PROBLEM SET 3 **MATH–171** **FALL 2006**

1. If $[(x-2)(x+2)(x+3)] \geq 0$ solve for x

2. Find the limits if they exist

a) $\lim\limits_{x\to 0}\dfrac{2x^2-3x+1}{2x^2-2}$ b) $\lim\limits_{x\to\infty}\dfrac{5x+2}{9x-3}$ c) $\lim\limits_{x\to 2}\dfrac{2-x}{x-2}$

d) $\lim\limits_{x\to 1} f(x)$ if $f(x)=\begin{cases} x^2 & \text{if } 0\leq x<1 \\ x & \text{if } x>1 \end{cases}$ e) $\lim\limits_{x\to\infty}\dfrac{x^{100}+(1/x^2)}{e-x^{98}}$

3. Consider the function $f(x)=\begin{cases} \dfrac{1}{x^2} & \text{if } x<-1 \\ 2 & \text{if } -1\leq x<1 \\ 3 & \text{if } x=1 \\ x+1 & \text{if } 1<x\leq 2 \\ -\dfrac{1}{(x-2)^2} & \text{if } x>2 \end{cases}$ and find;

a) $\lim\limits_{x\to -1^+} f(x)$ b) $\lim\limits_{x\to -1^-} f(x)$ c) $\lim\limits_{x\to -1} f(x)$ d) $\lim\limits_{x\to 1^+} f(x)$ e) $\lim\limits_{x\to 1^-} f(x)$ f) $\lim\limits_{x\to 1} f(x)$

4. Find the slope of $y=5-6x-2x^3$ when x=2

5. Find the equation of the tangent line to the curve $y=\dfrac{2x+3}{x^2}$ at the point (1, 5)

6. Find the following derivatives

a) $\dfrac{d}{dx}\left[(x^3-1)(x^3+1)\right]$ b) $\dfrac{d}{dx}\left[\dfrac{(x^3-1)}{(x^3+1)}\right]$ c) $\dfrac{d}{dx}(5x^2+2-\sqrt{x+4})$ d) $\dfrac{d}{dx}\left(\dfrac{5x^4-7x^2}{2x+5}\right)$

e) $\dfrac{d}{dx}\dfrac{e}{x+1}$ f) $\dfrac{d}{dx}x^e$ g) $y=\dfrac{2x^2+1}{2}$ y'=? h) $y=100x^{-3}+10\sqrt[3]{2x}$ y'=?

i) $\dfrac{d}{dx}\left[(x^3-1)(x^3+1)^{2006}\right]$ j) $\dfrac{d}{dx}\dfrac{-3}{(3x^2+1)^3}$ k) $\dfrac{d}{dx}\left(\dfrac{8x-1}{2x+1}\right)^3$ l) $y=\sqrt{\dfrac{x^2-2}{x^2+2}}$ $\dfrac{dy}{dx}=?$

Suppose $p=200-\sqrt{3q^2+10}$ is a demand equa

a) Find the rate of change of p with respect to q; $\dfrac{dp}{dq}$

b) Find the marginal revenue function, $\dfrac{dr}{dq}$ (remember: revenue = price . quantity)

8. The cost c of producing q units of a product is given by c=5500+12q+0.2q^2 If the price per unit p is given by the equation q=900-1.5p use the chain rule to find the rate of change of cost with respect to price, $\dfrac{dc}{dp}$.

Problem Set 3 Fall 2006

Math 171

1.) If $(x-2)(x+2)(x+3) \geq 0$, solve for x

$(x-2)(x+2)(x+3)=0$

$\boxed{x=2}$ $\boxed{x=-2}$ $\boxed{x=-3}$

		3	-2	2
$x-2$	$-$	$-$	$-$	$+$
$x+2$	$-$	$-$	$+$	$+$
$x+3$	$-$	$+$	$+$	$+$
$(x-2)(x+2)(x+3)$	$-$	$+$	$-$	$+$

Solution $\boxed{[-3,-2] \cup [2,\infty)}$

2) a) $\lim\limits_{x\to 0} \dfrac{2x^2-3x+1}{2x^2-2} = \dfrac{1}{-2} = -\dfrac{1}{2}$

b) $\lim\limits_{x\to \infty} \dfrac{5x+2}{9x-3} = \lim\limits_{x\to\infty} \dfrac{5\cancel{x}}{9\cancel{x}} = \dfrac{5}{9}$

c) $\lim\limits_{x\to 2} \dfrac{2-x}{x-2} = \dfrac{0}{0}$, so $\lim\limits_{x\to 2} \dfrac{-\cancel{(x-2)}}{\cancel{x-2}} = -1$

d) $f(x) = \begin{cases} x^2, & 0 \leq x < 1 \\ x, & x > 1 \end{cases}$, $\lim\limits_{x\to 1} f(x) =$

$\lim\limits_{x\to 1^-} f(x) = \lim\limits_{x\to 1^-} x^2 = 1$
$(x<1)$

$\lim\limits_{x\to 1^+} f(x) = \lim\limits_{x\to 1^+} x = 1$

$\lim\limits_{x\to 1} f(x) = \lim\limits_{x\to 1^-} f(x) = \lim\limits_{x\to 1^+} f(x)$

