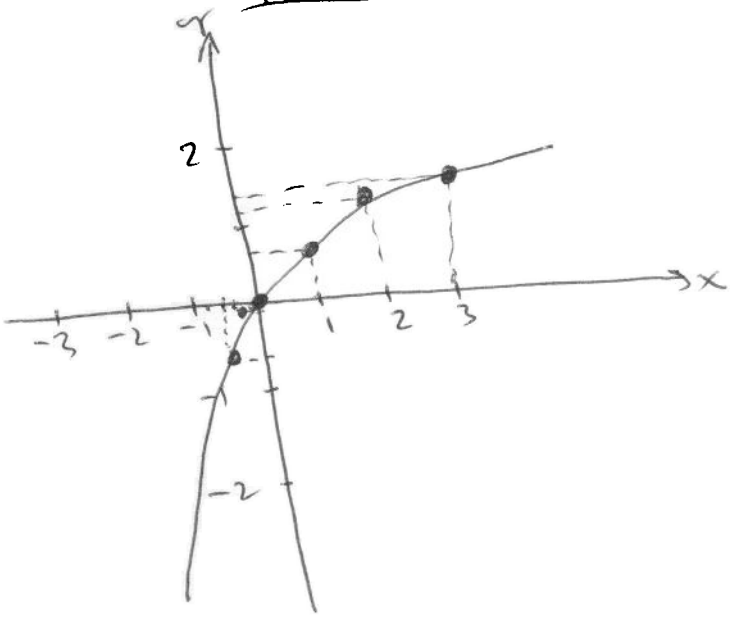
Range: $(-\infty, \infty)$

y-intercept: $x = 0 \Rightarrow y = \ln(0+1) = 0$

x-intercept: $y = 0 \Rightarrow 0 = \ln(x+1) \Rightarrow x + 1 = e^0 = 1$

$\boxed{x = 0}$

| x | y |
|---|---|
| $-1$ | $-\infty$ |
| $-1/2$ | $\ln\frac{1}{2} = -\ln 2 = -0.69$ |
| $-1/4$ | $\ln(3/4) = \ln 3 - \ln 4 = 1.10 - 1.39 = -.29$ |
| $0$ | $0$ |
| $1$ | $\ln 2 = 0.69$ |
| $2$ | $\ln 3 = 1.10$ |
| $3$ | $\ln 4 = 1.39$ |



5) Sketch $y(x) = \log_{10}(-x)$

(3)

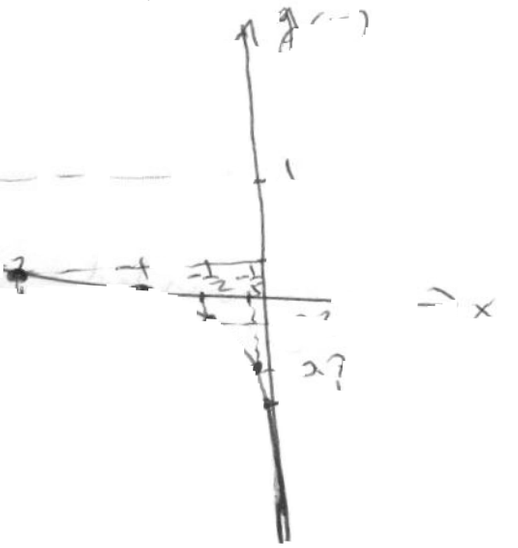
Domain: $x < 0$

Range: All real numbers

x-intercept $y = 0 = \log_{10}(-x) \Rightarrow -x = 1 \Rightarrow \boxed{x = -1}$

y-intercept: $x = 0 \Rightarrow y = \log(-0) \Rightarrow y \to -\infty$ so no y-intercept

| x | y(x) |
|---|---|
| $-10$ | $\log 10 = 1$ |
| $-5$ | $\log 5 = 0.7$ |
| $-2$ | $\log 2 = 0.3$ |
| $-1$ | $\log 1 = 0$ |
| $-\frac{1}{2}$ | $\log \frac{1}{2} = -\log 2 = -0.3$ |
| $-\frac{1}{5}$ | $\log \frac{1}{5} = -\log 5 = -0.7$ |
| $-\frac{1}{10}$ | $\log \frac{1}{10} = -\log 10 = -1$ |




6) $P = \$9000$ interest rate $r = 0.06$ %6

a) Compounded annually. $S = ?$ at the end of 5 years

$$S = P\left(1 + \frac{r}{k}\right)^{n \times k} = \$9000\left(1 + \frac{0.06}{1}\right)^{5 \times 1} = 1.06^5 =$$

$$= \$9000 \times 1.34 = \$12060$$

b) Compounded quarterly. $S = ?$ for 5 years

$$S = \left(1 + \frac{r}{k}\right)^{n \times k} = \$9000\left(1 + \frac{0.06}{4}\right)^{5 \times 4} = \$9000 \times (1.015)^{20}$$

