PROBLEM SET 4 MATH 171 SPRING 2010

1. Differentiate the functions. If possible, first use properties of logarithms to simplify the given function.

   a) $y=\ln(3x^2+2x+1)$ b) $y=x^2\ln x$ c) $y=x^2\log_2 x$ d) $y=\dfrac{x^2}{\ln x}$

2. A total-cost function is given by $c=25\ln(q+1)+12$. Find the marginal cost when q=6.
3. Differentiate the functions:

   a) $y=x^2e^{-x}$ b) $y=\dfrac{e^x-e^{-x}}{e^x+e^{-x}}$ c) $y=2^x x^2$

4. If $f(x)=ee^xe^{x^2}$, find $f'(-1)$
5. Find dy/dx by implicit differentiation

   a) $xy=4$ b) $2x^3+3xy+y^3=0$ c) $\ln(xy)+x=4$

6. Find an equation of the tangent line to the curve of $x^3+y^2=3$ at the point (-1, 2).
7. Find y' by using logarithmic differentiation.

   a) $y=(3x^3-1)^2(2x+5)^3$ b) $y=(x+2)\sqrt{x^2+9}\,\sqrt[3]{6x+1}$

   c) $y=\sqrt[3]{\dfrac{6(x^3+1)^2}{x^6e^{-4x}}}$ d) $y=4e^x x^{3x}$ e) $y=(\ln x)^{e^x}$

8. Differentiate a) $y=x^{x^3}$ b) $y=(x+1)^{x+1}$
9. Find dy/dx a) $\ln(xy^2)=xy$ b) $(\ln y)e^{y\ln x}=e^2$
10. If y is defined implicitly by $e^y=(y+1)e^x$, determine dy/dx as explicit functions of y only.
11. Determine when the function is increasing or decreasing, and determine when relative maxima and minima occur. Do not sketch the graph.

    a) $y=-x^5-5x^4+200$ b) $y=\dfrac{x^2-3}{x+2}$ c) $y=e^x+e^{-x}$

12. Sketch the graph of a continuous function f such that $f(1)=2$, $f(3)=1$, $f'(1)=f'(3)=0$, $f'(x)>0$ for $x<1$, $f'(x)<0$ for $1<x<3$, and f has a relative minimum when x=3.
13. For a manufacturer's product, the revenue function is given by $r=240q+57q^2-q^3$. Determine the output for maximum revenue.
14. Determine concavity and the x values for the function $y=-\frac{5}{2}x^4-\frac{1}{6}x^3+\frac{1}{2}x^2+\frac{1}{3}x-\frac{2}{5}$ where points of inflection occur. Do not sketch the graph.
15. Determine intervals on which the function $y=3x^4-4x^3+1$ is increasing, decreasing, concave up, and concave down; relative maxima and minima; inflection points; symmetry; and those intercepts that can be obtained conveniently. Then sketch the curve.
16. Sketch the graph of a continuous function f such that $f(4)=4$, $f'(4)=0$, $f'''(x)<0$ for $x<4$, and $f'''(x)>0$ for $x>4$.
17. Sketch the graph of a continuous function f such that $f(1)=1$, $f'(1)=0$, and $f''(x)<0$ for all x.
18. Test for relative maxima and minima for the function $y=x^4-2x^2+4$. Use the second derivative test if possible.
19. Find the horizontal and vertical asymptotes for the graphs of the functions. Do not sketch the graphs.

    a) $y=\dfrac{4}{x-6}+7$ b) $y=\dfrac{2}{9}+\dfrac{3x}{14x^2+x-3}$

20. Determine intervals on which the function is increasing, decreasing, concave up, and concave down; relative maxima and minima; inflection points; symmetry; horizontal and vertical asymptotes; and those intercepts that can be obtained conveniently. Then sketch the curve.

    a) $y=\dfrac{1}{x^2+1}$ b) $y=\dfrac{3x}{(x-2)^2}$

21. Let $f(x)=(x^2+1)e^{-x}$

    a) Determine the values of x at which relative maxima and relative minima, if any, occur. b) Determine the interval(s) on which the graph of f is concave down, and find the coordinates of all points of inflection.

22. Indicate intervals on which the function $y=x^3-12x+20$ is increasing, decreasing, concave up, or concave down; indicate relative maximum points, relative minimum points, points of inflection; horizontal asymptotes, vertical asymptotes, symmetry, and those intercepts that can be obtained conveniently and sketch the graph.