$= 1$

e) $\lim_{x\to\infty} \frac{x^{100}+\frac{1}{x^2}}{e-x^{98}} = \lim_{x\to\infty} \frac{x^{\cancel{100}\,2}}{\cancel{-x^{98}}} = -\infty^2 = -\infty$

3.) $f(x) = \begin{cases} \frac{1}{x^2}, & x<-1 \\ 2, & -1\le x<1 \\ 3, & x=1 \\ x+1, & 1<x\le 2 \\ -\frac{1}{(x-2)^2}, & x>2 \end{cases}$

a) $\lim_{x\to -1^+} f(x) = \lim_{x\to -1^+} 2 = 2$

$x>-1$

b) $\lim_{x\to -1^-} f(x) = \lim_{x\to -1^-} \frac{1}{x^2} = \frac{1}{(-1)^2} = 1$

$(x<-1)$

c) $\lim_{x\to -1} f(x)$ does not exist $\quad \frac{\lim_{x\to -1^+} f \neq \lim_{x\to -1^-} f}{2 \neq 1}$

) $\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} (x+1) = 2$

$x>1$

) $\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} 2 = 2$

$x<1$

f) $\lim_{x\to 1} f(x) = 2 = \lim_{x\to 1^+} f(x) = \lim_{x\to 1^-} f(x)$

but at $x=1$, it is discont... because

$f(x=1) = 3 \neq \lim_{x\to 1^+} f(x) = 2$

4) Find the slope of $y = 5 - 6x - 2x^3$ when $x=2$.

2

$$\frac{dy}{dx} = y' = (5 - 6x - 2x^3)'$$
$$= -6 - 6x^2$$

$$m = \left.\frac{dy}{dx}\right|_{x=2} = -6 - 6(2)^2 = -30$$

5) Find the eqn of the tangent line to the curve $y = \frac{2x+3}{x^2}$ at the point $(1,5)$

$$y' = \frac{(2x+3)'x^2 - (x^2)'(2x+3)}{(x^2)^2}$$

$$= \frac{2x^2 - 2x(2x+3)}{x^4} = \frac{2x^2 - 4x^2 - 6x}{x^4}$$

$$= \frac{-2x^2 - 6x}{x^4} = -\frac{2x(x-3)}{x^4} = -\frac{2(x-3)}{x^3}$$

The slope at $(1,5)$ is

$$m = y'(x=1) = -\frac{2(1-3)}{1^3} = 4$$

Tangent line with the slope $m=4$ at the point $(1,5)$

$$y - y_1 = m(x - x_1)$$
$$y - 5 = 4(x-1) \Rightarrow \boxed{y = 4x + 1}$$

6) a) $\frac{d}{dx}\left[(x^3-1)(x^3+1)\right] = (x^3-1)'(x^3+1)+(x^3-1)(x^3+1)'$

$= 3x^2(x^3+1) + 3x^2(x^3-1)$

$= 3x^2(\underbrace{x^3\cancel{+1}+x^3\cancel{-1}}_{2x^3}) = \underline{6x^5}$

b) $\frac{d}{dx}\left[\frac{x^3-1}{x^3+1}\right] = \frac{(x^3-1)'(x^3+1)-(x^3+1)'(x^3-1)}{(x^3+1)^2}$

$= \frac{3x^2(x^3+1)-3x^2(x^3-1)}{(x^3+1)^2}$

$= \frac{3x^2(\cancel{x^3}+1-\cancel{x^3}+1)}{(x^3+1)^2} = \frac{6x^2}{(x^3+1)^2}$

c) $\frac{d}{dx}(5x^2+2-\sqrt{x+4})$

$= 10x - \frac{1}{2}(x+4)^{\frac{1}{2}-1}$

$= 10x - \frac{1}{2}(x+4)^{-1/2} = 10x - \frac{1}{2\sqrt{x+4}}$

d) $\frac{d}{dx}\left(\frac{5x^4-7x^2}{2x+5}\right) = \frac{(20x^3-14x)(2x+5)-2(5x^4-7x^2)}{2x+5}$

e) $\frac{d}{dx}\left(\frac{e}{x+1}\right) = e\frac{d}{dx}(x+1)^{-1} = e(-1)(x+1)^{-2} = -\frac{e}{(x+1)^2}$

f)

7) $p = 200 - \sqrt{3q^2+10}$

a) $\frac{dp}{dq} = -\frac{1}{dq}(3q^2+10)^{1/2} = -\frac{1}{2}(3q^2+10)^{-\frac{1}{2}}(6q)$

$= -\frac{3q}{(3q^2+10)^{1/2}}$

) revenue function $= p \cdot q = (200q - \sqrt{3q^2+10})\,q$

marginal revenue

$= \frac{dr}{dq} = 200 - \left(\frac{d}{dq}\sqrt{3q^2+10}\right)q - \sqrt{3q^2+10}$

$= 200 - \frac{3q^2}{(3q^2+10)^{1/2}} - \sqrt{3q^2+10}$

8) $c = 5500 + 12q + 0.2q^2$ $\qquad q = 900 - 1.5p$

$\frac{dq}{dp} = -1.5$

$\frac{dc}{dp} = \frac{dc}{dq}\frac{dq}{dp} = (12 + 0.4q)(-1.5)$

$\boxed{\frac{dc}{dq} = -18 - 0.6q}$