$1.015$

$1.35$

$= \$12{,}150$

7) $r = 200{,}000e^{-0.2t}$

$t = 2$ years $\Rightarrow r = 200{,}000e^{-0.2\times 2} = 200{,}000e^{-0.4} = 134{,}000$

$e^{-0.4} \approx 0.67$

$t = 3$ years $\Rightarrow r = 200{,}000e^{-0.2\times 3} = 200{,}000e^{-0.6} \cong 110{,}000$

$e^{-0.6} \approx 0.55$

8) $P = \$4000$ $n = 5$ years $r = 0.10$

compounded semi annually ($k=2$)

$S = P\left(1+\frac{r}{k}\right)^{n\times k} = \$4000\left(1+\frac{0.10}{2}\right)^{5\times 2}$

$= \$4{,}000(1.05)^{10} = \$6{,}520$

$(1.05)^{10} \approx 1.63$

9) $S = \$1000$ due in 5 years

"Single payment now"

$r = 0.08$ (8%) compounded quarterly $k = 4$

$P = S\left(1+\frac{r}{k}\right)^{-n\times k}$

$= \$1000\left(1+\frac{0.08}{4}\right)^{-5\times 4} = \$1000(1.02)^{-20}$

$= \$\frac{1000}{1.02^{20}} = \$671$

$1.02^{20} \approx 1.49$

11) a) $\lim_{x\to 1} \frac{x^3-1}{x-1}$ $\frac{0}{0}$ so

$$\lim_{x\to 1} \frac{(x-1)(x^2+x+1)}{x-1} = 1+1+1 = 3$$

b) $\lim_{x\to 1} \frac{x^3-1}{x^2-1}$ $\frac{0}{0}$

$$\lim_{x\to 1} \frac{(x-1)(x^2+x+1)}{(x-1)(x+1)} = \frac{3}{2}$$

c) $\lim_{x\to\infty} \frac{x^2+x}{x^3+x} = \lim_{x\to\infty} \frac{x^2}{x^3} = \lim_{x\to\infty} \frac{1}{x} = \frac{1}{\infty} = 0$

d) $\lim_{x\to-\infty} \frac{3-2x-7x^3}{7+2x^2-5x^3} = \lim_{x\to-\infty} \frac{-7x^3}{-5x^3} = \frac{7}{5}$

e) $\lim_{x\to\infty} \frac{3x^3}{x^2+} = \lim_{x\to\infty} \frac{3x^3}{x^2} = 3\cdot\infty = \infty$

f) $\lim_{t\to\infty} (t-1)^3 = \infty^3 = \infty$

g) $\lim_{x\to-\infty} \frac{2}{(4x-1)^3} = \lim_{x\to-\infty} \frac{2}{4^3x^3} = \frac{\#}{(-\infty)^3} = \frac{\#}{-\infty} = 0^-$

h) $\lim_{t\to 2} \frac{t^2-4}{t+2} = \frac{2^2-4}{2+2} = 0$

(i) $\lim_{x\to 2^+} \frac{5x}{4-x^2} = \frac{5\cdot 2}{4-(2.000\ldots01)^2} = \frac{10}{4-4.000\ldots001}$

$$= \frac{10}{-0.00\ldots01} = -\frac{10}{0} = -\infty$$

$\infty$

→

(j) $\lim_{x\to 0^+} \frac{5}{x+x^2} = \frac{5}{0^+ + 0^{+2}} = \frac{5}{0} = +\infty$

k) $\lim_{t\to -2} \frac{t^2-4}{t+2} = \frac{0}{0} \Rightarrow \lim_{t\to -2} \frac{(t-2)(t+2)}{t+2} = -2-2 = -4$

l) $\lim_{x\to -\infty} \frac{ex^2-x^4}{30x-x^3} = \lim_{x\to -\infty} \frac{-x^4}{-x^3} = -(-\infty) = \infty$

12) Find all points of discontinuity for

$f(x) = \frac{x-7}{x^3-x}$

$x^3-x=0$ yapan değerler süreksizlik oluşturur

$x(x^2-1)=0$

$x=0$ $\quad x^2-1=0$

$x^2=1 \Rightarrow x=+1$

$x=-1$

$x\in\{-1, 0, 1\}$ değerlerinde $f(x)$ süreksizdir

13) $f(x) = \begin{cases} 2^{x-1} & \text{if } x>1 \\ 2^x - 1 & \text{if } x\le 1 \end{cases}$ find the points where $f$ is discontinuous

$x=1$'de bakmalıyız.

i) $f(x=1) = 2^1 - 1 = 1$ tanımlı ✓

ii) $\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} (2^x - 1) = 2^1 - 1 = 1$

$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} (2^{x-1}) = 2^{1-1} = 2^0 = 1$

} eşit olduğundan $\lim_{x\to 1} f(x) = 1$

iii) $f(x=1) = \lim_{x\to 1} f(x)$

$1 = 1$ ✓

süreksiz bir nokta yoktur

Fonksiyon süreklidir.